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THE
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AND ASTRONOMICAL PHYSICS

VOLUME XXIX JANUARY 1909 NUMBER 1

NOTES ON THE POSSIBILITY OF FISSION OF A CONTRACTING ROTATING FLUID MASS[1]

By F. R. MOULTON

I. INTRODUCTION

In the speculations on cosmogony there are two fairly definite hypotheses as to the manner in which a single body may give rise to two or more distinct masses without the intervention of external agencies. The first, as outlined by Laplace, is that possibly a rotating fluid may abandon an equatorial ring, which will subsequently be brought by its self-gravitation into an approximately spherical mass. The second, the fission theory, had its rise in Darwin's researches on tidal evolution, and in his speculations on the origin of the moon. It has found extensive application in attempts at explaining the great abundance of binary stars.

The hypothesis of Laplace has the support of no observational evidence, unless we regard the rings of *Saturn* as such, and rests upon no well-elaborated theory. On the contrary there are well-known considerations of the moment of momentum of our system which compel us to reject it as being an unsatisfactory hypothesis for the explanation of the development of the planets. But the fission theory of Darwin, even if the origin of the moon is left aside as being doubtful, has strong claims for attention because of its immediate application to explaining the origin of spectroscopic and visual binaries and certain classes of variable stars. Besides, it is in a general way confirmed by the investigations of Maclaurin, Jacobi, Thomson,

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[1] Abstract of a paper published by the Carnegie Institution.

Poincaré, and Darwin on the figures of equilibrium of rotating homogeneous fluids, and on their stabilities. In particular, considering a series of homogeneous fluid masses of the *same* density but of *different* rates of rotation, it is shown that there is a continuous series of figures of stable equilibrium beginning with the sphere for zero rate of rotation; then, with increasing rotation, passing along a line of oblate spheroids until a certain rate of rotation is reached; then, with decreasing rate of rotation but with increasing moment of momentum, branching to a series of ellipsoids with three unequal axes, and continuing until a certain elongation is reached; and finally, at this point, branching to a series of so-called pear-shaped figures. It has been conjectured that if it were possible to follow the pear-shaped figures sufficiently far, it would be found that they would eventually reach a point where they would separate into two distinct masses. From this line of reasoning it has been regarded as probable that celestial masses, through loss of heat and consequent contraction, do break up in this way often enough to make the process an important one in cosmogony.

Aside from the unanswered question as to what form the pear-shaped figures finally lead, there are two reasons for being cautious in accepting the conclusions. One is that the celestial masses are by no means homogeneous. When they have reached, or are in, that condition of steady motion of slow rotation postulated in the investigation, they are undoubtedly always strongly condensed toward their centers. The other, and probably more important, one is that isolated celestial masses do not change their rates of rotation except when they change their densities or distribution of densities. While the theoretical discussions to which reference has been made regard the rate of rotation as the single variable parameter, in the actual case there is a corresponding change in density. The importance of not neglecting the latter is easily seen.

Consider a slowly rotating homogeneous fluid having the form of a nearly spherical oblate spheroid. The eccentricity of a meridian section depends upon the quantity $\frac{\omega^2}{2\pi k^2\sigma}$, where ω is the angular rate of rotation, k^2 the gravitational constant, and σ the density of the mass. The eccentricity of the axial section increases with the increase of this function provided it does not go beyond a certain maximum. Now suppose the mass contracts in such a way as to remain homogeneous throughout, and so that it continues to rotate as a solid spheroid of equilibrium. Because the moment of momentum of an isolated mass is constant, the contraction implies an increase in ω, and therefore, so far as this factor alone is concerned, an increase in the oblateness of the mass. But the contraction also implies an increase in σ, and therefore, so far as this factor alone is concerned, a decrease in the oblateness of the mass. That is, keeping the moment of momentum constant, as the dynamical situation requires, we find the eccentricity acted upon by two opposing factors. If, under the influence of these factors, the figures should become less oblate, the fission theory would get no support from the discussion; if it should get more oblate, the question is at what rate the mass must rotate, and to what extent the contraction must proceed before there is a possibility of fission. This paper is devoted to a brief discussion of these questions.

II. THE ELLIPSOIDAL FIGURES OF EQUILIBRIUM OF ROTATING HOMOGENEOUS FLUIDS

For the applications which follow it will be necessary to review briefly the facts regarding the spheroidal and ellipsoidal figures of equilibrium and their conditions of stability.

Maclaurin[1] has shown that for very small values of $\frac{\omega^2}{2\pi k^2\sigma}$ there are two ellipsoids of revolution which are figures of equilibrium, one of them being nearly spherical, and the other very oblate, the limits for $\omega=0$ being respectively the sphere and infinite plane. For greater values of this quantity, the figure corresponding to the former is more oblate and that corresponding to the latter is less oblate. For $\frac{\omega^2}{2\pi k^2\sigma}=0.22467\ldots$ the two figures are identical. For $\frac{\omega^2}{2\pi k^2\sigma}>0.22467\ldots$ there is no ellipsoid of revolution which is a figure of equilibrium.

Jacobi has shown[2] that if $\frac{\omega^2}{2\pi k^2\sigma}<0.18709\ldots$ there is an ellipsoid of three unequal axes satisfying the conditions for equilibrium. When this quantity is very small the axis of rotation and one other are very small and nearly equal to each other, while the third is relatively very long. With greater values of this quantity the shorter axes are longer, and the longest axis is shorter. For $\frac{\omega^2}{2\pi k^2\sigma}=0.18709\ldots$ the figure becomes an ellipsoid of revolution and is identical with the more nearly spherical Maclaurin spheroid. For $\frac{\omega^2}{2\pi k^2\sigma}>0.18709\ldots$ the Jacobian ellipsoids of equilibrium do not exist.

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[1] *Treatise on Fluxions*, Edinburgh, 1742.

[2] Letter to French Academy, 1834.

In the case of the Maclaurin spheroids the relation between $\frac{\omega^2}{2\pi k^2\sigma}$ and the eccentricity, e, of an axial section is given by the well-known equation[1]

$$\frac{\omega^2}{2\pi k^2\sigma}=\frac{1}{\lambda^2}\left\{(3+\lambda^2)\frac{\tan^{-1}\lambda}{\lambda}-3\right\}. \qquad (1)$$

III. POINCARÉ'S THEOREMS RESPECTING FORMS OF BIFURCATION AND EXCHANGE OF STABILITIES

In a memoir[2] remarkable for its powerful methods and important results, Poincaré has proved the existence of an infinite number of other forms of equilibrium. Considering the equations for equilibrium as functions of the parameter ω, he showed that for an infinite number of values of ω other series of forms branch both from the Maclaurin spheroids and from the Jacobian ellipsoids. A form at which two series cross he called a *form of bifurcation*, and a form at which two series unite and disappear, a *limit form*. Starting with the nearly spherical oblate spheroid for small values of ω and following this series of forms for increasing ω, he found that the first form of bifurcation is that for which the Jacobian ellipsoids branch. Following along the Jacobian ellipsoids, he found that the first form of bifurcation is that for which the so-called pear-shaped figures branch.

Poincaré proved that if two real series of figures of equilibrium, A and B, cross, and if before crossing A is stable and B unstable, then after crossing A is unstable and B has at least one degree less of instability. Starting with the nearly spherical oblate spheroids, he proved that they are absolutely stable, and that they remain absolutely stable until the Jacobian ellipsoids branch. At this point they become unstable and the branching Jacobian ellipsoids are stable. These latter figures remain stable until the pear-shaped figures branch, where they lose their stability. Darwin concluded from a computation of the first terms of an infinite series[1] that the pear-shaped figure is stable. On the other hand Liapounoff asserts[2] it is unstable. If it is stable, apparently the mass proceeds at least somewhat farther toward fission into two bodies; if it is unstable, apparently this sort of fission is impossible, for when instability occurs the mass will undergo great changes of form and possibly lose its continuity if subject to the slightest disturbance. The conclusion that stability fails before fission is attained is strongly supported by Darwin's investigations on the stability of a liquid satellite,[3] in which he found that a satellite loses its stability before it can be brought near enough to its primary to coalesce with it.

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[1] Tisserand, *Mécanique céleste*, 2, chap. vi.

[2] "Sur l'équilibre d'une masse fluide animée d'un mouvement de rotation," *Acta Mathematica*, 7, 259–380, 1885.

This lengthy review of the case of homogeneous fluids has been given to show (1) that there is no possibility of fission before the pear-shaped figure is reached; (2) that in this simplest of all cases there is not only no proof that fission into two stable masses can ever take place, but that the results reached up to the present point strongly to the opposite conclusion; and (3) that it may be a guide in the more complicated case of heterogeneous compressible masses.

IV. FIGURES OF EQUILIBRIUM OF ROTATING HETEROGENEOUS FLUIDS

We now enter a field beset with formidable difficulties and in which there are but few positive results. We shall consider only those forms of equilibrium which are roughly spherical in form and which have central condensation. The theory of Clairaut[4] shows that for a given rate of rotation and mean density such bodies are more nearly spherical than they would be if they were homogeneous.

[1] "The Stability of the Pear-shaped Figure of Equilibrium of a Rotating Mass of Liquid," *Phil. Trans.*, A, 200, 251–314, 1903.

[2] *Acad. Imp. des Sci. de St. Petersbourg*, 17, No. 3, 1905.

[3] *Phil. Trans.*, A, 206, 161–248, 1906.

[4] See Tisserand, *Mécanique céleste*, 2, chap. xiii.

The existence of the sun and planets proves their stability, and observations show that they are actually considerably less oblate than the corresponding homogeneous masses would be with the same rates of rotation. The following table gives for the earth, *Jupiter*, and *Saturn* the computed eccentricities of axial sections found from equation (1) and the observed.

	Eccentricity If Homogeneous	Observed Eccentricity
Earth..............	0.096	0.083
Jupiter.............	.461	.347
Saturn..............	.607	.409

With changing rates of rotation the heterogeneous masses go through series of forms of equilibrium analogous to these for homogeneous bodies, and Poincaré's theorems regarding forms of bifurcation and exchange of stabilities still hold.

It is very probable that the central condensation and resulting nearer approach to sphericity tends toward stability. But the actual celestial bodies are composed of compressible fluids, and as Jeans has shown,[1] at least in certain cases, the compressibility tends toward instability. Thus, as compared with the ideal homogeneous masses, the stabilities of the actual masses are subject to two opposing factors. There are at present no quantitative results by means of which we may exactly measure their relative effects; but it is certain that there is no danger of fission until the compressible masses have become more oblate than those we actually find in our system. For purposes of safety, when we are comparing heterogeneous masses with homogeneous ones, we shall conclude there is possibility of fission in the former only when the latter are very far removed from all possibility of fission. Even if some danger as to the results should still remain, we are in the realm of practical certainty when we compare hypothetical masses with the actual forms given by observations.

V. THE CONDITIONS OF EQUILIBRIUM FOR CONSTANT MOMENT OF MOMENTUM

In the mathematical treatments of figures of equilibrium the rate of rotation, ω, is supposed to vary while all the other factors remain the same. In the actual physical problem as the body condenses the density, rate of rotation, and shape must always be related so that the moment of momentum shall remain constant. We shall impose this condition. Let m represent the mass of the body in question, a its axial radius, σ its density, ω its rate of rotation, $e=\frac{\lambda}{\sqrt{1+\lambda^2}}$ the eccentricity of an axial section, and M its moment of momentum. Then we have

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[1] *Phil. Trans.*, A, 201, 157–184, 1903.

$$\left.\begin{aligned} m&=\tfrac{4}{3}\pi a^3(1+\lambda^2)\sigma\,, \\ M&=\tfrac{2}{5}ma^2(1+\lambda^2)\omega\,. \end{aligned}\right\} \qquad (2)$$

Eliminating a and ω from (1) by means of these equations we have

$$\frac{25\left(\frac{4\pi}{3}\right)^{\frac{1}{3}}M^2}{6k^2m^{\frac{10}{3}}}\sigma^{\frac{1}{3}}=\frac{(1+\lambda^2)^{\frac{2}{3}}}{\lambda^2}\left\{(3+\lambda^2)\frac{\tan^{-1}\lambda}{\lambda}-3\right\}. \qquad (3)$$

When the figure of equilibrium is on the oblate spheroid series its density and oblateness must always satisfy this relation. It is easily shown that when the mass and moment of momentum are given, there is but one value of the density possible for a given oblateness, and but one oblateness possible for a given density. In particular, in any special case, equation (3) enables us to compute the density for which the Jacobian ellipsoids branch. In the homogeneous bodies there is no possibility of fission until long after these figures branch. At the time of branching the axial section of the spheroid has an eccentricity of 0.813; and when the Jacobian ellipsoid branches into the pear-shaped figure the eccentricity of the section through its longest axis and axis of rotation is 0.94, while the axial section at right angles to this is nearly circular. Because of its great elongation the figure which branches from it might better at first be called cucumber-shaped rather than pear-shaped.

In applying (3) we shall assume generally that an actual celestial body will not suffer fission or become unstable until the corresponding homogeneous body, i. e., a body having the same mass and moment of momentum, shall at least have reached the stage where the Jacobian ellipsoid branches. Though this is a very conservative assumption, we shall in some instances use the present oblateness of *Saturn* as the limit within which fission is certainly impossible and stability assured.

VI. APPLICATIONS TO THE SOLAR SYSTEM

Let us first consider the sun and find whether, under cooling and contraction, it can ever split into a binary star. Equation (3) shows that the greater M the smaller the density for which fission will occur. Consequently we shall favor the conclusion of ultimate fission if we introduce simplifying approximations so as to get the computed value of M larger than its true value. The moment of momentum is most easily computed if we assume that the sun is homogeneous, and since it is undoubtedly densest in the central part this assumption will give us a value of M in excess of its true value.

Taking the mean solar day, the mean distance from the earth to the sun, and the mass of the sun as the units of time, length, and mass respectively, we find for the density of water $\sigma = 1{,}567{,}500$, and that for the sun $M = 216 \times 10^{-8}$. The ellipsoids branch for $\lambda = 1.395$. Applying equation (3), we find that the sun cannot arrive at the stage where the Jacobian ellipsoids branch in the corresponding homogeneous mass until its density shall become 307×10^{11} on the water standard. This corresponds to an equatorial diameter of 22 miles (35 km). This inconceivable density, which is thousands of millions of times greater than matter is supposed ever to attain, means that the sun will never arrive at this stage. By a similar computation we find that the sun will not become so oblate as *Saturn* is now until its density shall become 148×10^{10} on the water standard. Its corresponding equatorial diameter is 75 miles (120 km). This means that the sun will always be less oblate and more stable than *Saturn* is now, and that it will never give rise to a binary star by fission. All of this is, of course, under the hypothesis that the sun never will come under the influence of powerful external disturbances, such as would operate upon it if it should pass near another star.

Apparently *Saturn* is more apt to suffer fission than any other member of our system. Taking its mass and period of rotation as $\frac{1}{3502}$ and 10.25 hours, respectively, we find by a similar computation that *Saturn* will not pass beyond limits of safety defined above until its density shall become 21 times that of water. For this figure its polar and equatorial diameters are 16,500 and 28,400 miles (26,600 and 45,700 km), respectively. The high mean density demanded, 28 times that at present, seems to be fatal to the hypothesis that *Saturn* will ever suffer fission. We find that the density will be 93 times that of water when the pear-shaped figures branch.

From this we see that no member of the solar system will ever suffer fission in the future. But the planets may have originated by fission from a parent nebula, or the moon from the earth in an early state. A quantitative examination of the formulas shows that the latter is the more favorable to the conclusion of fission. Hence we shall examine it.

We observe first that the moment of momentum of the earth moon system has remained constant except for outside influences. There is none readily assignable which could have increased it. Among those which may have decreased it, apparently solar tidal friction is the only one which can have produced sensible results. In a paper now being published by the Carnegie Institution the writer has computed the maximum effects the sun can have had in this direction in any time however long. Using these results, making simplifying approximations so as to get too large a value for M, then adding 25 per cent. of the computed value as insurance against the possible effects of unknown disturbing forces which may have been acting on it, it is found that in the units defined above $M=10^{-13}$. In the same units the mass of the earth-moon system is 3×10^{-6}. With these values we find from equation (3) that this hypothetical earth-moon mass would not pass the limits within which there can be no fission until its mean density had become 215 on the water standard, or 40 times the present mean density of the system. It would not even become so oblate as *Saturn* is now until its density had become 10.4 times that of water. Consequently we conclude that the moon and earth have not originated by fission from a common parent mass. In the paper just referred to, the same conclusion was reached by following backward in time the effects of the mutual tides of the system.

VII. APPLICATIONS TO BINARY STARS

Binary stars are relatively so numerous that their evolution is obviously a standard celestial process. We are raising the question whether the process is that of fission. Unfortunately the data at our disposal in treating these systems are so incomplete that our conclusions are not as final as we should desire.

Before taking up explicitly the question of fission, we shall write down some relations which must be satisfied. Consider a spectroscopic binary in which the lines of both stars are visible. For present purposes it will be sufficiently accurate to assume that the orbits are circular. Let i be the complement of the inclination of the plane of the orbit to the line of sight. Let v be the maximum observed relative radial velocity expressed in kilometers per second, and P the period expressed in mean solar days. Then the sum of the masses expressed in terms of the sun's mass must satisfy the relation

$$m_1+m_2=\frac{178Pv^3}{10^8\cos^3 i}\geqq\frac{178}{10^8}Pv^3. \tag{4}$$

Suppose the spectrum of m_1 alone is visible and let v_1 be the maximum observed radial velocity with respect to the center of gravity of the system expressed in kilometers per second. Let $\mu=\frac{m_1}{m_2}$, the ratio of the mass of the bright star to that of the dark. Then

$$m_1+m_2=\frac{178Pv_1^3}{10^8\cos^3 i}(1+\mu)^3\geqq\frac{178}{10^8}Pv_1^3(1+\mu)^3\ . \tag{5}$$

Suppose the two stars have the same density σ. Let a represent the distance between their centers and κa the distance between their surfaces. When P is expressed in mean solar days we have in terms of the density of water

$$\sigma=\frac{2}{100(1-\kappa)^3P^2}\quad\frac{(1+\mu^{\frac{1}{3}})^3}{(1+\mu)}. \tag{6}$$

Or, since this is a reciprocal equation in μ whose right member has a maximum for $\mu=1$,

$$\frac{8}{100(1-\kappa)^3P^2}\geqq\sigma\geqq\frac{2}{100(1-\kappa)^3P^2}\ . \tag{7}$$

When $\kappa=0$ the bodies are in contact and this is probably an unstable condition. From Darwin's work on the homogeneous satellite (*loc. cit.*) it seems probable that the smallest value κ can have is about $\frac{1}{3}$. Taking this limit we have

$$\sigma\geqq\frac{27}{400P^2}\ . \tag{8}$$

This formula can lead to little definite information except in the case of very short-period binaries. For example, in the case of *β Cephei*, whose period is 4.57 hours,[1] we find from this relation that $\sigma \geqq 1.86$ on the water standard.

Now let us return to the question of fission of binary stars. Denoting the periods of rotation of m_1 and m_2 by D_1 and D_2, their radii by a_1 and a_2, and assuming that they rotate in the direction of their revolution, the moment of momentum of the system is

$$M=\frac{m_1 m_2 k^{\frac{4}{3}} P^{\frac{1}{3}}}{(2\pi)^{\frac{1}{3}}(m_1+m_2)^{\frac{1}{3}}}+\frac{4\pi m_1 a_1{}^2}{5D_1}+\frac{4\pi m_2 a_2{}^2}{5D_2}\,. \tag{9}$$

The assumption of forward rotation of both masses made in writing this equation is allowable in testing the fission theory, for rotation in this direction would be a consequence of fission. Consequently

$$M>\frac{m_1 m_2 k^{\frac{4}{3}} P^{\frac{1}{3}}}{(2\pi)^{\frac{1}{3}}(m_1+m_2)^{\frac{1}{3}}}\,. \tag{10}$$

The more nearly equal the two masses are, the more nearly is this inequality an equality for given values of D_1 and D_2.

Let $\dfrac{m_1}{m_2}=\mu$. Then (10) and (3) give

$$\sigma^{\frac{1}{3}}<\frac{6(3\pi)^{\frac{1}{3}}(1+\mu)^4}{25k^{\frac{2}{3}}\mu^2P^{\frac{2}{3}}}\quad\frac{(1+\lambda^2)^{\frac{2}{3}}}{\lambda^2}\left\{(3+\lambda^2\frac{\tan^{-1}\lambda}{\lambda}-3\right\}. \tag{11}$$

Let us suppose that the two stars are equal, i.e., that $\mu=1$, as is certainly approximately true in many binary systems. Let us assume that fission will occur at those figures for which the corresponding homogeneous masses branch into the Jacobian ellipsoids. Then the density at the time of fission on the water standard must satisfy the relation

$$\sigma<\frac{0.016}{P^2}, \tag{12}$$

where P must be expressed in mean solar days. If fission should occur only when the pear-shaped figures begin in the corresponding homogeneous masses, we should have instead of (12)

$$\sigma<\frac{0.071}{P^2}. \tag{13}$$

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[1] Frost, *Astrophysical Journal*, **24**, 259–262, 1906.

It is obvious from these relations that in all visual binaries in which the periods are at least several years the fission must have taken place, if at all, while the star was yet in the nebulous stage. For example, the period of *a Centauri* is about 81 years, and formula (12) gives for the density of this star, on the water standard, at the time of possible fission $\sigma < 2 \times 10^{-11}$. These results indicate only the order of magnitude. It is significant that they agree with those found in the paper on tides mentioned above. In that paper it was shown that in the case of a binary of two equal masses originating by fission it is not possible for tides to more than double their initial distance, and hence that in such binary systems the initial distances of the two components cannot have been less than half their present distances.

VIII. CONCLUSIONS

Notwithstanding the lack of numerical precision because of the great difficulties of the actual problem in nature, we are justified in concluding that there has been in the solar system no example of the type of fission under consideration here, and that there will be none in the future. In the case of the binary systems the only conclusion reached is that in systems made up of approximately equal pairs and in which the periods are at least several years, fission has taken place, if at all, while the mass was yet in the nebulous state. It is not intended to affirm that any evidence has been brought forward that fission ever occurs. It seems to the writer that in masses with densities decreasing outward from their centers, such as those treated here, fission into two stable masses is extremely improbable. For, suppose a nebula of this type suffers fission. At the moment of separation each of the two parts will be rotating with the same angular speed that the united mass had just previous to the splitting. One of these two parts will have a density not greater than that of the original mass and, therefore, will have at least as great a tendency to undergo fission again, unless either it is in a more stable form or the tides of the other mass tend to keep it from breaking up. If the approximate spheroid is the most stable figure of equilibrium, as seems probable, and if the mass under consideration has suffered fission by evolution along this line of figures, as is assumed, then the former alternative is eliminated. It does not seem reasonable that tidal forces due to an exterior body could tend to stability, and Darwin's computations confirm this intuition. Besides all this, the binary stars are now bodies with considerable densities. This implies enormous contraction from the nebulous state which, as we have seen, tends still more toward fission. Consequently, it seems that if a nebulous mass with central condensation such as we are considering once suffered fission, it would break up into many fragments. If this view is sound the binaries cannot have originated in this way.

We may still retain the hypothesis that binaries and multiple stars of several members have developed from nebulae, but apparently they must have had originally well-defined nuclei. The photographs of many nebulae support this hypothesis. But we observe that if we are forced to this position we do not explain anything—we only push by an assumption the problem of explaining the binary systems a little farther back into the unknown.

In terminating these remarks it should be noted that in accordance with present-day thought it has silently been assumed throughout that the celestial evolution is from the nebulous state to the stars. This mental attitude has been brought about by a century of unquestioning faith in the Laplacian theory. There are now abundant reasons for rejecting the Laplacian hypothesis as applied to the solar system, and it may be the time is ripe for a serious attempt to see if the opposite hypothesis of the disintegration of matter because of enormous subatomic energies, which perhaps are released in the extremes of temperature and pressure existing in the interior of suns, and of its dispersion in space along coronal streamers or otherwise, cannot be made to satisfy equally well all known phenomena. The existence of such a definitely formulated alternative hypothesis would have a very salutary effect on the interpretation of the results of astronomical observations. We should then more readily reach what is probably a more nearly correct conclusion, viz., that both aggregation and dispersion of matter under certain conditions are important modes of evolution, and that possibly together they lead in some way to approximate cycles of an extent in time and space so far not contemplated.

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VERTICAL TEMPERATURE-GRADIENTS OF THE ATMOSPHERE, ESPECIALLY IN THE REGION OF THE UPPER INVERSION

By W. J. HUMPHREYS

GENERAL STATEMENT

The vertical temperature-gradients of the atmosphere, the condition and distribution of its moisture, the direction and speed of the winds, and other upper-air phenomena have long been objects of investigation. Meteorological stations located on hills and on plains, on mountain tops and in valleys, especially when near together, have furnished and continue to furnish much information both valuable and interesting. And for many years this has been supplemented in part by studies of the heights, of the forms, and of the velocities of clouds, and to some extent by records obtained in manned balloons, and still further by the aid of registering instruments sent up in kites. But all the information obtained in these several ways, though of the greatest value to the science of meteorology and to the art of forecasting, has thus far been limited by the upper level of the cirrus clouds. Within this region, however, observations soon made it certain that through a considerable distance next beyond three or four thousand meters above the surface of the earth, changes with altitude are more or less uniform; that in general the moisture constantly decreases; that the winds increase in velocity; and that the temperature fall approaches the adiabatic rate.

The conditions at higher altitudes could only be surmised, but many, by the uncertain process of extrapolation, assumed that the temperature must continue to decrease at about this same rate far out and until it was as low as adiabatic expansion could make it; that the moisture content must gradually decrease to zero; and that the winds in middle latitudes must remain easterly with speeds increasing with elevation. However, the splendid work with sounding balloons by Teisserenc de Bort,[1] by Assmann,[2] by Hoormann,[3] by Rotch,[1] by Peteval,[2] and by many others has shown these assumptions to be radically wrong.

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[1] *Comptes Rendus*, **135**, 897, 1902; **138**, 42, 1904; **145**, 149, 1908; *Quarterly Journal Royal Meteorological Society*, July 1908, p. 189; also many numbers of *Veröffentlichungen*.

[2] *Veröffentlichungen der Internat. Kom. für Wiss. Luftschiffahrt.*

[3] *Ciel et Terre*, 1907–1908; and *Veröffentlichungen*.

BALLOON RECORDS

During the past ten years hundreds of sounding balloons, equipped with suitable registering apparatus, have been sent up under widely different conditions to various heights; many to fifteen kilometers, some to twenty kilometers, and a few to even higher levels. The records represent flights by day and by night made in all sorts of weather and in every season of the year; flights over continental, island, and ocean regions; and flights at different latitudes from the tropics to beyond the arctic circle; and besides, the apparatus used has been by several makers and of different design. In this way disturbances due to location and to storm influence have been detected and systematic instrumental errors largely avoided. But the results of all the observations are in general accord, and show that the explored portion of the atmosphere consists of three more or less distinct regions.

I. The region of terrestrial disturbance, extending from the ground to an elevation of, roughly, 3000 meters above its surface, in which the temperature-gradient usually is very irregular and often locally reversed. This is also the principal region of cloud formation and of precipitation.

II. The region of uniform changes, lying, roughly, between the upper limits of I and the 10,000-meter level above the sea, in which the temperature-gradient, nearly constant, approaches the abiabatic. This region is comparatively free from condensation and precipitation, since it lies above the average nimbus cloud while the cirri float in only its topmost portion. At times it is the seat of vertical convections of varying extents and intensities, but its normal condition is one of stability as shown by its usual freedom from clouds, and as necessitated by the negative departure of its temperature-gradient from the adiabatic.

III. The region of permanent inversion, or all that explored portion above the 10,000-meter level or thereabouts. Here the temperature-gradient is small and usually positive, and vertical convection therefore is always impossible.

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[1] *Nature*, 78, 7, 1908.

[2] *Ibid.*, p. 56.

In the region of terrestrial disturbance, that is, in the lower or denser portions of the atmosphere, the winds are irregular, and commonly differ in both speed and direction above and below any, other than a mere surface, inversion. In all essentials they are chiefly determined by the locations and intensities of barometric highs and lows. The highs and the lows continue to play an important part, even far up in the region of uniform changes, but are less and less effective as the elevation is increased. In the upper portion of this region the great planetary circulation mainly prevails as shown by the movements of the cirrus clouds which are near its upper limit. It is known, too, from the drifting of balloons, and from observation on clouds, that in this region the wind moves faster and faster with increase in elevation, and easterly in extratropical countries. Above the inversion level the velocities of the winds are much less[1] than they are immediately below it. On passing up from the one region into the other this velocity usually is found to drop by from 25 to 80 per cent., and apparently the directions in the two places have but little interdependence. Besides, the air above the place of inversion seems always to be excessively dry[2] no matter what the humidity may be at lower levels.

The height, generally sharply defined, of the place where the upper inversion begins, like its temperature, is a function of season, of latitude, of barometric pressure, and probably of still other conditions. It is high up in a well-formed "high" and correspondingly low in a "low." It is at greater elevations and higher temperatures in summer than in the winter, and descends with increase of latitude from the tropic to the polar regions. The higher it is found above the earth, season and other things being equal, the lower in general is its temperature, and conversely the lower it lies, under similar conditions, the warmer it is.

We therefore have two distinct atmospheres that intermingle but slightly: a lower or inner turbulent one with a large negative temperature-gradient, and an upper or outer one with a small positive gradient, floating on the first like oil on water. The lower contains from two-thirds to three-fourths of the entire mass of such gases of the air as oxygen, nitrogen, and all the members, except helium, presumably, of the argon family; a still greater proportion of the carbon dioxide, and nearly all the water vapor. It is warm in its lowest portion, but cools, irregularly at first then rapidly and nearly regularly, with increasing elevation to a minimum of from −40° C. to −70° C. The upper atmosphere contains extremely little water vapor, and its temperature rises sometimes abruptly at the start and then slowly, but usually slowly all the way with elevation from the place of inversion to an unknown temperature at a height not yet reached. Occasional observations indicate an isothermal condition, and for this reason the outer atmosphere is often called the "isothermal layer," and a few observations have shown even a slow cooling with elevation, but both these conditions are unusual.

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[1] Bassus, *Beiträge zur Physik der freien Atmosphäre*, **2**, 92, 1908.

[2] Kleinschmidt, *ibid.*, **2**, 205, 1908.

The region above the upper inversion cannot appreciably be penetrated by convection currents from the air below, since this would cause a cooling by expansion of the rising mass to a temperature lower than that of the surrounding medium, so that under the same pressure or at the same level its density would be greater than that of the adjacent air; and therefore all storms, all condensation, all abnormal and irregular moisture distribution, and virtually all dust, except that of meteors and of violent volcanic eruptions, are limited to the lower atmosphere. The upper atmosphere, floating on the lower, will rise and fall as a whole with the latter, but with more or less mixing of the contiguous portions of the two. Upward currents of air, when they reach the inversion surface, are forced to spread out under it somewhat as warm air in a room spreads out next the ceiling, since, as explained, they cannot rise any farther; and doubtless this air carries with it the moisture for the highest cirrus clouds, because, like the inversion layer, they approach nearer the earth in colder regions, nearer in winter than in summer, and are highest toward the end of a clear warm spell of weather.

Much of the above is well illustrated by the temperature-gradient curves of Fig. 1, which is copied, with slight additions, i. e., curves 7 to 10, inclusive, from the *Annuaire météorolgique pour 1908* (Obser vatoire Royal de Belgique).

The flights giving these curves were made from Uccle near Brussels, only 100 meters above sea-level, and with a number of others made at other times (for these include all that were made within the dates given) have been reported in detail by Hoormann in *Ciel et Terre*.

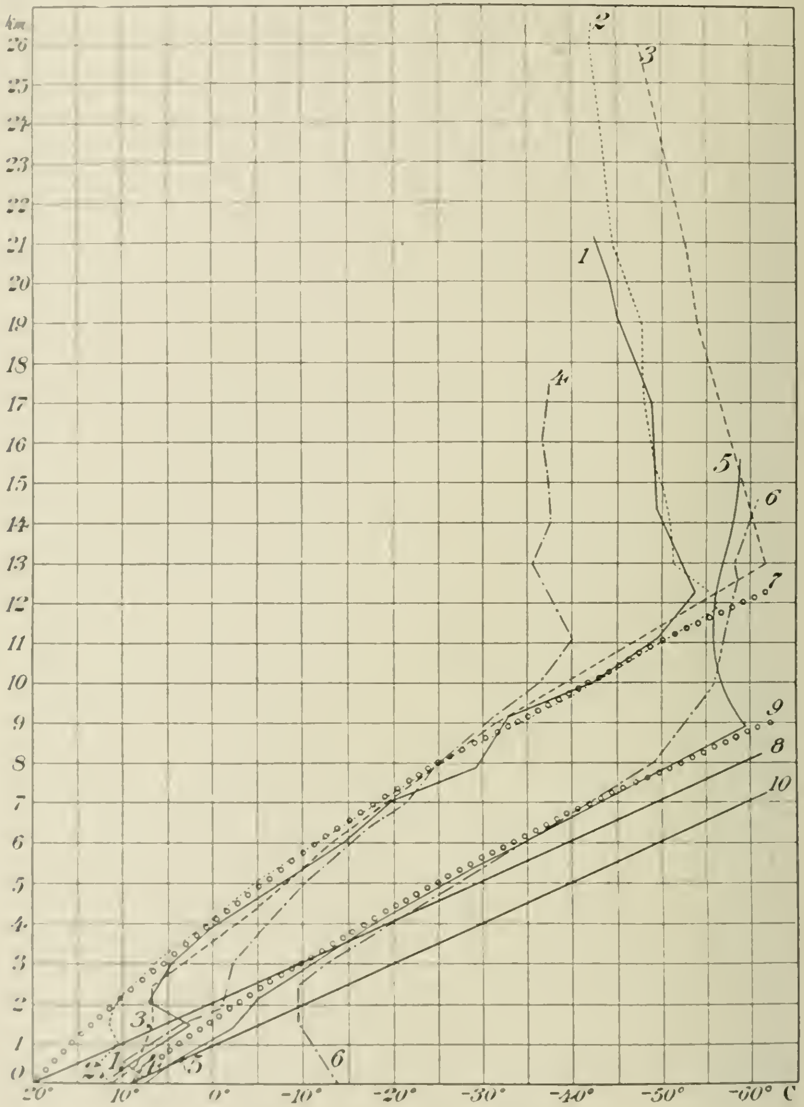

FIG. 1.—Temperature-Gradients.

The following table contains some of the more important data in connection with these ascensions. The temperature-gradient is for the region of uniform changes. The flight of July 25 lasted two and a half hours; the others about two hours each.

No. of Curve	Date	Hour of Starting	Condition of Sky	Barometer at Station	Weather	$\frac{\Delta t}{}$ from Region of Uniform Change
1...	July 24, 1907	7 A. M.	Covered; strato- and fracto-cum.	751.5	Neutral	0.68 C.
2...	July 25, 1907	7 A. M.	4 tenths cirrus	751.0	Neutral	0.70
3..	Sept. 5, 1907	7 A. M.	Covered; alto-strat. fracto-cum.	754.0	Neutral	0.66
4..	Oct. 3, 1907	7 A. M.	Covered; stratus	744.0	Cyclonic	0.50
5...	Dec. 12, 1907	after sundown	Covered (?)	740.0	Cyclonic	0.63
6...	Jan. 3, 1908	7 A. M.	Serene	760.0	Anti-cyclonic	0.71

Curve 8, a straight line, gives the adiabatic gradient for dry air, and curve 7 gives the temperature-gradient for saturated air, both starting at sea-level with a temperature of 20° C. Curves 9 and 10 are the same as 7 and 8, respectively, except that the initial temperature is assumed to be 9° C. Curves 1 to 3, inclusive, give a summer group, while 5 and 6 represent winter gradients. It will be noticed at once, and this is generally true, that the winter curves have a slope intermediate between the dry adiabatic and those of summer, and that the temperature-gradient is distinctly less in summer than it is in winter. It will also be noticed that the upper inversion is located higher in summer than in winter; that, of the summer gradients, No. 3 had the highest barometer and inverted at the greatest altitude, and that a cyclonic condition, or low barometer, lowers the place of inversion; further, that the higher the inversion, at any season, the less its temperature; and conversely. The place of inversion usually, and in all these curves except No. 6, is rather sharply defined.

THEORY

Irregularities in the temperature-gradient of the lower atmosphere.—The thermal irregularities and local inversions of the lower atmosphere are caused by wide temperature changes in the surface of the earth, by the over-running and by the under-running of warm and of cold masses of air, and by the action of cloud layers in changing the position of insolation from the earth and from the denser portions of the atmosphere to higher levels.

Morning inversions.—The inversion so commonly found in the early morning is due to the fact that the surface of the earth radiates more freely than does the atmosphere at the same temperature, and that therefore at night, especially when the sky is at all clear, it cools correspondingly lower, and also cools the atmosphere for some distance above it to an extent depending largely upon the humidity, degree of cloudiness, and wind velocity.

The more humid the air the less the depth to which earth radiation can directly penetrate, and therefore the more slowly, because of counter radiation, it gets away. Clouds check the loss of heat mainly by reflection; while the winds by mixing the air of different levels prevent excessive cooling at the surface.

The effects of this nocturnal cooling are well shown by the curves, as explained above. Flight No. 5, made just after sundown, of course shows no such disturbance.

Departure of gradient from the dry adiabatic.—Although the atmosphere constantly is in a state of more or less turmoil, its average temperature-gradient, even in the region of uniform changes where this is greatest, is less than the adiabatic of dry air; due partly to the heat it absorbs in the form of earth and of solar radiations—an amount which during the day is in excess of the corresponding loss—but chiefly to the fact that the air is not dry, and that it therefore receives much heat as it rises, from condensation of the moisture it contains. This liberated sensible heat, by all processes of distribution, conduction, convection, and radiation, extends to and warms the colder masses above, and decreases by counter-radiation the loss of heat from lower levels. The temperature-gradients are modified most in the lower atmosphere where the average percentage of saturation is greatest, and where, because of the temperature, its water capacity is large.

Curves 7 and 9 show how closely the actual normal gradients follow saturation paths, even though actual saturation seldom exists except in the midst of clouds, and how widely they depart from lines 8 and 10, that correspond to dry adiabatic expansion.

Seasonal difference in temperature-gradient.—The conditions that determine the departure of ordinary air gradients from those of dry air, insolation, and moisture content, are more pronounced in summer than in winter, and therefore summer gradients should differ from winter ones in being still farther removed, as they actually are, from the adiabatic curve of dry air.

Storm gradients.—A summer cyclone furnishes the extreme of departure from dry-air conditions, and therefore also yields the extreme gradient departure from the dry adiabatic, as illustrated by curve 4. It even sets free much heat in the rainy sections by condensation of moisture drawn in by the winds from distant regions. Two centimeters of rain, for instance, will free as much heat as the surface of the earth would receive in sixteen clear summer noon hours of sunshine; and practically all this heat goes to the higher atmosphere, as the lower levels and the ground are not warmed by the rainfall, but rather cooled by its evaporation, and because of the lower temperature it had when it left the clouds, and often by the under-running of masses of cold air.

The upper and the lower parts of curve 4 illustrate these effects.

Cause of the upper inversion.—Several attempts have been made to account for the upper inversion. Probably the earliest is that of Assmann,[1] who suggested that the warm air above the inversion level is a part of the world circulation from the equator to the poles.

Several objections can be offered to this theory; among them the fact that the level of the inversion surface and the temperature of the air above it are functions of the conditions of the lower atmosphere; and that it is not supported by the directions of the warm upper-air currents, which often are from northerly[2] points, and are in fact irregular[3] both as to direction and as to speed.

Nimführ[4] made a slight modification of Assmann's theory when he suggested that the isothermal layer is due to a downward component of the air, together with its movement from the south. But the above objections also apply to this theory.

Fényi[5] makes the plausible suggestion that it is due to the absorption in the outer atmosphere of an unknown amount of ultra-violet light. But observations by Abbot and Fowle[1] indicate that the absorbed ultra-violet portion of the sun's energy is of the order of only 2 per cent., or less, of the total insolation; and therefore even if there was no loss by radiation or otherwise, it would in twelve hours heat the air of the isothermal layer by less than half a degree C.; an amount insufficient to account for the observed temperatures.

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[1] *Sitzungsberichte der kön. preus. Akad. der Wiss.*, 1902, 495.

[2] Hergesell, *Beiträge zur Phys. der freien Atmos.*, 1, 143, 1905.

[3] Bassus, *ibid.*, 2, 92, 1906.

[4] *Meteorologische Zeitschrift*, 23, 245, 1906.

[5] *Ibid.*, 24, 355, 1907.

Trabert[2] holds that possibly the vertical temperature-gradient is less above oceans than above continents, and that therefore the upper inversion may be due to this relatively warm air as it sweeps over the land. But it is not clear how this can account for the increase of temperature beyond the minimum layer, nor is it supported by the irregular directions of the winds in the upper region, and finally the inversion is as definitely found over the oceans themselves as anywhere else.

Very[3] ascribes the isothermal condition to the absorption of lines and bands of water vapor. A fatal objection to this theory, however, if the observations are correct, is the fact that the water vapor is not there to absorb[4]—certainly there can be but little at an elevation of 26 kilometers, at which elevation the temperature is still increasing. Teisserenc de Bort[5] has suggested that the "isothermal layer" is due to the fact that there is in this region but little vertical convection. It seems, however, more probable that the absence of convection is a result rather than the cause of this warm layer.

Gold[6] claims that an explanation of the existence of the "isothermal layer" must take into account the radiation of the atmosphere, and gives conclusions, which he says follow from certain assumptions as to its absorption, so suggestive as to make a fuller paper from him on this subject very desirable.

Assuming balloon observations correctly to represent conditions as they actually exist, it is interesting to find their physical basis, and to the practical meteorologist extremely important to do so, since the chief hope for improvement in forecasting rests upon an increase in our knowledge of the causes and of the effects of atmospheric phenomena. And since these involve the transfer and the transformation of energy, it is necessary first to determine the whence and the whither, as well as the amount and kind of this energy supply.

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[1] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **2**, 56.

[2] *Metl. Zeitschrift*, **24**, 504 and 565, 1907.

[3] *Phil. Mag.* (6), **16**, 467, 1908.

[4] Kleinschmidt, *Beiträge zur Phys. der freien Atmos.*, **2**, 205, 1908.

[5] *Nature*, **78**, 551, 1908.

[6] *Ibid.*, **78**, 551, 552, 1908.

A little energy reaches the atmosphere by conduction from the heated interior of the earth; some from the omnipresent radioactive elements; a little as a result of meteoric bombardment, and possibly an additional small amount through solar electrons, but the sum total from these and from similar sources is negligibly small. Even the combined radiations of all the stars, exclusive of the sun, is estimated to produce in space an effective absolute temperature of only a few degrees C.; not less than two, probably, nor more than ten.

Hence the supply of energy to the earth is almost wholly of the radiant form from the sun, and, neglecting trifling losses otherwise, equilibrium is maintained by a correspondingly intense reflection and radiation from the earth to space. The causes therefore of many atmospheric conditions, among them the existence of the upper inversion, are to be found in the phenomena of absorption and of radiation as they pertain to the substances concerned and to the particular limits of wave-length and of temperature involved.

Solar energy, excepting a small amount, as it reaches the earth's atmosphere, is confined to wave-lengths ranging from $2.5\,\mu$ down, with the maximum intensity somewhere between $0.4\,\mu$ and $0.5\,\mu$. The extent to which this energy is absorbed by the gases surrounding the earth is not an easy matter to determine, since it depends upon the wave-length, upon the kind of gas and its temperature, upon the amount of the gas through which it passes, or its partial pressure, and even upon the total[1] pressure to which it is subjected; nor is it certain that these are all the modifying conditions. Still, due mainly to the Astrophysical Observatory of the Smithsonian Institution, very much is known about this problem.

On the violet side, as it reaches us, the energy curve is very steep, and vanishes for ordinary levels at about $0.3\,\mu$. It is also steep on the red side, so that roughly three-fourths of the total energy lies between $0.4\,\mu$ and $1.1\,\mu$. The extreme ultra-violet portion of the insolation, of which there can be no reasonable doubt, is wholly absorbed by the outer dry portions of the atmosphere, as are also certain narrow regions, such as make up the oxygen lines in the red. And besides, a general, though less and less absorption extends to regions nearer and even into the visible spectrum.

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[1] Ångström, *Arkiv för Matematik, Astronomi och Fysik*, 4, No. 30, 1908.

Lower down in the atmosphere water vapor absorbs particular groups of wave-lengths amounting in all to from 5 to 10 or 12 per cent. of the whole energy. The rest is absorbed or reflected by clouds, by dust, or by the surface of the earth. That portion of course of the reflected light which returns to space retains its original wave-lengths. The absorbed part of the energy, however, escapes as earth radiation of long wave-lengths from 4 μ to 20 μ mainly, with the maximum intensity not far from 12 μ. For all these water vapor, according to Rubens and Aschkinass,[1] has a large coefficient of absorption, and for a few of them so has carbon dioxide; so that radiation does not escape to any great extent directly from the earth, nor from the air or clouds of lower levels, but chiefly through a step-by-step process, as each layer radiates to those above and these in turn more and more freely to space through the remaining water vapor and the outer dry atmosphere, where the coefficient of absorption certainly is small. For a discussion of this point see *Annals of the Astrophysical Observatory of the Smithsonian Institution*, Vol. II.

Since the horizontal layers of the moisture may be regarded as absorbers of energy, and as radiators, it is easy in a general way to consider the action of a given layer in sending radiation to say a unit horizontal area at any position above or below it.

Let S be a unit area parallel to, but any distance from, a flat radiating surface of infinite extent; and let I be the radiation intensity of the plane, that is, the total upward radiation in all directions from a unit surface, divided by 2π, and let θ be the angle of incidence of a ray on this unit area. Then, in the absence of any absorption, the total radiation normal to S is

$$R = 2\pi I \int_0^{\frac{\pi}{2}} \sin\theta \cos\theta d\theta = \pi I.$$

Let now a uniformly absorbing layer of constant thickness t be placed parallel to the radiating plane, but between it and S. Then the length of that portion of any ray within this layer on its way to S is

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[1] *Annalen der Physik*, 64, 598, 1898.

$$l=t\sec\theta.$$

If a is the coefficient of absorption, that is, in the case of normal radiation, the ratio of the energy absorbed by a layer of unit thickness to the incident energy, then the portion absorbed by a layer of thickness l is $1-(1-a)^l$, but when a is small and l not too large, this is approximately $la=at\sec\theta$; and therefore in this case,

$$R=2\pi I\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta d\theta-2\pi Iat\int_0^{\frac{\pi}{2}}\sin\theta d\theta=\pi I(1-2at).$$

These equations give the intensity of the radiation from the initial surface normal to A in terms of experimentally determinable quantities and show that while it is a function of the absorbing layer, it is, nevertheless, in each case independent of the distance from the radiating surface. The absorbed energy, however, is re-radiated, so that the sum total of the outgoing energy is a constant.

Therefore above the plane of the upper inversion, that is, above the moisture of the atmosphere, or the region in which solar energy is mainly absorbed, the radiation normal to a horizontal area from any plane below is, neglecting the slight absorption of the dry air through which it passes, approximately independent of its position, so long as its distance above this plane is small in comparison with the radius of the earth, as of course that of every point so far reached really is. And, as explained above, when equilibrium is established the total radiation to this surface is independent of absorption.

In considering the temperatures in the region of the upper inversion the effects of earth radiations, $\lambda\epsilon$, and of solar radiations, $\lambda\varsigma$, will be examined separately, because the latter, besides being active only half the time, produce results other than heating alone, and therefore the Stewart-Kirchhoff law of radiation and absorption is not applicable to them; while so far as known it is applicable to the former, which also are continuous in their action both day and night.

Since convection necessarily is absent above the upper inversion, and since the thermal conductivity of gases is very small, therefore anything beyond this plane is in the position, nearly, so far as earth radiation is concerned, of a body receiving radiation from a flat, black surface of infinite extent; because the water vapor below is black to long waves, the only kind given off by objects at the prevailing low temperatures.

Consider first the condition of an object placed between two such surfaces, parallel to each other and close together in comparison to their linear dimensions. Let their temperature be T_2 absolute; then whatever the object placed between them, its temperature will become T_2 also, because it is practically surrounded by walls at this constant temperature.

Now let a slab of this substance, say one centimeter square, δt thick, be placed parallel to the large planes, and let H_2 be the number of heat units of wave-length $\lambda\epsilon$ they supply to it per second, and let A_2 be the number of units of this energy it absorbs per second. Then if E_2 is the number of units of energy of the same wave-length it radiates per second, as a result of its hotness only, the following equation is given by the Stewart-Kirchhoff law:

$$\left(\frac{E_2}{\frac{A_2}{H_2}}\right)_{\lambda\epsilon,\,T_2}=(R_2)_{\lambda\epsilon,\,T_2}=\left\{\begin{array}{l}\text{The number of heat units a black body radiates per second}\\ \text{within the given range of wave-lengths (under the given con-}\\ \text{ditions the total radiation) when at temperature } T_2.\end{array}\right.$$

But the state is a steady one, and therefore $E_2=A_2$, and $R_2=H_2$.

Now let one of the parallel surfaces be removed and the other retained at its old temperature T_2. The heated body, small slab, will come to a new state of equilibrium at a temperature T_1, and give the relations

$$\left(\frac{E_1}{\frac{A_1}{H_1}}\right)_{\lambda\epsilon,\,T_1}=(R_1)_{\lambda\epsilon,\,T_1}, \quad E_1=A_1, \quad \text{and} \quad R_1=H_1.$$

Since H_2 is the radiation per second incident on the heated body from the two planes jointly, while H_1 is the corresponding amount from but one of them at the same temperature, then evidently

$$\frac{H_2}{H_1}=\frac{R_2}{R_1}=\frac{2}{1}.$$

But from the Stefan-Boltzmann law we know that

$$\frac{R_2}{R_1}=\frac{T_2^4}{T_1^4}=\frac{2}{1};$$

and hence

$$\frac{T_2}{T_1}=\frac{119}{100}, \quad \text{approximately.}$$

The second condition above described, that is, the single radiating black surface heating an object in its neighborhood, is analogous to the effective radiating surface of the earth at temperature T_2 and the dry diathermanous atmosphere above the surface of the upper inversion at temperature T_1.

Since the absorption of the dry atmosphere is minute the value of T_1, if determined by earth radiations alone, would be nearly constant—though of slowly decreasing value—with increase of elevation. A knowledge of these two temperatures should, therefore, show to what extent the temperature of the upper air is determined by that of the effective radiation surface. And both are known: the latter calculated by Abbot and Fowle[1] from the results of many observations made chiefly by them at the Astrophysical Observatory of the Smithsonian Institution, and the former directly obtained with sounding balloons. The temperature of the upper inversion varies through a considerable range, but averages something like $-55°$ C. Therefore, assuming this to be due entirely to earth radiation, we get

$$\frac{T_2}{218}=\frac{119}{100} \quad \text{or} \quad T_2=259° \text{ Abs., approximately.}$$

But this is about the temperature[2] required by the solar constant for the effective radiating layer, that is, a layer of dense water vapor—the equivalent nearly at this temperature of a black surface—that radiates the given amount of energy; and therefore it seems that in the main the temperature of the upper air is due to radiation from the moisture of the lower levels and that the dry air is extremely diathermanous.

The effective temperature, or temperature of the effective layer, is found in middle latitudes about three kilometers above the surface of the earth in the winter and five in the summer, or say four kilometers for the year.

Consider now the effect of the solar radiation on the outer atmosphere. The oxygen of this region absorbs the ultra-violet and a little of the red, and besides slight heating, produces, it is believed, electrical and chemical effects. That is, it is thought that the air is ionized and that both ozone[1] and nitrogen pentoxide[2] probably are produced. The blue of the sky would then be due in part, as many have suggested, to this ozone. Each of these compounds, but especially ozone,[3] has absorption bands scattered through a wide range of the spectrum from well within the ultra-violet to 12 μ or farther in the infra-red. It has a strong band at and below 0.3 μ that may account in part for the sharp limitation near this wave-length of the observed solar spectrum. It has another well-defined band at 0.6 μ, which possibly adds something to the great depression found in the solar intensity curve at this place. There are several other bands, but in the infra-red, one of which 8.5 μ to 10.5 μ is very strong, and of course would absorb earth radiations.

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[1] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, 2, 174.

[2] *Ibid.*

Spectroscopically then, assuming the existence of ozone in the high atmosphere, for which there is much evidence, the known part of the earth's atmosphere is divisible into three regions: the black body, the diathermanous, and the selectively absorptive. The first is the water-vapor region which totally, or nearly so, absorbs long-wave radiations. The second is the dry air just above this region; and the third is the high atmosphere, presumably rich in ozone, in which absorption is marked but selective. There is reason to believe, however, that the outer atmosphere shades away in density with elevation and changes in composition, until at a height of one hundred kilometers or less it is again diathermanous, with nothing but a little hydrogen and possibly some helium left.

Since both ozone and water vapor transmit solar radiation fairly well, most of it reaches the lower levels and the earth, where it is absorbed, and there emitted again as long waves for which water vapor is nearly opaque. *In this way the lower atmosphere becomes heated and rises, in which act it cools by expansion and by radiation until it reaches a minimum temperature determined mainly by the*

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[1] Lyman, *Astrophysical Journal*, **27**, 87, 1908; Henriet et Bonyssy, *Comptes Rendus*, **146**, 977, 1908.

[2] Warburg und Leithäuser, *Annalen der Physik*, **23**, 209, 1907.

[3] Ladenburg und Lehmann, *ibid.*, **21**, 305, 1906.

sum total of the energy emitted from lower levels. Above this place the temperature slowly rises, presumably because of the absorption, as explained, due to oxygen and to ozone.

The lower levels of the upper atmosphere should be comparatively free from ozone, partly because it is less rapidly produced at these levels, and partly because it slowly reverts into ordinary oxygen. Besides, a small amount of water vapor must diffuse across the inversion plane, since it extends in appreciable quantities to this level, as we know by the cirrus clouds; and it is probable that this water vapor, as it comes in contact with the ozone of the inversion region, converts it by way of hydrogen peroxide into ordinary oxygen. These two processes, the spontaneous and the chemical reduction of ozone to oxygen, would keep this place constantly diathermanous. Presumably, too, above the limits of the water vapor, where the oxygen absorption of the ultra-violet is greatest, the largest amounts of ozone are found. If so, then this region will absorb a larger amount of solar energy as well as more of the earth's radiation than will the diathermanous layer below, and thus be kept constantly warmer. Besides this, difference in temperature may be somewhat accentuated, or at least maintained, at night by the rapid radiation through the outer atmosphere of the upper levels of the water vapor, causing it to cool more rapidly than the air above it, in a manner analogous to the radiation phenomena involved in the formation of anchor ice, and of morning inversions at the surface of the earth.

Cause of abrupt change of temperature-gradient.—The sharp definition of the place of inversion probably is due to the existence at that level of a more or less well-formed cloud veil, which becomes to that extent the locus of insolation. If so, then a sky with extensive cirrus clouds should give immediately above the inversion—presumably the upper surface of the cirrus—a rapid increase of temperature with elevation, while a serene sky should furnish a curve of gradual transition from the one gradient to the other. Both these conditions and results are shown by the curves, which are in harmony with other flights under similar circumstances. No. 2 shows the effect of cirrus clouds, and No. 6, the gradient when the sky is perfectly clear. In the one the change is gradual; in the other it is abrupt and the increase large.

Cause of seasonal difference in height of inversion.—During the summer the moisture content of the air, because of increased temperatures, is greater than it is in winter, and extends to greater elevations, and therefore the effective radiating surface is correspondingly elevated. This combined with the difference in the gradients for the two seasons—less in summer than in winter—puts the inversion level at its greatest elevation during the warmer weather.

Cause of seasonal difference in temperature at inversion surface.—The lower atmosphere and the surface of the earth receive more solar energy during the summer than in the winter, and consequently must radiate more at this season. Therefore, if the inversion temperature is determined mainly, or even largely, by earth radiation, it should be greatest in summer and least in winter; and this is supported by the published observations from Trapps, from Uccle, and from Strassburg, and presumably, therefore, is generally true.

Cause of latitude effect on height of inversion.—Change in latitude at any given season is analogous, meteorologically, to change of season at any given latitude, and might be expected to lead, as it does, to a corresponding change in the height of the inversion.

Cause of relation between barometric pressure and height of inversion.—Normally the region of high barometer is one of clear weather, while a low barometer is accompanied by precipitation, which correspondingly heats the upper air, and decreases its temperature-gradient. As a result of this heating, the place of inversion is shifted to one of lower level and higher temperature, as is well illustrated by curve No. 4.

CONCLUSIONS

1. Considered from the standpoint of temperature-gradients, the explored portion of the atmosphere is divisible into three parts: (*a*) the region of terrestrial disturbance, extending from the ground to an elevation of about 3000 meters above its surface; (*b*) the region of uniform changes, from the top of (*a*) to roughly 10,000 meters above sea-level; (*c*) the region of permanent inversion, or all that explored portion, at least, that lies above the plane of the upper inversion.

2. Spectroscopically the known atmosphere is divisible into

three parts: (*a*) the black body portion, coincident with the region of and due to the relatively dense water vapor; (*b*) the diathermanous, or the dry air next above the water vapor; (*c*) the selectively absorptive, or the air of the isothermal layer, presumably rich in ozone.

3. The isothermal layer is due mainly to earth radiations. The lower heated air rises and cools by expansion until it reaches a minimum temperature fixed by the radiation from below at that time and place.

4. The differences in the temperature-gradients of the region of uniform changes, the differences in the heights of the plane of inversion, and in the temperatures at which inversion takes place, as determined by season, by latitude, and by storm conditions, are all accounted for by the corresponding differences in the moisture content and in the radiation of the atmosphere.

5. The temperature and the height of the upper inversion are given by the equations

$$T_1=0.84T_2\,,$$

and

$$h_1=h_0+\int_{T_0}^{T_1}\frac{dh}{dT}\,,$$

in which

h_0=any initial height. It may be the surface of the earth.

h_1=the height of the inversion surface.

$\frac{dh}{dT}$=the reciprocal of the temperature-gradient where the temperature is T.

T_0=the absolute temperature at the elevation h_0.

T_1=the absolute temperature at the level of inversion, h_1.

T_2=the absolute temperature of the effective radiating surface, or of a dense layer of water vapor radiating the given amount of energy.

SUGGESTIONS

Much needs yet to be done in observing temperature-gradients during special types of weather, particularly in the centers and about the borders of cyclones. It would also be well to take the height of the high cirrus at the time the ascension is made.

Many laboratory investigations are also desirable in connection with the absorption and radiation of the several gases and vapors of the atmosphere, singly and variously combined, at different pressures and different temperatures. The investigations indicated would necessarily be long, and some of them most difficult, but only in so far as they and kindred ones are completed, can meteorological problems of the nature of those discussed in this paper be fully solved.

The substance of this paper was read July 1, 1908, before the Physics Section of the American Association for the Advancement of Science,[1] and again on August 27, 1908, before the Astronomical and Astrophysical Society of America.

MOUNT WEATHER OBSERVATORY
VIA BLUEMONT, VA.
October 1908

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[1] *Science*, **28**, 256, 1908.

THE CONSTITUTION OF THE SUN[1]

BY J. F. HERMANN SCHULZ

My theory is based upon the lines laid out long ago by Kirchhoff thus:

> The most probable assumption which can be made is that the sun consists of a solid or liquid nucleus in the highest state of incandescence, surrounded by an atmosphere of somewhat lower temperature.

This assumption later on was modified and extended by Zöllner thus:

> The sun is a glowing *liquid* body, surrounded by a glowing atmosphere; in this latter floats a sheet of luminous, cumulus-like clouds, constantly being renewed, at a certain distance above the liquid surface.

When Zöllner in 1873 expressed this view, it appeared nearly impossible to speak in earnest of a *liquid* solar nucleus At that time the most extravagant figures were still mentioned as to the temperature of the sun.

Zöllner himself found 1 to 2 millions of degrees Centigrade as a minimum temperature for the gases of the prominences before their escape out of the liquid body into the open solar atmosphere, whereas Secchi went still farther, and came up to 40 million degrees C.[2]

Now at such temperatures, or even at only a hundred thousand degrees, there is practically no basis for the assumption of a liquid sun, but today the matter stands at a widely different point.

The readers of this *Journal* will be well aware that the latest calculations, based upon the most reliable determinations of the solar constant, combined with the Stephan's law of radiation, give us less than 6000° C. as *mean temperature of the radiating layers.* Millochau and Féry find 5620° absolute, or 5347° C.[3] We do not speak of the temperature of the sun, for it is clear that the measured solar radiation on the earth is not simple but is the combined radiation (1) of the true gaseous atmosphere; (2) of the photospheric clouds—condensed particles; and (3) of the liquid nucleus

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[1] About twenty years ago I advanced a theory of the sun, at first in the *Gaea*, edited by Dr. Hermann J. Klein of Cologne, in 1885 and 1886, afterward in the *Astronomische Nachrichten*, **119**, 1887 and 1888, which latter papers were reprinted in Exner's *Repertorium der Physik*, **25**. As this theory may appear more plausible now than in those days, I venture to bring it again before the notice of astrophysicists.

[2] *Die Sonne*, p. 573.

[3] *Comptes Rendus*, **143**, 570, 1906.

From this assumption we are able to explain correctly the fact that the intensity of solar radiation diminishes from the center of the disk toward the limb. Near the limb we get radiation only from the atmosphere and the tops—the least heated parts—of the photospheric clouds, whereas in the central portions of the disk, we are looking down into the gaseous interstices between these clouds, and thus here get also radiation from the lower, hotter parts of same, as well as also from the still hotter liquid nucleus. This is a real explanation of the observed fact, whereas the usual absorption hypothesis does not stand a closer test. I pointed out in *Gaea*, p. 430, 1886, and in *Astronomische Nachrichten*, Nos. 2817, 2818, 1888, that absorption does not mean simply annihilation, and that, since the solar atmosphere certainly is not absolutely diathermous, a certain absorption must also originally have taken place in the outer cooler layers. These layers thereby were heated and their proper radiation increased until a state of equilibrium was reached, where absorption and their own radiation became equal throughout the entire atmosphere, and thus no diminution of the radiation as a consequence of absorption could be spoken of.

As generally the absorbed energy is transformed into radiation of longer wave-length, it is clear that the luminous, and still more the ultra-violet radiation must be weakened by the process described, but the total value of the radiated energy cannot be affected after the state of equilibrium between absorption and radiation is reached.

Recently also Julius[1] and W. Wundt[2] have come to the same conclusions as those I advanced twenty years ago.

From these considerations follows also necessarily a very important fact, namely, that the solar atmosphere cannot be in the adiabatic state, but that the upper strata must possess a relatively higher temperature—thence would also follow a greater extension of the atmosphere than would be otherwise possible—and finally, that it is impossible to explain any cooling in lower levels—such as sun-spots would require—by descending currents in the atmosphere. The

[1] *Astrophysical Journal*, **23**, 323, 1906.

[2] *Physikalische Zeitschrift*, **7**, 385, 1906.

differences in temperature between various levels must be far less than usually anticipated.

We now come to consider whether at a mean temperature of about 5400° C. we are allowed to imagine reasonably a truly liquid nucleus of the sun.

If about 5400° C. be the *mean* temperature, the upper surface of the photosphere must be at a lower, the liquid nucleus, however, at a higher temperature, the latter thus perhaps still above 6000° C. The late Henri Moissan expressed his view in the *Comptes Rendus* (**142**, 673, 1906) that the temperature of the liquid nucleus of the sun—which he adopts in conjunction with Birkeland—might not differ much from 3000° C., since in the electric furnace at perhaps 3500° C. all known elements volatilize. I cannot share this view, but think that the temperature of the sun is higher, and probably as high as mentioned before, but, in spite of that, we may very well assume that *the bulk of the sun is in the liquid state.*

The temperature *in* the electric furnace must, without doubt, reach considerably higher than 3500° C., which is the generally admitted temperature of the electric arc in air.[1] Moissan considered this also as the minimum, for he said on this point, rather *too* cautiously, in his book *Der electrische Ofen* (German edition, Berlin, 1897, p. 25):

> Wir hatten aber immer den Eindruck bei zahlreichen, hierauf bezüglichen Untersuchungen, unter sehr verschiedenen Spannungen, *dass die Temperatur in dem Masse stieg, als die Leistungsfähigkeit der Maschinen, mit denen wir arbeiteten* [. . . . that the temperature rose in the same measure as the efficiency of the machinery at our disposal].

In his latest experiments Moissan had at his command an electric current equal to 300 horsepower, and this amount of energy was utilized for a time from 5 to 20 minutes to heat a clear space *in* the electric furnace of perhaps 13×13=169 cm. His largest crucibles measured 9 cm in diameter, and he says it were well to have still a free space of 2 cm between the crucible and the sides of the furnace. The supplied energy of 300 HP for about 169 cm equals about 18,000 HP for one square meter, whereas the sun's radiation from one square meter at the solar surface is calculated to equal 130,000 HP.[2]

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[1] Lummer and Pringsheim found this temperature to be from 3500° to 4200° C.

[2] Young, *Manual*, p. 251.

This proportion corresponds to a temperature of about 3500° C. for the electric furnace, and of about 5400° C. for the sun, and further to the supposition that the heat generated is freely dissipated by radiation in the surrounding space. If, however, to a certain space, such as the interior of the electric-furnace, energy is continuously supplied for some time, and this energy cannot dissipate by radiation, and likewise not by conduction (which Moissan tells us), then necessarily the temperature in the interior of the furnace must increase, until the increase is checked by the melting and evaporation of the limestone of which the furnace is constructed. Therefore I think that we are well justified in supposing *that the temperatures attainable in a Moissan electric furnace are comparable to the sun's temperature, as we know it today.*

Now Moissan has shown that even at the enormous temperatures attained in his electric furnace, we have not yet reached the point at which all terrestrial elements are truly boiling. In this respect the following table is very instructive, which Moissan gave in *Comptes Rendus* of February 19, 1906 (**142**, 430). I quote Moissan's own words as follows:

> Nos principales expériences sont résumées dans le tableau suivant, qui montre bien la différence des points d'ébullition des métaux de la famille du fer:

Métal	Poids: grammes	Temps, minutes	Ampères	Volts	Métal distillé, grammes
Nickel..................	150	5	500	110	56
	200	9	500	100	200
Fer.....................	150	5	500	110	14
	825	10	1000	55	150
	800	20	1000	110	400
Manganèse.............	150	3	500	110	38
	150	5	500	110	80
Chrome................	150	5	500	110	38
Molybdène.............	150	10	700	110	0
	150	20	700	110	56
Tungstène (Wolfram)....	150	20	800	110	25
Uranium...............	150	5	500	110	0
	150	5	700	110	15
	200	9	900	110	200

Moissan further adds the following remarks:

Molybdenum. The 150 gr were not fused by a current of 500 amperes and 110 volts. After applying 700 amperes and 110 volts for 7 minutes, the metal was fused, but nothing evaporated. After 20 minutes 56 gr were distilled.

Tungsten (*Wolfram*). After applying 500 amperes and 110 volts for 5 minutes, the metal was not yet fused. After applying 800 amperes and 110 volts for 20 minutes, boiling commenced, but only 25 gr distilled.

Another highly interesting paper of Moissan is: "Sur la distillation des corps simples."[1] We here find the following statement:

Gold commences to evaporate *in vacuo* at 1070°;
it boils *in vacuo* at 1800°;
and should boil at 760 mm pressure at 2530°;

thus showing how much depends upon the pressure under which the boiling takes place. Now all Moissan's experiments, tabulated above, are made at ordinary atmospheric pressure, and we are entirely at a loss to say how much the evaporation of the various metals would have been retarded under increased pressure, such as we may expect at the very base of the solar atmosphere, close to the liquid nucleus.

Moissan tried also the metalloid *titanium* in his electric furnace. Five hundred gr were heated by a current of 500 amp. and 110 volts; after 4 minutes vapor appeared, but after 5 minutes the stuff was fused only on the surface, and carbide of titanium had formed. Then 300 gr were treated with 1000 amp. and 55 volts for 7 minutes; 110 gr were distilled; the stuff itself, however, had been only viscous, the surface had not become horizontal. In his book *Der electrische Ofen*, p. 238, he says that even with a current of 2200 amperes and 60 volts the stuff in the crucible is not completely fused.

The conclusion which we must draw from Moissan's investigations and experiments is, therefore, that it is a grave error to consider the sun as a completely gaseous body, as it was considered for a long time but erroneously, because at relatively low temperatures all terrestrial elements and compounds were not only molten, but totally evaporated and entirely dissociated. Moissan has shown that such views are incorrect.

The fact that we observe iron-vapor in the atmosphere outside the photosphere does not exclude the possibility that this element could exist in lower, thus hotter, levels, still in the *liquid* state. Aqueous vapor exists in the earth's atmosphere up to altitudes of 70 to 80 km where the temperatures must be far lower than −100° C. An astrophysicist, say on the moon, who would therefrom draw the conclusion that liquid water, or solid ice, could not exist at the base of the earth's atmosphere, would, of course, be greatly mistaken.

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[1] *Annales de Chimie et de Physique* (8), 8, 145–181, 1906.

Professor A Ritter, who during the period 1880 to 1889 published in *Wiedemann's Annalen* a series of articles on "Die Constitution gasförmiger Weltkörper," says (**36**, 567, 1889) at the beginning of the closing section 19 (when he had read my paper in *Astronomische Nachrichten*, Nos. 2847, 2848, 1888):

> Die Hypothese des *gasförmigen* Zustandes der Sonne müsste dagegen zur *zweiten* Gruppe—welche bei voller Anerkennung ihrer Unzuverlässigkeit, *nur versuchsweise* aufgestellt wurden—gerechnet werden; *denn niemand ist im Stande, zuverlässige Auskunft darüber zu geben, wie das Innere der Sonnenmasse beschaffen ist.*

It is generally claimed for the theory of the entirely gaseous nature of all self-luminous stars, that only this state affords the possibility of the constancy of radiation. A liquid star, it is said, would be covered rapidly by a non-luminous crust. This, however, is not correct under all circumstances. At the high temperature of the sun, its liquid masses must be very fluid, quite in contrast to gases, which are probably of greater viscosity at high temperatures. Thus we are justified in assuming very entensive convection-currents in the cooling, liquid sun, provided the bulk of the same consists of material of uniform density. As further the Helmholtz contraction-hypothesis holds as well for the liquid as for the gaseous state, we find that the latter state is in no way superior to the liquid state concerning the maintenance of solar radiation.

As pointed out, it is only necessary for my hypothesis that the sun—or a similar star—to a depth of perhaps half the radius be of uniform density, whereas the remaining innermost nucleus may contain masses of greater density. If this condition is not fulfilled, and the outer layer of least density is relatively thin, then this layer will cool down rapidly, and the star will be covered really by a non-luminous crust. This would explain the fact that in some binaries one of the components, in spite of a considerable mass, may be of far less brilliancy, or even quite dark, as compared with the other component. If the outer, dark layer is incomplete, or very thin, so that it may be broken up at times, this would afford an explanation of the wide, irregular range in brilliancy of certain variables.

These are the chief reasons for which I hold that the sun, including, perhaps, most other stars, is not entirely gaseous, but only *in the liquid state*, as assumed by Kirchhoff and Zöllner, a state which enables us also to explain all solar phenomena upon generally recognized physical principles

HAMBURG
1908

INTERACTION OF SUN-SPOTS

By PHILIP FOX and GIORGIO ABETTI

If a review be made of the records of sun-spot positions, considering the spots individually and regarding especially their longitude, a seeming interrelation of the spots of the two hemispheres appears, for they will be found not infrequently to travel in pairs, paralleling each other on opposite sides of the equator. To decide whether there is a physical foundation for this coincidence in longitude, or whether the coincidence is due to chance, would require, in the absence of an actual, visible demonstration of interaction, an extended statistical study.

An examination of the sun-spot charts of Carrington[1] and of Spoerer[2] indicates the number of coincidences to be about equal to the probability of coincidence from chance arrangement. We counted the coincidences on Spoerer's charts, regarding spots as coincident when some part of either group was between the meridians bounding its neighbor, and found 28 per cent. of the spots traveling in pairs. The average length in longitude of a group is 5°66. The space allowed for coincidence on the average is then 17°0. The average number of spot groups per revolution was 11.27, which gives for the average space occupied by each spot in each hemisphere 64°0. The probability of coincidence is then $17.0 \div 64.0 = 26.5$ per cent. We cannot affirm from this test that a spot in one hemisphere tends in general to accompany a spot in the opposite hemisphere.

We have also examined the Rumford spectroheliograms, counting not only coincidences of spots with spots but of spots with disturbances indicated by flocculi. This test yielded a percentage of coincidences somewhat larger than the probability would call for.

However the general case may stand, in certain individual cases a physical connection seems evident. Concerning this, Maunder states:[3] "Nothing is more striking in the case of a considerable disturbance

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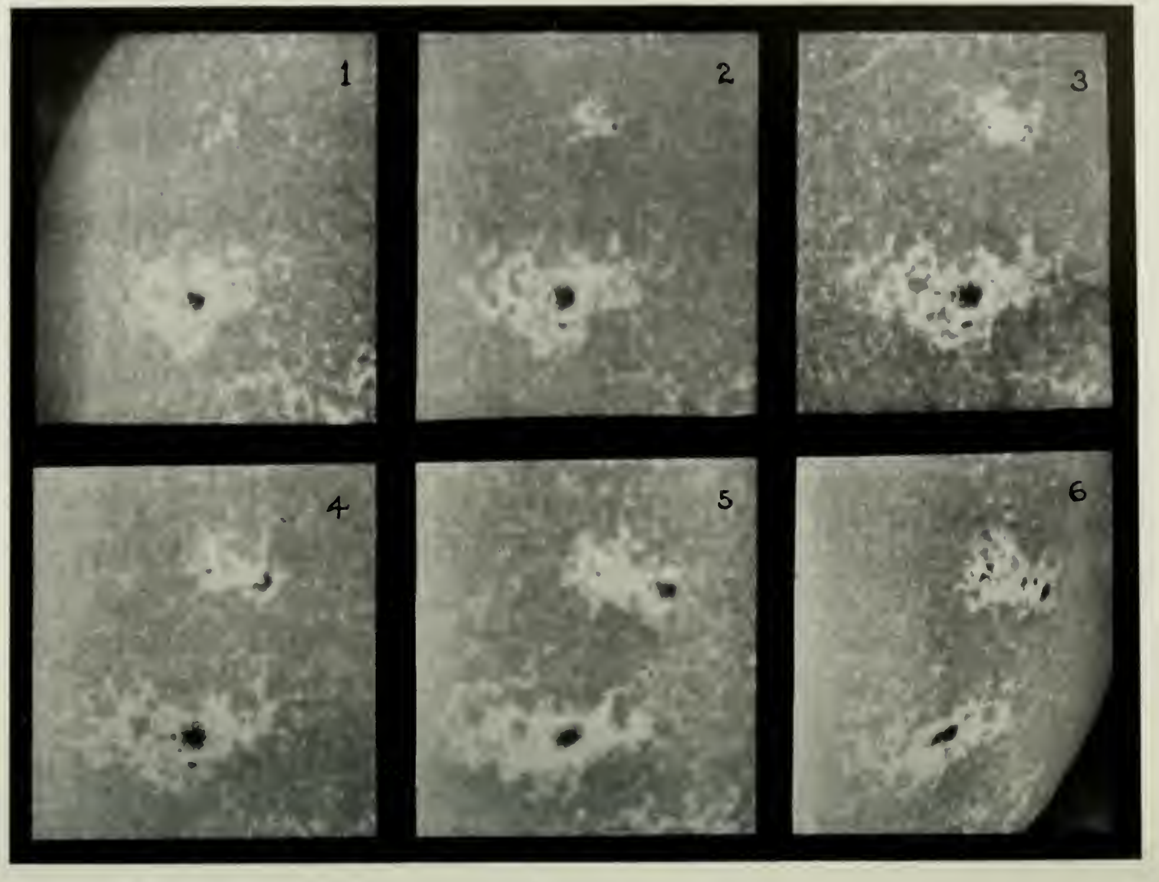
[1] *Observations of Solar Spots.*

[2] *Publicationen der Astronomischen Gesellschaft*, 13; *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, 1, 2, 4, and 10.

[3] *Knowledge*, 17, 199, 1894.

PLATE I



N

E

W

May 6, 3h 40m G. M. T.

May 7, 3h 37m

May 8, 3h 23m

May 9, 3h 10m

May 11, 4h 19m

May 13, 4h 19m

Spectroheliograms Showing Development of Companion Spot than to see it repeat itself, at it were, in another latitude, and an active train will often be accompanied by a feebler copy of itself a few degrees north or south." This is well illustrated on Spoerer's charts for the revolutions Nos. 32 and 43, also in Plates XIX and XX (**28**, 256, 1908) of this *Journal*. Plate I, made from Rumford spectroheliograms of May 1907, shows the development of a spot, Greenwich No. 6185, as a northern companion of a southern spot here making its second appearance on the disk, Greenwich No. 6184. The plate needs no further explanation than the appended data supplies, except to point out that the coming spot was, as usual, antedated by the flocculus.[1] The east limb is seen on the photograph of May 6 and the west on the plate of May 13, the spots having nearly crossed the disk.

As the spots approach the equator toward the end of the sun-spot maximum and the distance between north and south neighbors lessens, it seemed hopeful that if there was communication between them, perhaps normally confined to action below the surface, some visible demonstration of connection might be observed upon the surface. Fortunately on September 10 we obtained a series of plates with the Rumford spectroheliograph showing positive proof of such interaction. Certain of these plates illustrating the various stages of the events are reproduced in Plates II and III. Figs. 2 and 11 are from calcium (H line) spectroheliograms, the others are hydrogen with the line $H\alpha$, using "pan-iso" plates.

The spots concerned were the pair, lying in opposite hemispheres, which were near the center of the disk on September 9, 1908. A study of the plates on previous days revealed no unusual features. The southern spot was first seen on August 8 as a small spot near the east limb. On August 10 it had disappeared but was again present on August 13. From this date it developed rapidly. The northern spot was first seen on the plates of August 13. It too died out but was again present and rapidly developing when the region came into view on the east limb on September 4. On September 8 and 9 spots were present in the isolated flocculus preceding the northern spot.[2]

The appearance of the groups on September 9 is shown in Fig. 1

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[1] Fox, *Astrophysical Journal*, **28**, 255, 1908.

[2] Hale, *Astrophysical Journal*, **28**, 323, Plate XXVII, 1908.

of Plate II. The first plates of September 10 were the routine calcium exposures, Fig. 2. These showed considerable activity in the spot groups, principally in the eruptions in the northern group. The *Hα* plates of $5^h 36^m$ and $5^h 58^m$ G. M. T. revealed an intensification of the eruptions in the northern group,[1] a brightening of the flocculi following the southern group, and a slight strengthening of the isolated flocculus preceding the northern group, but gave no intimation of the pending outburst. Visual, spectroscopic examination at $7^h 30^m$ G. M. T. indicated a violent uprush of hydrogen immediately behind the southern spot and also at some distance following. A plate was immediately exposed, $7^h 52^m$. On this and subsequent plates tremendous changes were found. A brilliant eruption had occurred behind and in contact with the southern spot. Somewhat more remotely following, a still greater and more striking eruption was in progress, extending east and south like a great leg, brilliant to its toe, and stretching to the north in two branches, one nearly to the neighboring spot, the other a chain of brilliant eruptions reaching toward the isolated flocculus. This flocculus, the remains of an expired spot, had increased greatly in intensity and had extended arms toward the approaching chain of eruptions and toward the northern spot. From the northern spot an eruptive arm stretched to meet the eruptions advancing from below and from the flocculus. This is well seen at $8^h 13^m$. At $8^h 25^m$ the connection of the northern and southern spots was completed.

After this consummation the subsidence to relative tranquillity was as rapid as the rise. Examination of the line *Hα* at $9^h 30^m$ revealed descent in the eruptive regions behind the southern spot. Measurement of the displacement of the line on the photographs made at this time gives velocity of descent behind the spot as about 100 km per second and 170 km in the great eruption due east of the spot. The result of this is seen on the plates at $9^h 39^m$ and $9^h 48^m$ in the obliteration of the eruptive feature behind the spot and in the decline in the great eruption. Other plates exposed later showed a continued decline. The whole display lasted less than four hours and fortunately the history of it as shown on our plates is nearly complete.

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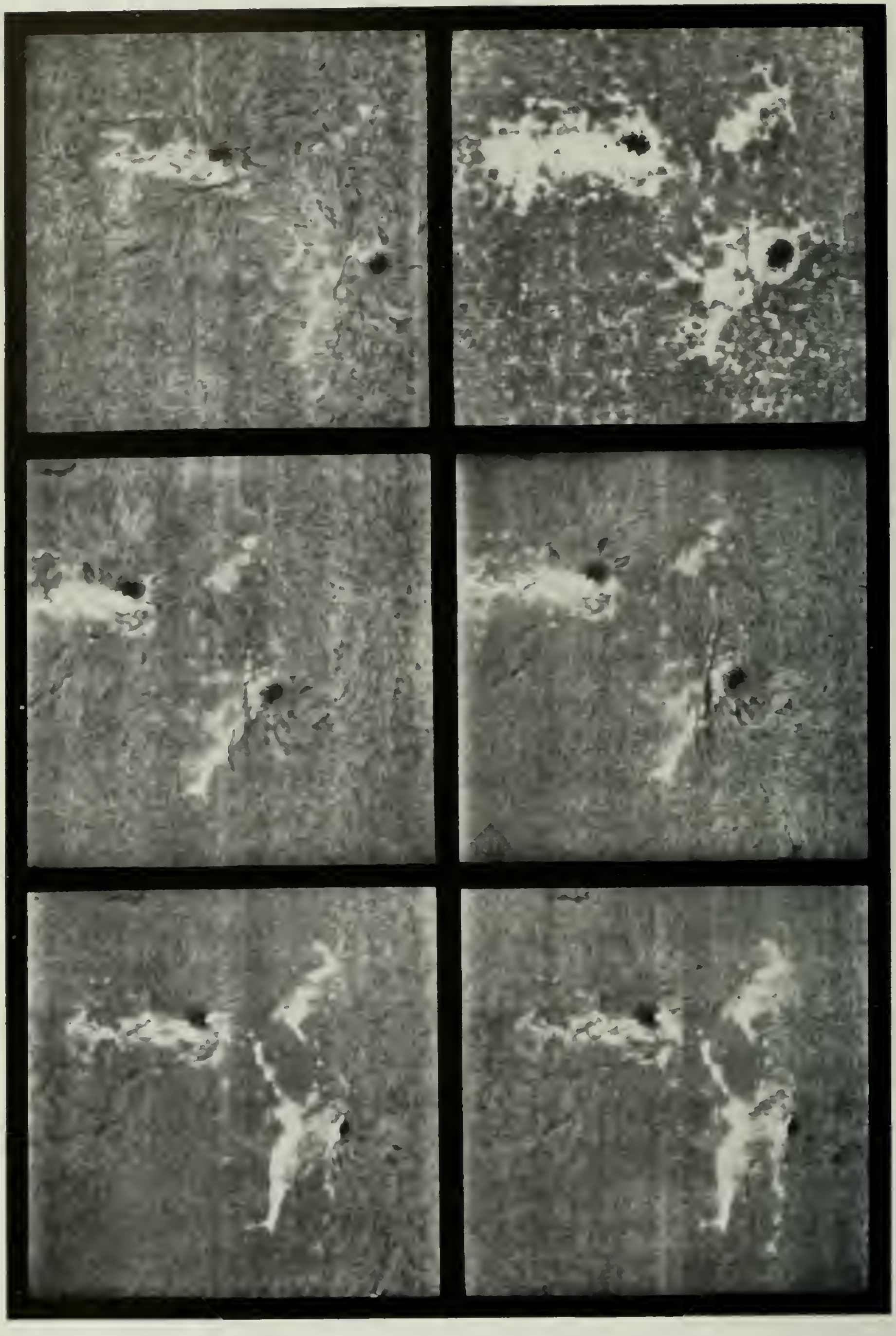
[1] Dennett observed an eruption in this group on September 9 lasting from 19^h45 to 20^h30 G. M. T. *Observatory*, 31, 388, 1908.

PLATE II



Sept. 9
$7^{h}\ 56^{m}.0$
Hα

Sept. 10
$3^{h}\ 39^{m}.2$
Ca H_2

Sept. 10
$5^{h}\ 36^{m}.5$
Hα

Sept. 10
$5^{h}\ 58^{m}.0$
Hα

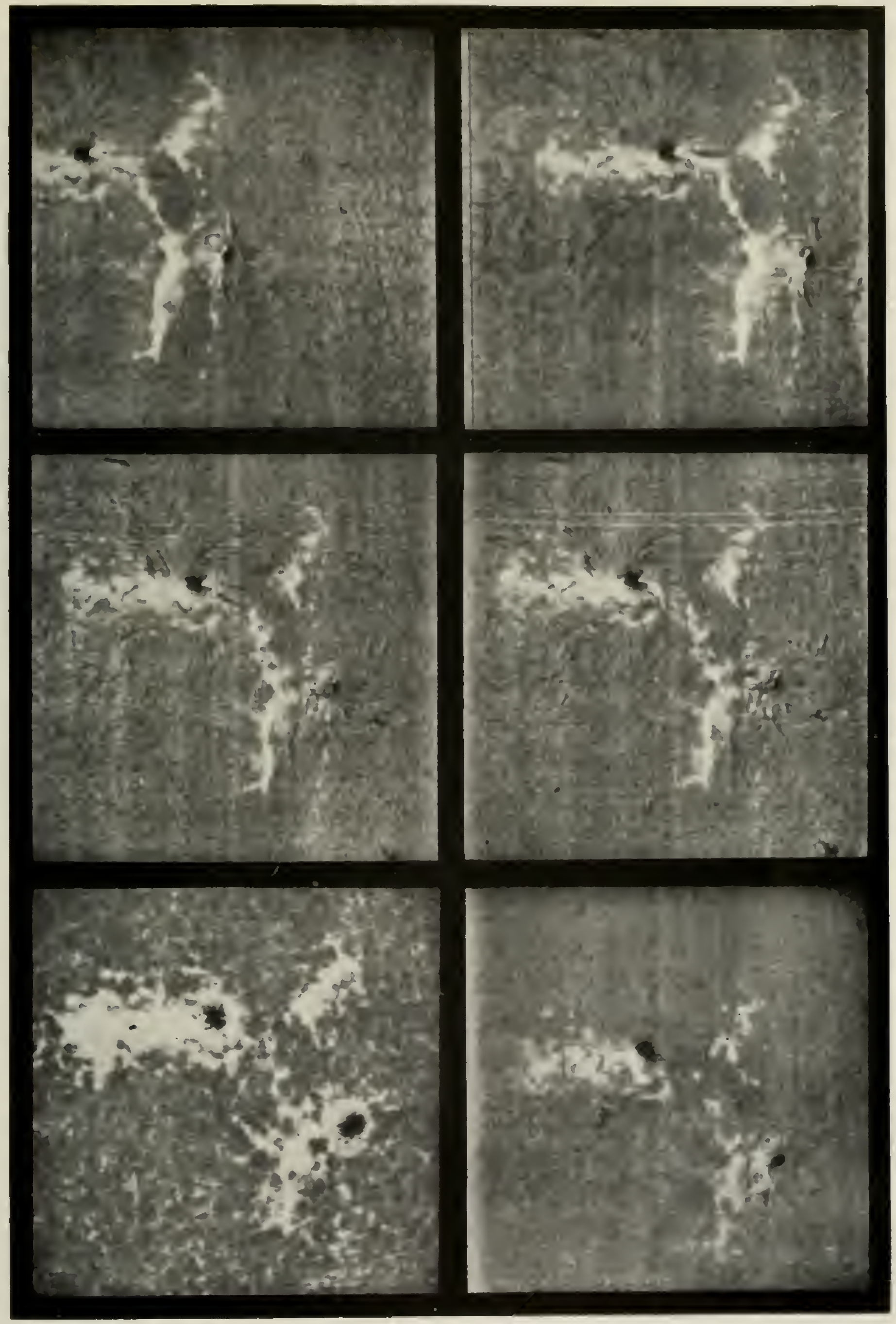
Sept. 10
$7^{h}\ 52^{m}.4$
Hα

Sept. 10
$8^{h}\ 2^{m}.7$
Hα

Eruption Connecting Sun-Spots
September 9–10, 1908. Scale: Sun's Diameter = 150 mm

PLATE III

N



pt. 10
16^m5

Sept. 10
$8^h 25^m5$
Hα

pt. 10
39^m1

Sept. 10
$9^h 48^m5$ W
Hα

pt. 10
50^m8
H_2

Sept. 11
$4^h 19^m0$
Hα

Eruption Connecting Sun-Spots
September 10–11, 1908. Scale: Sun's Diameter = 150 mm

It is to be regretted that we obtained no calcium plate when the disturbance was at its height. A plate at $10^h 51^m$ furnishes no startling changes when compared with the calcium plate at $3^h 39^m$. Certain minor changes, visible in the arrangement of the flocculi between the spots, are of such character (the chain, for example, is nearly continuous), that there seems little room to doubt that the calcium fully participated in the disturbance. Intensification in the eruptions following the northern spot is also noticeable. On the following day many small spots had developed in the flocculi train. An $H\alpha$ plate of September 11 is added to show the return to a more quiet existence.

The further history of these groups, so far as we have observed, was uneventful. Only one thing need be mentioned. On the next return of the spots to the center of the disk the space between the two groups as shown on the calcium plates was filled with small scattered flocculi.

The heliographic positions of the spots of the groups and of the flocculi about them were measured on plates before and after September 10 to determine if any notable drifts occurred. We reduced the plates using the constants employed at Greenwich. The positions of the leader spots are collected in the accompanying table. Between September 8 and 12 the southern spot remained unmoved in its position, but it is perhaps significant that the northern spot had advanced toward the southern $3.^\circ3$ in longitude and $1.^\circ7$ in latitude. The flocculi showed no consistent motions either of translation or rotation.

Hale has pointed out[1] the resemblance of the pattern made by the hydrogen flocculi and that of iron filings in a magnetic field. It is of interest, though of doubtful significance, that the hydrogen flocculi following the northern spot group have the heart-shaped configuration of the lines behind the weaker of two interacting electrical masses of unlike sign. We have considered the possibility of explaining this configuration as a section through an inclined vortex but this necessitates assigning far too great depth to the vortices. An indraught from the rear along the line of brilliant flocculi following the spot would be a sufficient, though perhaps not probable, mechanical explanation.

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[1] *Astrophysical Journal*, 28, 114, 1908.

Several investigators have expressed the opinion that the source of terrestrial magnetic storms is in the eruptions about the spots rather than in the spots themselves. Hale in discussing the magnetic fields which he has recently detected in sun-spots concludes, from the rapid diminution of strength of field with altitude, that terrestrial magnetic storms are probably not caused directly by these fields[1] and states: "Their origin may be sought with more hope of success in the eruptions shown on spectroheliograph plates in the regions surrounding spots." A disturbance of the magnitude of this one of September 10 might have been expected to produce noticeable effects on the earth. In *Nature* of September 24 Chree describes a large magnetic storm which occurred on September 11, beginning at $9^h 47^m$. The lag in

Plate No.	Date	G. M. T.	North Spot		South Spot		North Spot—South Spot	
			Heliographic Long.	Heliographic Lat.	Heliographic Long.	Heliographic Lat.	Long.	Lat.
	1908							
Σ*s* 2949...	Aug. 13	4^h $2^m.2$	265°.1	+11°.7	272°.1	−4°.3	+ 7°.0	+16°.0*
2955...	Aug. 14	5 48.2	266.9	+11.8	274.3	−5.0	+ 7.4	+16.8†
3057...	Sept. 8	4 37.2	267.2	+12.9	280.0	−5.6	+12.8	+18.5‡
3059...	Sept. 8	4 55.0	267.1	+12.8	279.7	−5.7	+12.6	+18.5
3063...	Sept. 9	4 40.5	268.7	+11.7	280.0	−5.6	+11.3	+17.3
3069...	Sept. 10	3 39.2	269.9	+11.3	279.7	−5.7	+ 9.8	+17.0¶
3071...	Sept. 10	3 56.1	270.1	+11.4	279.9	−5.6	+ 9.8	+17.0¶
3080...	Sept. 10	10 45.3	270.3	+11.4	279.7	−5.3	+ 9.4	+16.7
3082...	Sept. 11	2 24.5	270.7	+11.3	279.9	−5.6	+ 9.2	+16.9
3084...	Sept. 11	2 40.8	270.6	+11.1	279.9	−5.6	+ 9.3	+16.7§
3091...	Sept. 12	4 41.3	270.5	+11.1	280.0	−5.8	+ 9.5	+16.9
3141...	Oct. 2	3 37.4	271.5	+11.5	278.1	−5.5	+ 6.6	+17.0**
Σ*y* 520...	Oct. 2	9 10.7	271.6	+11.8	278.1	−5.4	+ 6.5	+17.2
Σ*s* 3153...	Oct. 5	3 53.2	271.7	+12.0	278.5	−5.4	+ 6.8	+17.4
3170...	Oct. 10	3 32.4	271.6	+12.5	280.3	−4.6	+ 8.7	+17.1

* First rotation.—The north spot is the leader of an oval group.

† South spot scarcely visible. ‡ Second rotation. ¶ Bridge across the north spot.

§ Many new spots in the train following the north spot. ** Third rotation.

REMARKS

Measurements are for the leader spot.

The plate Σ*y* 520 is a direct photograph with the 40-inch, all others (Σ*s*) are calcium spectroheliograms.

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[1] *Astrophysical Journal*, 28, 341, 1908; see also Schuster, *Nature*, 78, 663, 1908; *M. N.*, 65, 187, 1904.

time between the solar and terrestrial storms, something over twenty-six hours, is not consistent with the view of Young[1] and of Nordmann[2] that the magnetic impulse travels with the speed of light; but is more in harmony with the observations of Maunder[3] and of Riccò,[4] who find retardations of 26^h and $45^h.5$ respectively. The lags found by Maunder and Riccò, though both rest on the central meridian passage of sun-spots, are not directly comparable, for Maunder determined the lag to the middle of the magnetic storm and Riccò to the maximum.

In conclusion we express our indebtedness to Mr. Wallace for his instruction in the preparation and manipulation of the pan-iso plates.

YERKES OBSERVATORY
November 10, 1908

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[1] *Nature*, 7, 109, 1872.

[2] *Journal de Physique* (4), 3, 115 1904.

[3] *Monthly Notices*, 64, 207, 1903–1904.

[4] *Memorie Spet. Ital.*, 33, 38, 1904.

ON THE ABSORPTION OF LIGHT IN SPACE[1]

By J. C. KAPTEYN[2]

Undoubtedly one of the greatest difficulties, if not the greatest of all, in the way of obtaining an understanding of the real distribution of the stars in space, lies in our uncertainty about the amount of loss suffered by the light of the stars on its way to the observer. For the sake of brevity and in accordance with what has been done by other astronomers, I will designate this loss by the name of *absorption* of light, though the loss caused by *scattering* of light will be included.

Elsewhere (*Astronomical Journal*, No. 566, **24**, 115, 1904) I tried to show how fundamentally our results for the arrangement of the stars in space are changed by admitting even an absorption of light considered as small by some astronomers.

Now there can be little doubt, in my opinion, about the existence of absorption in space and I think that even a good guess as to the order of its amount can be made. For, first, we know that space contains an enormous mass of meteoric matter. This matter must necessarily intercept some part of the star-light.

In the second place, if, assuming the universe to be completely transparent, we apply the method explained in *Publications of the Groningen Laboratory*, No. 11, we find that the stars gradually thin out as we recede farther and farther from the sun. In *Astronomical Journal*, No. 566, I showed that the data used in this investigation are about as well represented if we assume that the star-density is the same at different distances from the sun, provided that we introduce an absorption of light of a determined small amount.

The choice between the two probabilities does not seem to be very difficult. For the data brought together in *Publications of the Groningen Laboratory*, No. 18, show that there is little difference in the distribution of the stars over the different magnitudes in diametrically opposite regions of the sky.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 31.

[2] Research Associate of the Mount Wilson Solar Observatory.

From this we may conclude that, if there is a thinning-out of the stars for increasing distance from the sun, it must be so in whatever direction from the sun we proceed.[1] This would assign to our sun a very exceptional place in the stellar system, viz., the place of maximum density.

On the other hand, if we assume that the thinning out of the stars is simply apparent and due to absorption of light, the apparent thinning-out in any arbitrary direction is perfectly natural.

The latter alternative is therefore undoubtedly the most probable one, and as there is, at least for every zone of galactic latitude, only one value (a) of the amount of absorption which would lead to a nearly constant density, whereas a greater amount would lead to the result that the star-density would *increase* on all sides of the sun (that is, would lead to the same difficulty as before), it would seem probable that the amount of absorption must be of the order a.

In *Astronomical Journal*, No. 566, I found for a the value of 0.016 magnitude for a distance of 33 light-years (corresponding to a parallax of 0".1) as an average for the whole of the sky. Using better material,[2] the result will doubtlessly be somewhat different. It is hardly necessary to say that I do not propose a value obtained in this way for acceptance. The method is by no means fundamental. I merely think that there is some reason to believe that the average absorption must be of this *order of magnitude*.

In the same paper I proposed a more fundamental method for the derivation of the absorption. The data in existence at the time, however, proved to be still inadequate. I have now collected far more extensive material which must lead to a considerable improvement. The plan of selected areas will, in due time, furnish still better data. Elsewhere I suggested another independent way of determining the absorption by extended observation of nebulae.[3]

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[1] The proof is not complete because different galactic latitudes were not separately treated. I have, however, convinced myself, in a rough way, that thinning-out of the stars is found for every zone of galactic latitude. This, together with the fact that the distribution of the stars varies very little with the galactic longitude, proves our conclusion.

[2] A beginning has been made in *Proceedings of the Academy of Science at Amsterdam*, 1908, p. 626.

[3] *Plan of Selected Areas*, p. 57.

Meanwhile it cannot be denied that the truly fundamental importance of the absorption of light makes it highly desirable to find still other methods,. depending on quite different data. Such a new method could probably be obtained if it turns out that the absorption in interstellar space is more or less *selective*. If the absorption and scattering of light by meteoric matter is really sensible, then there can be no reasonable doubt but that the violet end of the spectrum must be more strongly affected than the less refrangible rays.

But even the probability of gas-absorption, producing absorption lines or bands in the spectrum, cannot be denied a priori. Owing to the gas of the corona lost by the sun, to similar loss presumably suffered by the other stars, to that lost by comets, etc., interstellar space must contain, at every moment, a considerable amount of gas. Might not this gas, in a thickness of hundreds of light-years, cause an appreciable absorption of light?

It thus seems that there is reason to inquire whether or not light suffers in space (*a*) a general absorption, particularly strong in the violet; (*b*) a selective absorption, widening or strengthening lines or bands in the spectrum.

To my regret I have never had the opportunity of undertaking the observations necessary to settle these questions. Recently, however, I obtained some results from the Harvard observations, which can be explained most readily by an absorption of the first kind.

In the observations of the spectra of the bright stars (*Harvard Annals*, **28**, Part I) Miss Maury distinguishes, in Class XV*a*, two divisions. One of these has the remark 184, the other the remark 185. The parts of these remarks to which I wish to call attention are as follows:

"184. This star resembles *a Bootis* in the absorption shown in the region having wave-length shorter than 4307."

"185. This star resembles *a Cassiopeiae* in the general absorption shown in the region having wave-length shorter than 4307."

In the detailed description of the classification (p. 39) the difference is described in these words:

> The stars of this group (XV*a*) appear to fall into two divisions, exhibiting a slight difference in the degree of general absorption in the violet regions. Of these divisions *a Bootis* and *a Cassiopeiae* are respectively typical. In the first

the general absorption is slight; in the second, it is more conspicuous both in the regions of the violet above mentioned and beyond wave-length 3889, where the photographic spectrum generally appears to be suddenly cut off.

The distinction here made is a very difficult one in practice, owing to the fact that different photographs of the *same* stars show considerable difference in the general absorption of the violet. The difficulty is so great that Miss Cannon, in the classification of the southern stars, gave up this division altogether. She says (*Harvard Annals*, **28**, Part II, p. 159), "These effects were found to vary so much in different photographs of the same star taken under different conditions that they have been assumed to be photographic effects, rather than real."

On the other hand, Miss Maury contends (*ibid.*, Part I, p. 39), "The degree of absorption indicated also varies in photographs of different density so that, *while there seems to be sufficient ground for believing that the distinction is a real one, there is more or less liability to error in assigning individual stars to one or the other division.*"

I have taken the liberty of italicizing the words which. in order to appreciate what follows, we must keep well in mind. I have been led to think, with Miss Maury, *that the distinction is probably a real one* and that the most natural, though not the only way to account for it is to attribute it to *space-absorption*.

The suspicion that it might be due to space-absorption was of course at once raised by the preceding considerations. It is easy to see how it may be tested.

If it is well founded, then the stars belonging to the *a Cassiopeiae* division must be farther removed from the solar system than the stars of the *a Bootis* division. Our knowledge about individual parallaxes is far too defective to be of any use in the present question. In the absence of the parallaxes the amount of proper motion will be the safest guide in our estimate of the evidence. If our suspicion is in accord with nature, we have to expect that the stars in the *a Cassiopeiae* division will in general have smaller proper motions than the stars in the division of *a Bootis*.

The following summary shows the proper motions for all the stars in the two divisions. With very few exceptions they were taken from Newcomb's *Fundamental Catalogue*. Those which are not contained in that catalogue were taken from Auwers-Bradley as corrected in *Groningen Publications*, No. 9. All the proper motions may thus be considered as being very reliable.

To the stars in Class XV*a* were added nine stars from Class XIV and *XIV* for which, according to the remarks of Miss Maury, the

α *CASSIOPEIAE* GROUP

No. in *H. P.*	Star	Mag. in *H. P.*	α 1900	δ 1900	Spectrum	Cent. P. M.
45	*ι Ceti*	3.6	0^h14^m	− 9°23′	XV. *a*	3″
92	*δ Andromedae*	3.4	34	+30 19		17
94	*α Cassiopeiae*	2.2	35	+56 00		6
103	*β Ceti*	2.1	38	−18 32		23
122	*62 Piscium*	6.0	43	+ 6 45		7
155	*43 H Cephei*	4.5	55	+85 43		8
183	*η Ceti*	3.6	1 3	−10 43		25
220	*θ Ceti*	3.8	19	− 8 42		23
251	*υ Persei*	3.7	32	+48 07		13
333	*α Arietis*	2.0	2 2	+23 00		24
498	*κ Persei*	4.0	3 3	+44 29		23
560	*σ Persei*	4.4	24	+47 39		3
750	*δ Tauri*	4.0	4 17	+17 18		11
775	*θ′ Tauri*	3.9	23	+15 44		7
1452	*26 Monocerotis*	4.2	7 36	− 9 19		8
1453	*σ Geminorum*	4.1	37	+29 07		23
1629	*ζ Hydrae*	3.3	8 50	+ 6 19		9
1695	*1H Draconis*	4.6	9 23	+81 46		3
1760	*μ Leonis*	4.1	47	+26 29		24
1800	*λ Hydrae*	3.9	10 6	−11 51		22
1902	*46 Leon. min.*	3.9	48	+34 45		30
2056	*14 H′ Draconis*	5.8	12 0	+77 28		16
2063	*ε Corvi*	3.1	5	−22 03		7
2208	*ε Virginis*	3.0	57	+11 30		27
2569	*ι Draconis*	3.3	15 23	+59 19		1
2627	*α Serpentis*	2.7	39	+ 6 44		14
2962	*β Ophiuchi*	2.9	17 38	+ 4 36		16
3003	*ξ Draconis*	3.9	52	+56 55		13
3007	*ξ Herculis*	3.9	54	+29 15		9
3084	*δ Sagittarii*	2.8	18 15	−29 52		5
3328	*τ Draconis*	4.5	19 17	+73 10		17
3519	*19 Vulpeculae*	5.8	20 8	+26 31		0
3732	*ζ Cygni*	3.5	21 9	+29 49		6
3836	*ε Pegasi*	2.4	39	+ 9 25		2
3923	*ζ Cephei*	3.5	22 7	+57 43		2
4182	*γ Cephei*	3.4	23 35	+77 04		17
994	*β Leporis*	3.0	5 24	−20 51	XIV. *a*	9
1312	*ο¹ Canis majoris*	4.0	6 50	−24 04	*XIV. a*	4
1474	*ξ Argus*	3.4	7 45	−24 37	XIV. *P*	0
1747	*ε Leonis*	3.1	9 40	+24 14	XIV. *P*	5
2249	*γ Hydrae*	3.3	13 14	−22 39	*XIV. a*	8
2515	*β Bootis*	3.6	14 58	+40 47	*XIV. a*	6
3779	*ζ Capricorni*	3.8	21 21	−22 51	*XIV. P*	2
3780	*35 Capricorni*	6.0	21 22	−21 37	*XIV. a*	6
4020	*η Pegasi*	3.1	22 38	+29 42	XIV. *a*	4

α *BOOTIS* GROUP

No. in *H. P.*	Star	Mag. in *H. P.*	α 1900	δ 1900	Spectrum	Cent. P. M.
..	54 *Piscium*	6.1	0h34m	+20°43′	XV. *a*	60″
93	55 *Piscium*	5.5	35	+20 54		3
812	53 *Eridani*	3.9	4 34	−14 30		18
1086	δ *Leporis*	4.0	5 47	−20 54		69
1093	δ *Aurigae*	3.8	51	+54 17		16
1459	β *Geminorum*	1.1	7 39	+28 16		62
1823	γ *Leonis*	2.2	10 14	+20 21		34
1926	α *Urs. maj.*	2.0	58	+62 18		14
1961	ν *Urs. maj.*	3.8	11 13	+33 39		4
1963	δ *Crateris*	3.9	14	−14 14		23
1999	2 *Draconis*	5.5	30	+69 52		16
2400	α *Bootis*	0.0	14 11	+19 44		228
2541	δ *Bootis*	3.5	15 12	+33 42		16
2589	γ *Librae*	4.0	30	−14 27		7
2736	ι *Ophiuchi*	3.4	16 13	− 4 27		9
2774	β *Herculis*	2.8	26	+21 42		11
3037	70 *Ophiuchi*	4.1	18 00	+ 2 32		115
3090	η *Serpentis*	3.4	16	− 2 56		90
3447	ε *Draconis*	3.9	19 48	+70 01		9
3450	β *Aquilae*	4.0	50	+ 6 10		48
3648	ε *Cygni*	2.7	20 42	+33 36		49
3656	η *Cephei*	3.6	43	+61 26		83
4034	μ *Pegasi*	3.7	22 45	+24 05		16
4114	γ *Piscium*	3.8	23 12	+ 2 44		75
4174	λ *Andromedae*	3.9	33	+45 56		45

absorption for wave-lengths below 3889 approaches that of *a Cassiopeiae*.

From this summary we see that neither are the proper motions of the stars in the *a Cassiopeiae* division exclusively small, nor the motions of the *a Bootis* stars invariably considerable. After the words quoted from both Miss Cannon and Miss Maury such a thing could not be expected. Moreover, at least some of the proper motions of the *a Bootis* stars may only *appear* to be small because they are seen strongly foreshortened, and at least a few of the proper motions of the *a Cassiopeiae* stars must be large, *not* owing to small distance, but in consequence of an exceptionally large linear velocity. Finally, in classifying the stars into two divisions gradually merging into one another, there must always be a certain proportion so near the line of division that they might almost equally well be put in either class. In this respect it is worthy of notice that for Class XIV Miss Maury made no division on account of absorption in the violet. For this class we may thus presume that she will have drawn attention *only* to the well-marked cases. For this reason the fact that in *all* these cases, without exception, the proper motion is small seems very significant. Notwithstanding the contrary cases, our list shows clearly that, *as a rule*, the proper motions in the *a Bootis* division exceed those in the other group very considerably.

The following averages may show this yet more forcibly.

		a Cassiopeiae div.	*a Bootis* div.
General mean of centennial motion		11″.4 (45 stars)	47″.1 (25 stars)
Partial means	$\alpha = 0^h$ to $\alpha = 6^h$	13.6 (15 stars)	33.0 (5 stars)
	6 to 12	12.9 (10 stars)	25.4 (6 stars)
	12 to 18	11.9 (10 stars)	54.1 (6 stars)
	18 to 24	6.1 (10 stars)	58.9 (9 stars)
Number of stars with centennial p.m. $< 10''$		58 per cent.	20 per cent.
Number of stars with centennial p.m. $> 30''$		0 per cent.	48 per cent.

The percentages become still more favorable to our supposition if we include only the cases in which two or more, or three or more plates have been taken. As the number of stars becomes rather small in these cases I will not further insist on this point.

The objection which might be based on the fact that the subdivision here discussed was made only in the case of Class XV*a*, whereas, if the interpretation by space-absorption is correct, it must show in all classes, loses greatly in force by the fact that all other classes of Miss Maury contain so small a number of stars that the contrast between the two divisions had small chance of being noticed.

If we grant the reality of the difference in the distance of the stars of the *a Bootis* and *a Cassiopeiae* divisions, it does not necessarily follow that the interpretation of the difference in the spectrum as being due to space-absorption must be correct. For, as the average magnitude of the stars in the two divisions is about the same, the total light-power (luminosity) of the more remote *a Cassiopeiae* stars must be greater than that of the *a Bootis* stars. The meaning of our result may therefore also be: for stars of the same spectrum, the light of the more luminous objects is relatively weaker in the violet part of the spectrum. Whichever view we think the more probable, the phenomenon seems well worthy of being more closely and carefully investigated.

Of course care will be taken to make the spectra to be compared as equal as possible in the less refrangible part. Moreover, we shall choose exclusively extreme cases in regard to proper motion. In particular, fainter stars ought to be included in order that we may have a more extensive choice of stars having large proper motions and at the same time a larger difference of distance of the two groups to be compared. It seems unnecessary to enter into further details on this point. But a few lines may perhaps be allowable in order to show the importance of a determination of the space-absorption which might eventually result from such an investigation.

There is first, of course, the contribution we may expect for the determination of the *total* amount of absorption (the absorption-*constant* in the case the universe is fairly uniformly filled with the absorbing matter). As was pointed out above, such a determination would probably enable us to get rid of the main obstacle in the way of a reliable determination of the real distribution of stars in space. Then there is the fact that we shall get a *criterion* or even a *measure* of distance, which promises to be of particular value where other means begin to fail. To see this clearly we have only to consider how uncertain our determinations of average parallaxes, even if obtained by means of the parallactic motion, become for stars for which the distance exceeds, say, 3000 light-years. In the present state of science the discrimination between a distance of 3000 and a distance of 6000 light-years must be considered as practically impossible. On the contrary, if space is somewhat uniformly filled with matter, the difference between the absorption for the two classes of stars must be equal to that between stars at the respective distance zero and 3000 light-years.

Further, there is the importance of getting an insight into the true spectrum of the stars, freed from the changes brought about by the medium traversed by light on its way to the observer. Thus, for instance, is it not possible that the somewhat strange phenomenon, that, according to Huggins, stars of Secchi's second type often show spectra which, in the more refrangible part, are more intense and more extensive than in the spectra of the first type, is simply due to space-absorption? For it is well known that the average distance of the stars of the second type is considerably smaller than that of the stars of the first type of equal magnitude.

Even more important than the general absorption here discussed would be a gas-absorption, producing *space-lines* or *space-bands.* Indications of such an absorption might be obtained as above by comparing stars having spectra of the same class but of widely different proper motion. There would be, however, a decisive criterion by which we might prove or disprove the reality of our results. If there are space-lines, they must not share in that part of the radial motion which is due to the motion of the stars themselves. The space-lines must thus show their nature in somewhat the same way as the atmospheric lines of the solar spectrum in Cornu's well-known experiment.

As, however, I have no evidence as to the real occurrence of such lines or bands, no more need be said about them at present. I am permitted to say in conclusion, that in the program of the stellar work for the five-foot reflector of the Mount Wilson Solar Observatory some stars chosen with special reference to the subject of the present paper will be included.

Mount Wilson Solar Observatory
October 1908

SPECTRUM OF COMET MOREHOUSE (1908 *c*)

By EDWIN B. FROST and J. A. PARKHURST

Our intention to study this spectrum with the Zeiss doublet and objective-prism was put into effect on October 28. The ease with which a strong impression was obtained on a photographic plate showed that we might have begun observations considerably earlier, when the comet was much less bright. At that date an exposure of fifteen minutes was adequate to give distinctly the spectrum of the head as a row of seven knots. With an exposure of half an hour most of these extended out into the tail for at least 3°. On many of the later exposures the images of the tail ran off the plate, the edge of which was 3°5 from the head.

The scale of the spectrum is exceedingly small, as the focal length of the objective is only 81.4 cm (aperture 14.5 cm) and the prism has an angle of but 15°. The distance from $H\beta$ (λ 4861) to $H\theta$ (λ 3798) in a stellar spectrum is only 3.0 mm, whence it is evident that but a slight degree of accuracy can be expected in the determination of wave-lengths. All the optical parts are of the ultra-violet ("U.V.") glass of Schott & Co., so that the transparency is excellent for the more refrangible portions of the spectrum.

Twenty-one satisfactory plates were obtained with the objective-prism on eleven dates, ending with December 2, when the comet was too far west for further observations.

As the identification of the knots in the spectrum could only be made by assuming the wave-length of some one of them, we attempted to get plates with a slit-spectrograph, attaching to the same telescope the small quartz spectrograph designed for use with the two-foot reflector. This instrument has collimator and camera lenses of "U.V." glass, of aperture 2 cm and focal length 15 cm, and a Cornu quartz prism of 60° angle. The scale of the plates is about one-sixth smaller than with the objective-prism.

Comparison spectra of helium and hydrogen were tried; the small tube of helium specially adapted for the apparatus unfortunately gave very strongly the spectrum of the negative pole of nitrogen, largely masking the helium lines; the hydrogen tube could not be attached very conveniently. Metallic comparison spectra, for which the apparatus is arranged, cannot be well employed, as the lines run together too much with the wide slit necessary for the cometary spectrum.

We obtained our best results with the use of *Vega*, then not far from the comet, by allowing the star to trail along the portion of the length of the slit not occupied by the comet's head. The cross-wires in the guiding telescope made it possible to put the star at a desired position in the slit with considerable accuracy. The wave-lengths of the cometary bands were thus established with a better degree of precision than we had expected in view of the wide slit and small scale. Satisfactory plates were obtained with the quartz spectrograph on four nights, with exposures of from one to three hours, as indicated in the journal of observations. All of the plates were taken by Parkhurst with such assistance from Frost as was occasionally necessary.

On the evening of October 30 an exposure of 158 minutes was made by Frost and Barrett with the Bruce spectrograph (used with one prism) attached to the 40-inch refractor. Guiding was very difficult from the extreme faintness of the comet in the long-focus finder ($a=$ 10 cm, $f=19$ meters). No trace of an impression due to the comet could be detected on the plate. The great refractor, with its long focal length, is obviously unsuited for such an object.

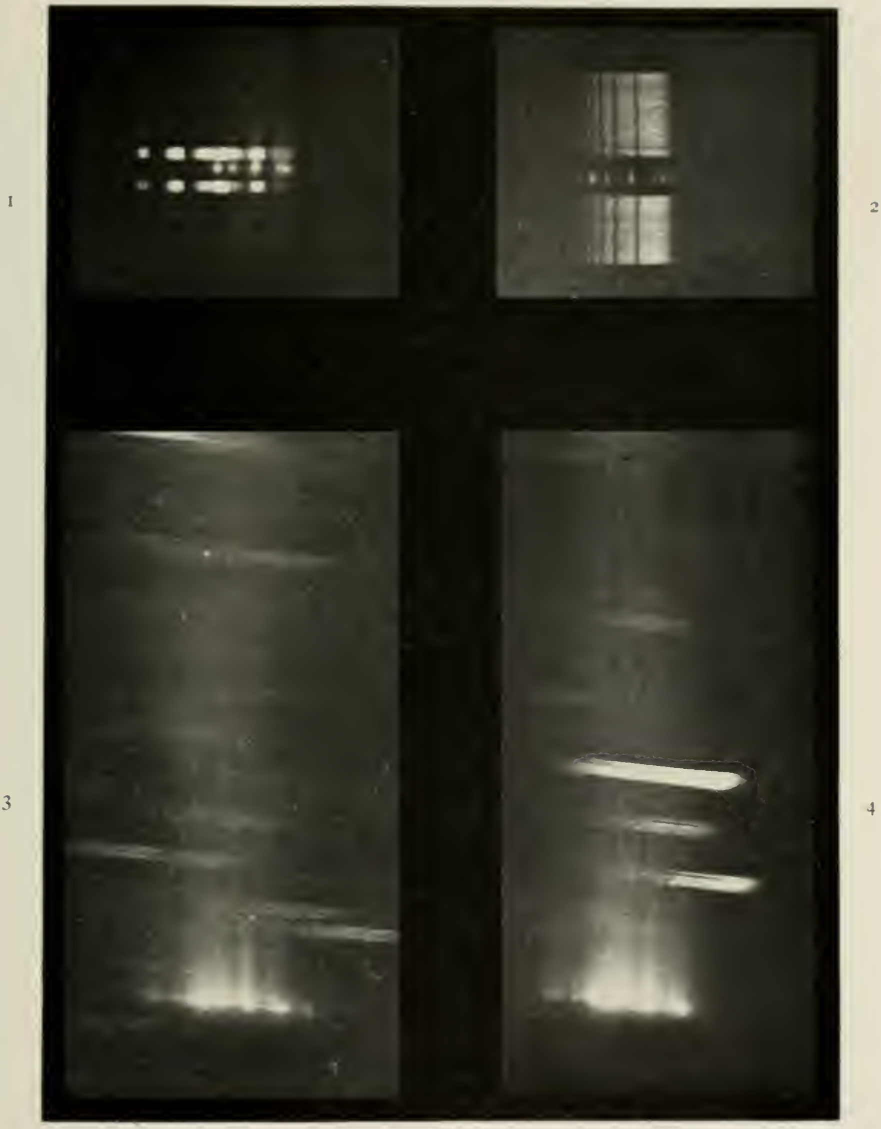
In the column "Plate," S refers to the Seed "27;" Σ to the Lumière "Sigma" plate, of which recent emulsions show very high speed; and P-i denotes a Seed "27" bathed according to the Wallace "Pan-iso"formula.

In examining the first plate we were struck by the lack of a continuous spectrum, and a careful search for it has been made on all the plates. We can state that there is no certain evidence of its presence on any plate, whence we must infer that at this period the amount of reflected light was very small relatively to the intrinsic light from the radiations of carbon, cyanogen, and in the unidentified bands.

The general appearance of the spectrum is shown by Plates IV and V. The knots do not present the impression of bands, being images

PLATE IV

SLIT-SPECTOGRAPH



OBJECTIVE-PRISM

Spectrum of Comet Morehouse

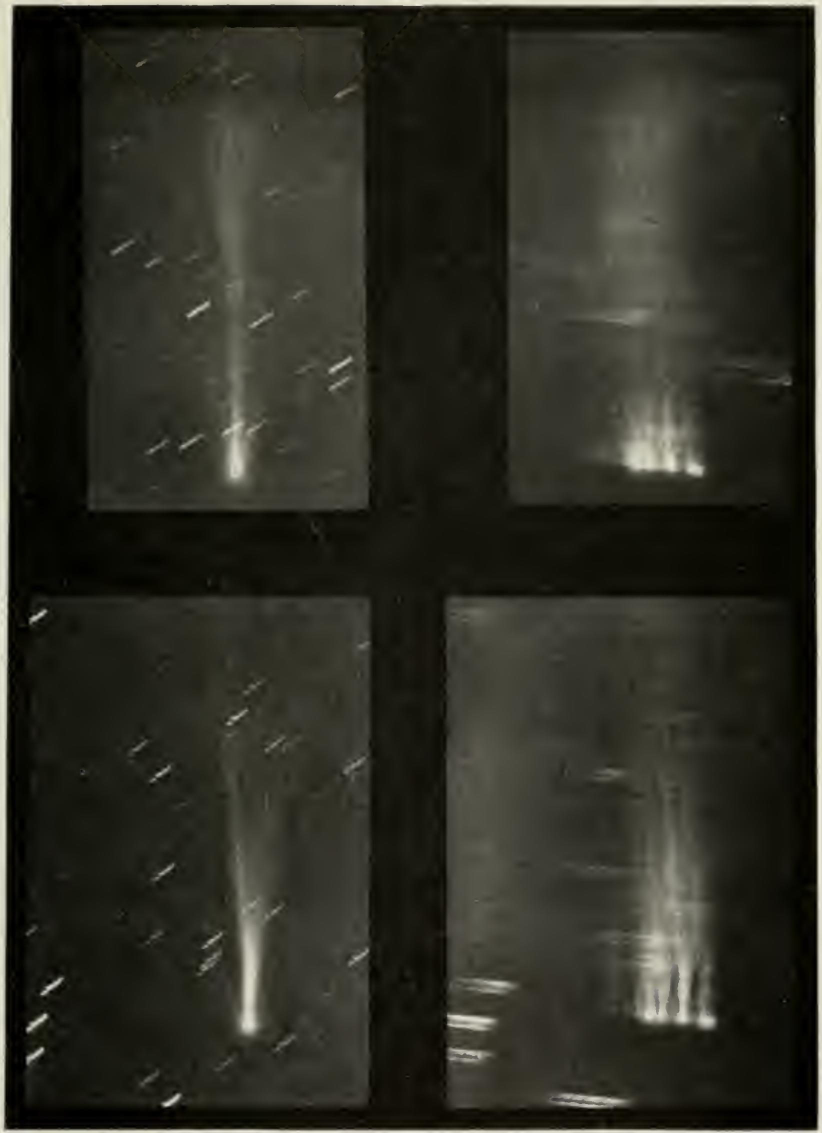
1. Nov. 4, $6^h\ 0^m$ to $9^h\ 0^m$
Comparison: *He* and *Ni*

2. Nov. 18, $5^h\ 30^m$ to $8^h\ 0^m$
Comparison: *Vega*

3. Oct. 30, $6^h\ 35^m$ to $7^h\ 11^m$
"Pan-iso" plate

4. Nov. 19, $5^h\ 40^m$ to $6^h\ 40^m$
Σ plate

PLATE V



BRUCE TELESCOPE, 6-INCH LENS OBJECTIVE PRISM

DIRECT AND SPECTRAL IMAGES OF COMET MOREHOUSE

3. Oct. 30, 9^{h} 20^{m} to 10^{h} 30^{m} 4. Oct. 30, 8^{h} 56^{m} to 9^{h} 57^{m}

1. Oct. 29, 8^{h} 56^{m} to 10^{h} 0^{m} 2. Oct. 29, 8^{h} 47^{m} to 9^{h} 21^{m}

of the comet's entire head; with the quartz spectrograph and narrower slit, they become elongated, and also extend into the tail. Plate IV, Fig. 1, reproduces spectrogram Q 221, with comparison spectrum from helium tube, yielding chiefly the nitrogen bands. On the negative the tail appears above the comparison spectrum. In Fig. 2, from plate Q 226, the comparison spectrum was supplied by *Vega*. $H\beta$ is just visible at the right edge of the star's spectrum. The slit was here 0.07 mm wide, or about one-fourth as wide as in Fig. 1. The comet's bands here look like tadpoles, the extension upward being of course due to the comet's tail. Fig. 3 of Plate IV is given to show the knot in the green at λ 5165 as it appears on OP 220, taken on a plate bathed by the Wallace "Pan-iso" formula.

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SLIT-SPECTROGRAPH PLATES

No.	Date	Begun C. S. T.	Ended C. S. T.	Plate	Slit-Width	Comparison Spectrum
	1908				mm	
Q 221....	Nov. 4	$6^h\ 0^m$	$9^h\ 0^m$	Σ	0.27	Helium tube
224....	16	5 39	7 49	Σ	0.07	*Vega* trails
225....	17	6 0	8 15	Σ	0.07	*Vega* trails
226....	18	5 30	8 0	Σ	0.07	*Vega* trails

OBJECTIVE-PRISM PLATES

No.	Date	Begun	Ended	Plate	Remarks
	1908				
OP 212...	Oct. 28	$6^h\ 15^m$	$7^h\ 15^m$	S	Good, streamers traced 1°
213...		7 25	8 25	Σ	Strong, streamers traced 2°
214...		8 52	9 17	S	Weak, thick sky
215...	29	6 22	6 40	S	Weak, thick sky
216...		7 5	7 40	Σ	Strong, fair
217...		8 47	9 21	Σ	Best plate
218...		9 25	9 58	P-i	Knot λ 5165 shown
219...	30	6 8	6 29	Σ	Strong good
220...		6 35	7 11	P-i	Strong, good
221...		8 23	8 53	S	3 exposures, 5, 10, 15^m
222...		8 56	9 57	Σ	Very strong
223...	Nov. 5	6 7	6 22	Σ	Weak, moonlight
224...		6 24	6 34	Σ	Weak, moonlight
226...	9	7 23	7 35	Σ	Weak, moonlight
227...	11	6 22	6 57	Σ	Strong, good
229...	19	5 40	6 40	Σ	Strong, good
230...		6 42	7 6	Σ	2 exposures, 7, 16^m
232...	21	5 36	6 6	S	Very good plate
235...	28	5 27	5 49	S	Fair, short exposure, moon
238...	Dec. 1	5 30	5 50	S	Fair, short exposure, moon
239...	2	5 24	5 44	S	Fair, short exposure, moon

MEASUREMENTS

The plates were measured independently by the two observers with different machines; those taken with the slit-spectrograph were given the greater attention. The wave-lengths are referred to the hydrogen lines of *Vega*. Owing to photographic irradiation, dependent upon the intensity of the image, the edge of a knot is a very uncertain point to set upon, in no case a sharp edge. It was often impossible to set upon anything but the middle of a knot, but where feasible, settings were made on both edges.

The mode of reduction was also independent for the two observers. Frost employed a Hartmann formula derived from settings on $H\beta$, $H\gamma$, and $H\theta$, for the spectrograph plates, and similar formulae for the objective-prism plates from stellar spectra; for the ultra-violet region the formula was derived from $H\epsilon$, $H\eta$, and $H\mu$. The wave-length 4268, as obtained with the spectrograph for the center of the most symmetrical knot, was assumed for all objective-prism plates, of which Frost measured the best seven. The deviation of the separate values from the mean given in the table is less than might be expected; for instance, on the edge at λ 474, it is 7 tenth-meters; for the centers of the four best-defined knots (λ 4268 necessarily excluded), it is 3.5 tenth-meters.

Parkhurst derived his wave-lengths from a curve drawn from the stellar hydrogen lines. He measured ten of the best objective-prism plates in one direction, setting on the centers of the knots, and reduced by a curve from lines in the spectrum of *Vega*, then adjusting the wave-lengths to the system of the slit-spectrograph by using the wave-lengths of the five best-defined knots, λ 4716, 4601, 4268, 4015, and 3883. The agreement of these measures of the objective-prism plates may be regarded as satisfactory, the residuals from the mean of the ten plates averaging 5 tenth-meters, and the largest value, for a poor line, being 17 tenth-meters. The wave-lengths at the extreme ends of the spectrum are extrapolations and subject to some additional uncertainty, but they have been checked by measures on stellar plates extending to those regions.

In addition to the bands tabulated above, faint and doubtful lines were measured as follows on some of the objective-prism plates: by F. on four plates at λ 411, 415, 413, 414; by P. on three plates at λ 413, 413, 413, mean λ 413. F. also measured a marking on three plates at λ 4369, 4390, 4376, mean λ 4378. We do not identify these.

In forming the "adopted wave-length," we have given to the determinations with the slit-spectrograph a weight of 3, the mean of the results of the two observers with the objective prism receiving a weight of 1.

We avoid the still unsettled question of the precise origin of the "carbon" bands (of the so called "Swan" spectrum) which have been so often ascribed to a hydrocarbon, specifically acetylene, and we use for them the simple designation "carbon."

The presence of the third and fourth bands on our plates seems to be beyond a doubt; the absence of the fifth band seems equally sure. The third cyanogen band is unquestionably present, and the fourth and first also appear to be assured; but the second is certainly lacking.

MEASURES OF PLATES WITH SLIT-SPECTROGRAPH

Measures on	Q 224		Q 225		Q 226		Mean
	F.	P.	F.	P.	F.	P.	
Edge	4744	4753	4734		4731	4743	4741
Middle		4720	4718	4721	4715	4709	4717
Edge	4679	4680			4697	4665	4680
Edge	4591	4602			4584	4592	4592
Middle		4571	4544		4568	4560	4561
Edge	4548	4542				4520	4537
Edge	4286	4291	4279	4290	4284	4293	4287
Middle	4267	4268	4265	4270	4262	4271	4267
Edge	4248	4248	4252	4250	4242	4240	4247
Edge	4020	4031	4020		4022	4041	4027
Middle	4014	4016	4012	4012	4013	4020	4014
Edge	4004	4002				3992	3999
Edge	3919	3924					3922
Middle		3915			3907	3921	3914
Edge		3905					3905
Edge	3894	3900			3886	3909	3897
Middle	3877	3884		3882	3875	3893	3882
Edge	3866	3869			3866	3872	3868
Edge	3801				3810		3806
Middle	3792	3790	3799		3796	3800	3795
Edge							

SUMMARY OF RESULTS

Band	Slit-Spectrograph Mean of F. and P.	Objective-Prism		Adopted Wave-Length	Probable Identification
		F.	P.		
Edge		5138			Third "Carbon" Band
Middle		5064	5086	5075	Edge at λ 5165.3
Edge		5013			(Kayser and Runge)
Edge	4741	4739		4741	Fourth "Carbon" Band
Middle	4717	4688	4699	4711	Edge at λ 4737.2
Edge	4680	4647		4672	(Kayser and Runge)
Edge	4592	4601		4594	First *CN* Band
Middle	4561	4556	4565	4561	Edge at λ 4606.3
Edge	4537	4518		4532	(Crew and Basquin)
Edge	4287	4310		4293	
Middle	4267	4267	4276	4268	?
Edge	4247	4224		4241	
Edge	4027	4049		4032	
Middle	4014	4024	4027	4017	?
Edge	3999	3998		3999	
Edge	3922	3931		3924	
Middle	3914			3914	?
Edge	3905			3905	
Edge	3897	3916		3902	Third *CN* Band
Middle	3882	3887	3887	3883	Edge at λ 3883.6
Edge	3868	3862		3867	(Kayser and Runge)
Edge	3806	3811		3807	
Middle	3795	3792	3790	3794	?
Edge		3776			
Edge		3703			
Middle		3687	3686	3686	?
Edge		3677			
Edge		3609	3594	3602	Fourth *CN* Band
Middle		3540	3528	3534	Edge at λ 3590.5
Edge		3510	3509	3510	(Kayser and Runge)

Attention may properly be drawn to the fact that the only lines in this region in the "line spectrum" of carbon are at λ 4267 and λ 3920, but this cannot be regarded as an identification. The edge of the first of these is at about 431, in agreement with a line photographed by Campbell[1] in comet 1893 II, falling at the edge of the violet carbon band at λ 4312 (Ångström and Thalén). The line at about λ 437 may be identical with a bright line photographed by Campbell in that comet at λ 4366.

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[1] *Astronomy and Astrophysics*, **12**, 652, 1893.

We are aware that our results differ considerably from those published while our observations were in progress by MM. de la Baume Pluvinel and Baldet,[1] and by MM. Deslandres and Bernard,[2] who also employed prismatic cameras. The first named observers agree with us in detecting no trace of continuous spectrum (dates reported, October 4, 5, and 7), and in seeing on the plate seven distinct monochromatic images of the comet. Their wave lengths differ, however, materially from ours: for the first two knots being more than 100 tenth-meters less. For the knots of shorter wave-length the agreement is better. Their values are: λ 465–458; 448; 421; 397; 388–385; 376; 367. They regard the "carbon" bands as absent: "Ce qui frappe tout d'abord dans ce spectre, c'est l'absence des raies du spectre des hydrocarbures. La comète Morehouse semble donc faire exception a la règle générale, car on sait que les spectres cométaires présentent toujours les bandes des hydrocarbures." Per contra, they regard the cyanogen bands as fully represented: "Si le spectre des hydrocarbures fait défaut, le spectre du cyanogène se trouve au complet." We differ from them here in being unable to detect a trace of the second *CN* band, with its edge at λ 4216.

When they used a prism giving increased dispersion they found that all the images of the comet became double. We have examined our plates carefully for such duplication, particularly since these papers appeared in *Comptes Rendus*, but we cannot detect it, although on one or two plates there was a suggestion of duplicity at the band with center at λ 402.

MM. Deslandres and Bernard are unable to establish with certainty any trace of the "carbon" bands: "mais nous n'aurons pu constater aucune trace certaine des bandes des hydrocarbures, émises en général fortement par les comètes, surtout dans le vert." They also find a continuous spectrum on all of their plates (observations beginning on October 14): "il est visible dans la tête, et dans la queue jusqu'à une grande distance; mais il a une intensité relativement bien moindre que dans le comète Daniel."

The wave-lengths published by MM. Deslandres and Bernard are in excellent agreement with our determinations: our difference in interpretation or identification therefore represents a difference in

[1] *Comptes Rendus*, **147**, 666, October 19, 1908.

[2] *Ibid.*, **147**, 774, November 2, 1908.

judgment. As it is well established that measurements on cometary bands are likely to be systematically too small, we regard it as highly probable that a band measured at 4713 really has its edge at λ 4737, hence is the fourth carbon band.[1] Their values are as follows:

4713	4279	3881
4688	4256	3869
466	4232?	380
		3784
4572	4023	368
455	4003	356
4524	3914	333?

The bands with centers at λ 4561, 4267, and 4013 were also observed by them in comet Daniel, and similarly by Chrétien and by Evershed.

The relation between the knots and streamers shown on the objective-prism plates, and the corresponding direct photographs, is quite clearly shown in Plate V, where the spectrum plates of October 29 and 30 and the nearly simultaneous direct photographs taken by Professor Barnard with the Bruce camera are placed side by side. The focal lengths of the Bruce and Zeiss lenses are nearly the same, 78.4 cm and 81.4 cm respectively; and the engravings are on the same scale. The pictures would then be directly comparable were it not for the fact that the exposures with the Bruce were somewhat longer than with the Zeiss, and the light was concentrated in one image; so that the Bruce images are enlarged to a greater extent by photographic irradiation, which must be allowed for in making the comparison.

The relative intensities of the brighter head-knots cannot be inferred from the engravings, as the exposures were too long; but shorter exposures, for example on plates 221, 230, and 232, show that the knot at λ 3883 is the strongest photographically, followed in order by the knots at λ 4268, 4711, 4017, 4561, 3914, 3794, 3534, and 3686.

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[1] A reference to Campbell's paper on the spectrum of Daniel's comet (*Astrophysical Journal*, **28**, 234, October 1908) shows that on plates taken with a slit-spectrograph with much higher dispersion, the same description "very bright, well defined" is applied to the first four edges of the fourth carbon band at λ 4737, 4715, 4697, and 4684. If these four bright radiations were run together by a low dispersion, the wave-length of the center would fall quite close to 4710. Is it impossible that some of the duplicities or complexities reported in the bands are due to the resolution of the additional edges?

This order also holds for the slit-spectrograph plates. On the Paniso plates the knot in the green at λ 5165 is about as bright as that at λ 4601. The photographs give no grounds for suspecting any change in the above order during the period of 33 days covered by the observations.

In tracing the relation of the tail-streamers to the knots, we are struck at first glance by the fact that the streamer proceeding from knot λ 4711, though certainly present, is very weak compared to the streamers from the fainter knots λ 4017 and 4561. A closer examination shows that the strongest knot of all, λ 3883, is also deficient in tail-material, the streamer which seems to accompany it in reality flowing mainly from the weaker knot λ 3914. Also these two knots deficient in tail-material extend farther on the side opposite the tail (i. e., toward the sun) than the other knots. The chemical origin given for the spectrum bands causing these knots does not explain the reason for these peculiarities.

The tracing of the separate streamers out into the tail is made difficult by their interlacing, but on the best negatives, OP Nos. 217, 222, and 229, they can be traced more than 2° from the head, *following the bends in the tail*. In Fig. 1, Plate V, the direct photograph shows two sharp streamers diverging 20° from the main tail on each side. These streamers can be seen on the corresponding objective-prism negative, diverging from the knot λ 4267, thus showing that particles of the same constitution are thrown out from the head at angles at least 40° asunder. The bearing of these facts on the theory of the constitution of the tail is evident.

With the two exceptions already noted, the relative intensities of the knots and their corresponding streamers are similar if not identical. Also the different plates which are similar in quality show practically identical spectra; so that, in spite of the extraordinary activity shown by this comet, the spectrum seems to have remained unchanged throughout the period covered by these observations.

It is obvious that comet Morehouse exhibited considerable differences in its spectrum from comet Daniel of the preceding year, one important effect of which was to make it photographically active. Such differences in comets are to be expected from past experience, and this emphasizes the importance of securing observations of faint comets with objective-prisms at the earliest date when an impression can be secured. M. de la Baume Pluvinel seems to have been the first in recent years to call attention to the utility of this mode of observation, in his excellent paper[1] on comet 1902 *b*. His example does not seem to have been sufficiently followed, however, until Daniel's comet was investigated in this way by the observers already mentioned in this paper.

With a sufficiently long guiding telescope, it is possible before or after the exposure on the comet to place a neighboring star in a position where it may furnish with the prismatic camera a comparison spectrum, from which cometary wave-lengths may be inferred with some degree of accuracy without any assumptions. Of course the accuracy of the wave-lengths cannot compare with that yielded by a powerful slit-spectrograph. Besides having great light-power, the prismatic camera is alone capable of depicting the spectral images of the comet's tail and of showing the distribution therein of the constituent radiations. This is particularly important when internal changes are occurring rapidly, and masses of material are moving in the tail. This simple instrument is then wholly competent to test Bredichin's theory of a different chemical composition of differently curved tails. It may be remarked, incidentally, that the comparatively straight tail of comet Morehouse should have shown the spectrum of hydrogen, according to that theory, instead of that of "carbon" and cyanogen.

A further test of the theory will be readily given if plates were obtained anywhere with the prismatic camera on September 16 (or possibly on the days just preceding) when the tail was violently curved at a large angle from its direction before and after that date. The manner in which the "carbon" and cyanogen radiations follow the convolutions of the tail, as shown on Plates IV and V, make it seem a priori unlikely that an element not then showing its characteristic spectrum should do so when the deflection of the tail changes.

In the preparations for Halley's comet, it is therefore important that objective-prisms should be provided for the photographic doublets intended for its observation.

Yerkes Observatory
December 1908

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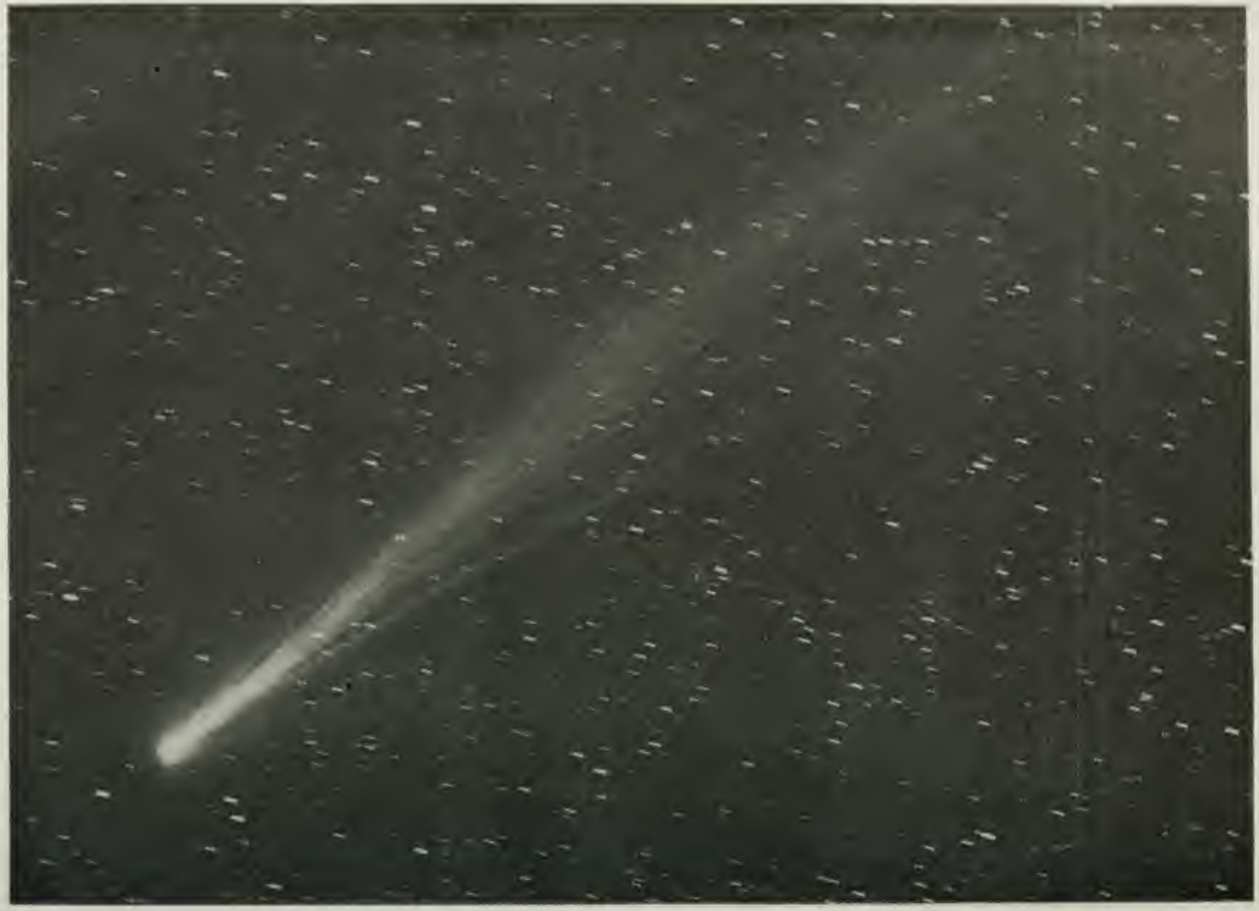
[1] *Comptes Rendus*, 136, 743, 1903.

PLATE VI

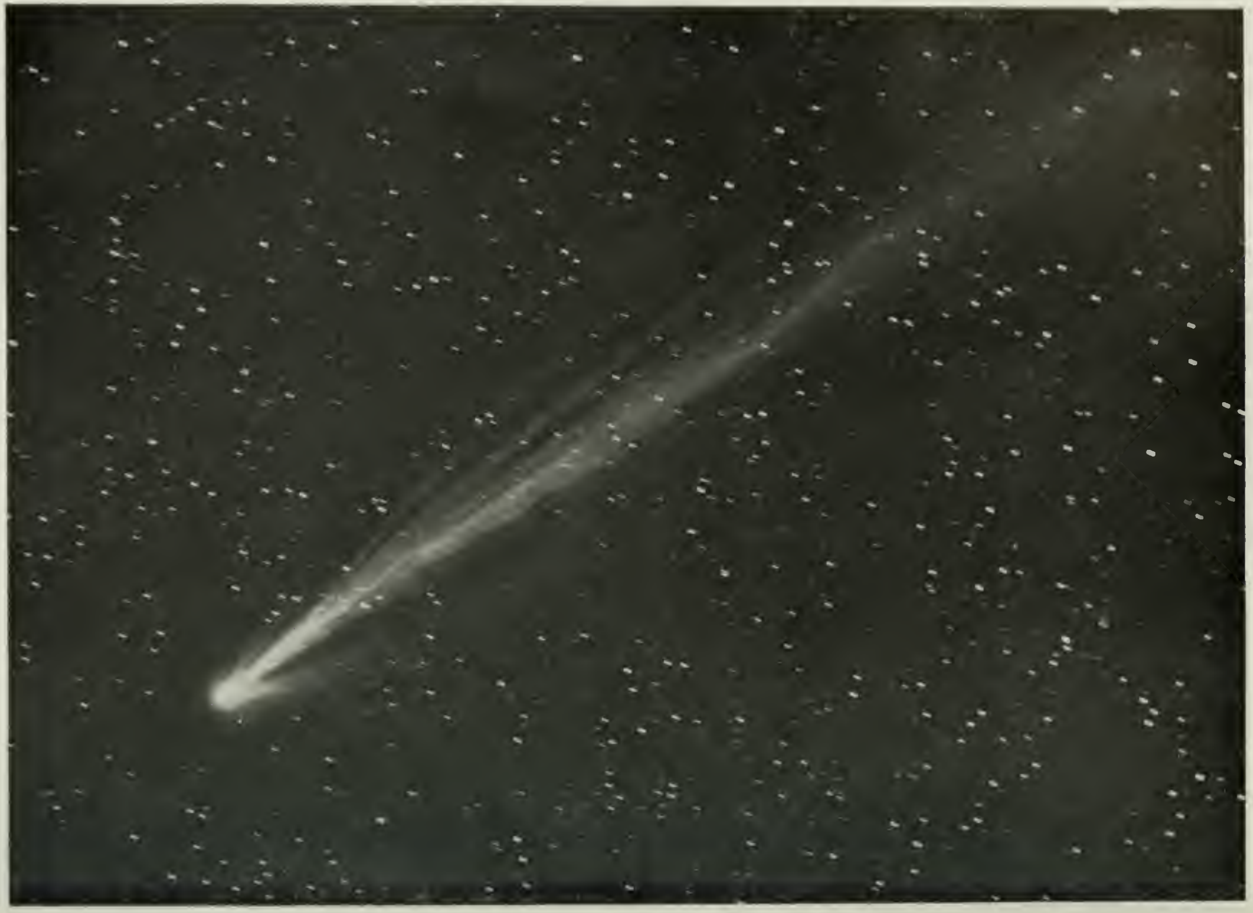
South



COMET 1908 *c* (MOREHOUSE) ON NOVEMBER 14, 1908, AT 6^{h} 47^{m} C. S. T. EXPOSURE 1^{h} 2^{m}
10-Inch Lens of Bruce Telescope. Scale: 1 cm=$0\overset{\circ}{.}30$

PLATE VII

South



Comet 1908 *c* (Morehouse) on November 15, 1908, at 6^h 6^m C. S. T. Exposure 0^h 40^m
10-Inch Lens of Bruce Telescope. Scale: 1 cm=0°.30

PHOTOGRAPHIC OBSERVATIONS OF COMET *c* 1908 (MOREHOUSE)

THIRD PAPER

By E. E. BARNARD

Since the two previous papers were written (see this *Journal* for November and December) the comet has presented a rather different series of phenomena, marked mainly by long, slender streamers at small angles to the tail and by convoluted forms in the tail itself. The previous pictures of it were not remarkable for individual long streamers or narrow rays. The new phenomena appeared about the middle of November. They gave the comet a very beautiful appearance, quite different from almost any of its previous aspects, and have added much to the interest of this remarkable body.

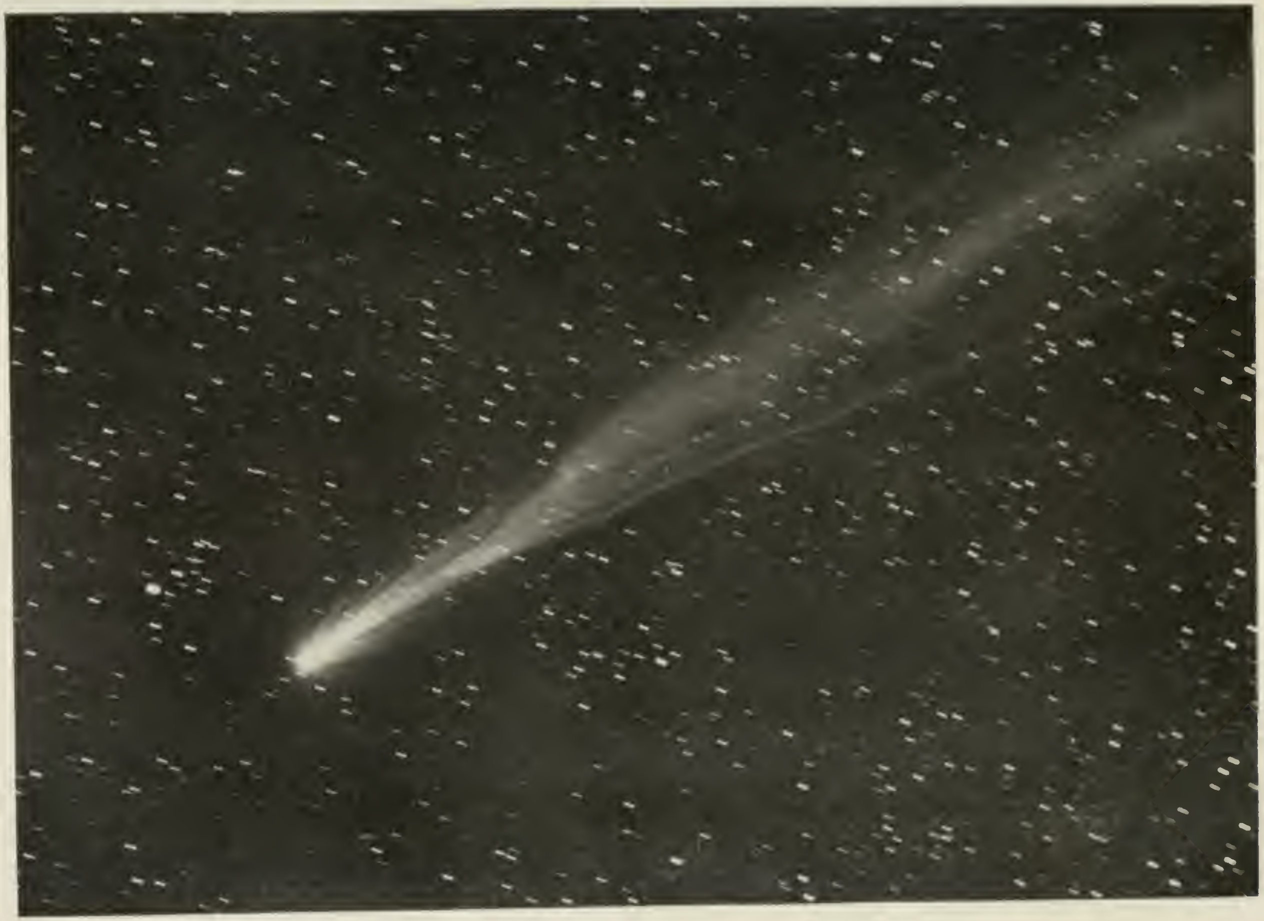
These new features are perhaps best shown on the pictures of November 14, 15, 16, 17, 18, and 19. Some of these rays are so straight and slender that they give the impression of being shot out with immense velocity. This is especially the case in the photograph of November 14, where a slender straight streamer apparently emanating in the north side of the tail, near the head, shoots out for about five degrees at an angle of 10° to the axis of the tail. The body of the comet is strongly undulating, like a ribbon waving in the wind, and has almost the shape of a corkscrew near the head.

The most remarkable of these photographs, however, though not the most beautiful, was obtained on November 15. In this picture the main tail widens out from a narrow neck attached to a rather small head, from each side of which broad diffused brushes of light issue at considerable angles to the tail. The ones on the north side extend for several degrees. On the south side of the comet two slender streams, apparently originating in the main tail, some distance back of the head, run back at a slight angle for several degrees. The outer one of these, at a distance of about 2°, rather suddenly makes a considerable bend southward as though something had deflected its course; it then seemingly runs back again. A similar appearance is presented by the inner stream at a greater distance out. This last streamer is streaked along its middle with brighter strips. The main tail is brighter and full of structure, and at a distance of 5° from the head it bends northward at an angle of about 10° in a great curve that suggests a possible return to its original direction. Its north side is strongly undulating. At about two or three degrees from the head, in the middle of the tail, is a long, narrow, curved, bright strip, which is probably a streamer seen in projection; this has the same kind of deflection as the two south streamers.

In the photographs of November 17 the tail is full of irregular bright masses and there is a general convexity on the south side, in the direction of motion. A very slender curved line runs parallel to the north side and seems to begin at the edge of the tail 2° from the head. It can be traced to a distance of about 8°. On each side of the head a slender streamer is visible for about 2°. The tail connects with the head by a very slender sinuous neck. There are decided changes in the comet in the interval of 1¼ hours between the two photographs on this date.

The original photograph of November 19 is an exquisite picture. The tail seems to be made up of sheetlike streams which are very sharp at their edges; the neck, near the head, has a twisted aspect. Of a similar nature and equally beautiful is the photograph of November 16. That of November 18 is also very beautiful. On this last date great numbers of delicate streamers shoot out symmetrically from the head for several degrees. The main body of the tail is streaked and undulating and several degrees back it brightens up in curved masses. The entire comet is rich with sharp details and is a very striking object. Indeed, one of the most remarkable things about the later pictures of this comet has been the extreme sharpness of the structural details near the head. In this respect they are quite different from our general ideas of a comet.

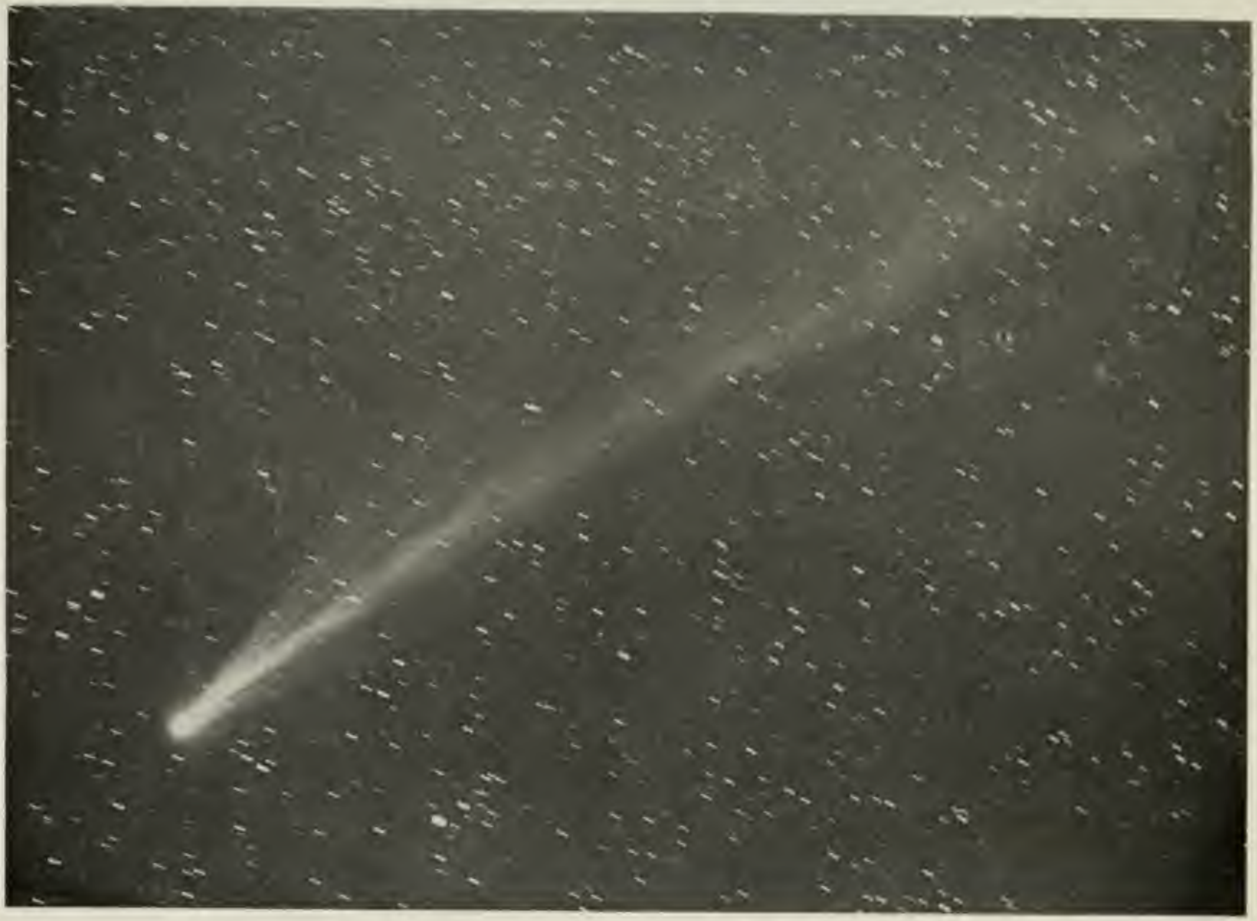
In any attempt to explain certain of these features of the comet we are forced to tread on very dangerous ground, for there is nothing in the adopted theories of comets that will explain them, and we must assume unknown quantities that perhaps violate all our ideas of the conditions existing in space in the vicinity of the sun and planets. But, as there seems to be no other explanation, the very fact that we



Comet 1908 *c* (Morehouse) on November 16, 1908, at $7^h\ 10^m$ C. S. T. Exposure $1^h\ 3^m$
10-Inch Lens of Bruce Telescope. Scale: 1 cm = 0°.30

PLATE IX

South



COMET 1908 *c* (MOREHOUSE) ON NOVEMBER 18, 1908, AT $7^h\ 25^m$ C. S. T. EXPOSURE $1^h\ 10^m$
10-Inch Lens of Bruce Telescope. Scale: 1 cm = $0^{\circ}.30$

are driven to extremes in the search for a possible cause may lead to a knowledge of interplanetary conditions that would never become known without the aid of the wide sweeping tail of a comet.

What caused the deflection in the streamers of November 15? Certainly not the sun or the comet. As I have suggested long ago, in connection with another comet, there must be some influence at work independent of the pressure of the sun's light and of the forces at work in the head of the comet to make these sudden changes in the direction of a comet's tail or of its streamers. If we should suppose that there are electrical conditions existing in the tail of the comet—which is highly probable—a possible repulsive condition might exist locally at points in the tail which would force the flow of the particles outward; or if the conditions were right they might be attracted toward the seat of disturbance. This is a subject, however, for the electrician and physicist.

Are there currents in interplanetary space across which the tail may sweep? This is strongly suggested in previous cases—especially in the case of Brooks's comet of 1893 on November 2, where the end of the tail is bent suddenly backward almost at a right angle, and where the main tail is strongly concave in the direction of motion. (See *Astrophysical Journal*, **22**, 251, 1905.) The appearance on that date strongly suggests that the tail was beating against a resistance of some sort in its onward flight. A similar case occurred in Morehouse's comet on September 16 where the tail is violently curved from its original direction. In this case, however, the direction of deflection is in the direction of motion—ahead of the radius vector; or in other words, the tail as it left the comet *rapidly moved forward faster than the comet itself*. There must have been some force that carried the tail forward. The resultant of the outward flow of the particles and the onward motion of the comet might have curved the tail but it could only make it move more slowly than the comet and hence deflect it in the opposite direction to that shown in the photograph of September 16. I have already called attention, in *Popular Astronomy* for December 1908 (**16**, 593), to the fact that the masses in the tail of the present comet on October 15 were moving faster through space (perpendicular to the radius vector) than the comet itself.

In pursuance of this subject I may be permitted to quote the following from a paper of mine in *Monthly Notices* of the Royal Astronomical Society, **59**, 355, March 1899:

> Unlike the planets, the comets often traverse the entire solar system. They are, therefore, our only means of exploring the regions between the planetary orbits. Instead of ponderous bodies like the planets, they are but flimsy creations of enormous dimensions. They are thus likely to be easily subject to disturbances in their forms that would produce no perceptible effect on their motions. What these influences may be we do not know: probably swarms or streams of meteors, which we know do exist in space, or possibly some other cosmical matter yet unknown. Such objects might be (and possibly have been) revealed to us by their effect upon the form of the comet's tail as it sweeps through space.

I believe these remarks are just as pertinent today as they were ten years ago.

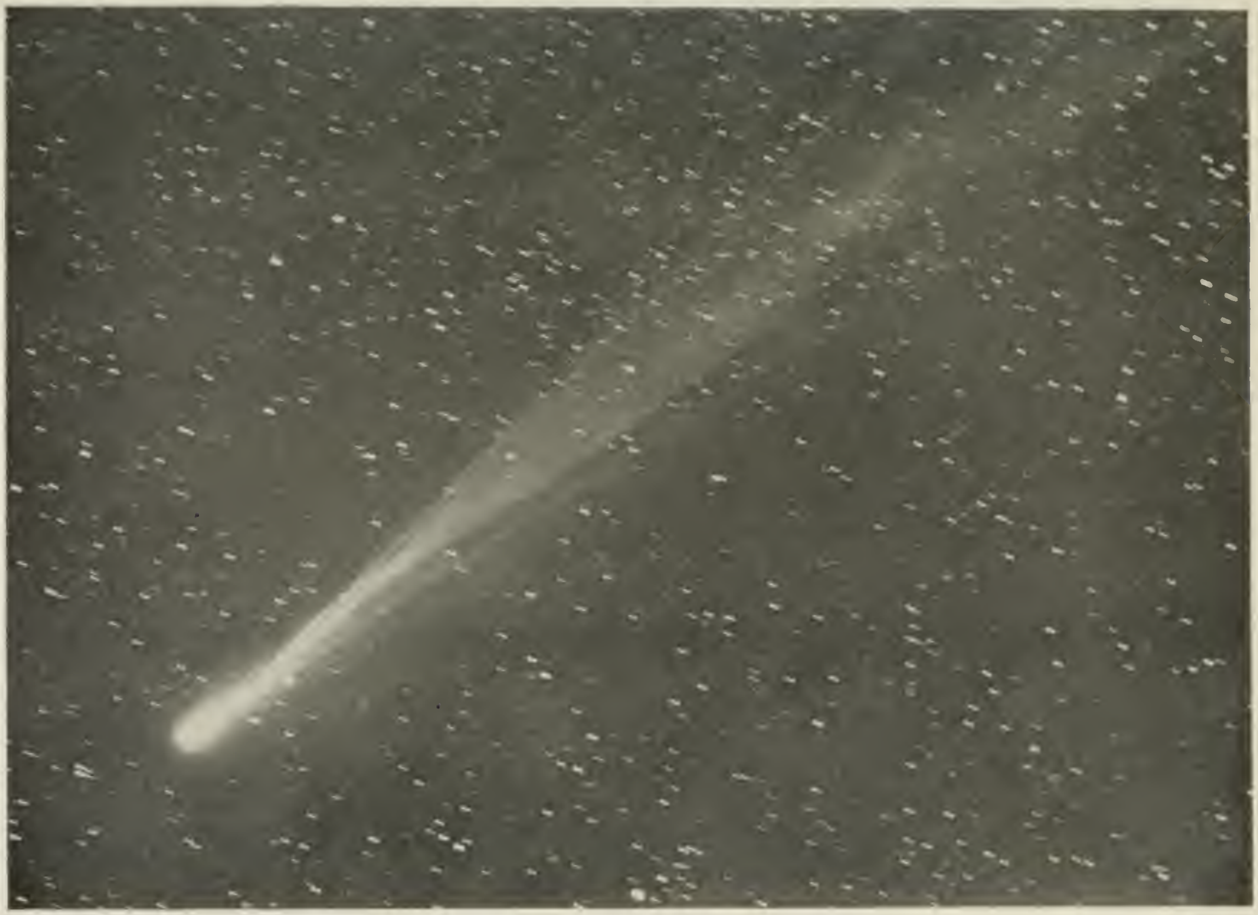
After November 21 continued cloudy weather prevented any observations of the comet until November 28 when an exposure was secured in a moonlit and densely hazy sky. Clouds again cut off the view until December 1 and 2 when a bright moon interfered badly, and after this cloudy weather occurred again.

LIST OF NEGATIVES OF THE COMET

(Continued from the December number)

Central Standard Time	Middle of Exposure	Duration
1908 Nov. 13	$6^{h}\ 25^{m}$	$0^{h}\ 59^{m}$
13	7 58	1 5
14	6 47	1 2
15	6 6	0 40
16	6 7	0 48
16	7 10	1 3
16	8 19	0 44
17	6 21	1 13
17	7 37	1 0
18	6 7	1 1
18	7 25	1 10
19	6 9	1 3
19	7 25	1 9
1908 Nov. 20	$6^{h}\ 35^{m}$	$1^{h}\ 50^{m}$
20	7 55	0 40
21	6 9	1 11
28	6 8	1 18
Dec. 1	5 55	0 41
2	5 43	0 22
2	6 15	0 24
4	6 14	0 26
7	5 43	0 20
9	5 56	0 36
11	5 59	0 38
13	5 55	0 33

With the exception of November 16, 17, and 20 and December 2 the exposures with the 3.4-inch lens were simultaneous with the other two lenses. The following table gives the exposures with this lens for the above dates:



Comet 1908 *c* (Morehouse) on November 19, 1908, at 6^h 9^m C. S. T. Exposure 1^h 3^m
10-Inch Lens of Bruce Telescope. Scale: 1 cm = 0°.30

Central Standard Time	Middle of Exposure	Duration	Central Standard Time	Middle of Exposure	Duration
1908			1908		
Nov. 16	$6^h\ 14^m$	$1^h\ 5^m$	Nov. 20	$6^h\ 57^m$	$2^h\ 35^m$
16	7 46	1 50	Dec. 2	6 0	0 55
17	6 56	2 22			

In the following table is given the approximate length of the comet's tail taken from the photographs made with the small lens. Clouds, moonlight, or bad sky, in some cases, will have prevented all the tail being shown, but in general I think the full length is visible on these plates. On November 15, by averted vision, I could faintly trace the tail with the eye for 10°, as measured on a star chart. The exposure on this night was cut short by clouds which would doubtless account for the less length of the tail on the photographs. Yet on the picture with the 6-inch lens it can be traced readily for 10°—to the edge of the plate.

ESTIMATES OF THE LENGTH OF THE TAIL OF THE COMET FROM NEGATIVES WITH THE 3.4-INCH LENS

1908		1908		1908	
Sept. 2	3°	Oct. 9	4°	Nov. 9	7°
6	4	11	9	10	9
7	2	12	7	11	10
8	4	13	6	13	7
16	4	14	7	14	8
18	2	15	8	15	8
19	3	16	5	16	10
21	6	17	6	17	9
23	6	21	7	18	10
25	6	28	9	19	10
26	6	29	9	20	8
29	6	30	11	21	8
30	9	31	9	28	8
Oct. 1	8	Nov. 1	8	Dec. 1	7
2	6	2	7	2	4
3	8	3	8	4	3
4	4	4	9	7	2
5	7	5	9	9	7
6	8	6	6	11	8
7	4	8	7	13	3
8	3				

In reference to the great disturbance of September 30, remnants of the tail discarded by the comet on that date are also visible on the plates with the small lens on October 2 and 3 (besides, of course, on October 1, where the old tail is conspicuous).

The beginning of the disturbance shown in the tail on October 6, which so much resembles that of October 15, is doubtless visible on the plates of October 5, where there is a large mass—really the rear end of a disconnected tail—at a distance of about $\frac{1}{2}$° from the head. Evidences of this disturbance are still seen on the photographs of October 7.

Yerkes Observatory
December 5, 1908

ADDENDUM

Since the above was in type there have appeared in *Astronomische Nachrichten*, No. 4287, a photograph of the comet taken on October 15 at the Observatory of Geneva by M. Pidoux, and in *Bulletin de la Société Astronomique de France* for December a photograph taken on the same date by M. Quénisset at Juvisy (M. Flammarion's observatory). These pictures add a remarkable interest to the comet, for they clear up any uncertainty as to the origin of the masses shown in the tail of the comet on that date (see Plate XXXII in the *Astrophysical Journal* for December 1908). The first of these photographs was made about 8 hours and the other 7 hours earlier than my photograph. On the Geneva picture at about 20′ from the head there is a strong bend in the tail but no masses. On the Juvisy plate this bend is stronger and more suggestive of the appearance shown on the photographs taken in this country. It is very clear from these pictures that the masses were not thrown off as such from the comet, but had their origin in a disruption of the tail which must have occurred a short time previous to Greenwich $7^h\ 0^m$ on October 15. This accords exactly with my idea that the tail of the comet suffered encounters with some sort of disturbing medium in space. If there should be photographs made several hours previous to the picture by M. Pidoux, it will doubtless be seen that the tail was straight and that it subsequently encountered something that bent and distorted it. What that something is I have tried to show in a paper read before the meeting of the Astronomical Section of the American Association for the Advancement of Science, at the Baltimore meeting in December, present. In that paper I have shown that there must be some influence, perhaps of a local and temporary nature, in the planetary spaces that can retard or accelerate, and disrupt a comet's tail when struck by it. It was suggested that this may come from energies originating in disturbances on the sun—perhaps of a similar nature to those that cause magnetic disturbances and auroral effects on the earth.

The comet has now passed to the southern hemisphere and is out of reach of all northern observatories. Cloudy weather here in the first half of December permitted but few observations of the comet. Exposures were made, however, every time that it could be seen. The last picture was made on December 13, when the comet was close to the horizon. This showed a tail about 3° long. The plates of December 9 and 11 are very interesting. The one of the 11th is very remarkable—indeed it is one of the most remarkable of the entire series. Though the comet was close to the horizon on that date, the tail shows for a length of 8°. It is convex on the south side—in the direction of motion—and looks as if it were encountering a resistance. About 2° from the head is a mass 30′ long; while at a distance of 2° 50′ a considerable portion of the tail is apparently shattered and thrown away to the north. On December 9 a peculiar mass appears in the middle of the tail, while the outlines of the tail are definitely bounded by two diverging streamers.

By additions to the proofsheets I have made the above list complete for my series of photographs of the comet. It is to be hoped that other observers will print similar lists so that the photographic history of the comet may be known and located.

ERRATUM.—In my paper in the December number, in the fifth line from the foot of p. 387, for October 27 read October 29.

December 31, 1908

ON THE COLORS OF SOME OF THE STARS IN THE GLOBULAR CLUSTER M 13 *HERCULIS*

By E. E. BARNARD

Over eight years ago in the *Astrophysical Journal* for October 1900 (**12**, 176–181) I called attention to some "abnormal" stars in the globular cluster M 13 *Herculis*. It was shown that certain stars of the cluster which were visually faint were relatively very bright on a photograph taken with the Potsdam Astrographic refractor. So great was this difference that I thought the stars in question were variable, but they did not prove to be so. The most remarkable of these is Scheiner 148 which, though faint visually, is the brightest star on the Potsdam photograph. It is evident that these photographically bright stars are bluer and hence their light is more actinic than the average stars of the cluster.

Since the publication of the above paper I have found other stars of this class in M 13 and also in M 5 *Librae*. About six months ago it occurred to me to compare a photograph of M 13, made with the Potsdam refractor, with one made with the 40-inch telescope and a yellow color-screen. The two pictures were copied to the same scale (in the negative form). They were then placed in the "Blink Microscope" of the Zeiss stereo-comparator. With a single eyepiece and two sets of prisms this instrument permits two negatives of the same object to be seen at once superposed. By the aid of a small movable blind, one picture is alternately made instantly to take the place of the other. If any object is on one photograph and not on the other (or if there in an altered condition), it will at once attract the eye in a most striking manner.

When these two plates were thus examined with the blink microscope, the result was very remarkable. It was seen at once that there were more of these "blue" stars in the cluster than I had found visually. It was also shown that there was a considerable number of yellow stars. These last were faint on the Potsdam plate but relatively bright on that with the 40-inch. Of course, visually, it is not possible to detect any color in stars as faint as those forming M 13. They all look alike except in relative brightness. From such an inspection one would think that the stars in the cluster were all white. The above experiment shows, however, that in reality M 13 consists of stars of different colors and hence of different spectral types.

I am making a catalogue of these blue and yellow stars in M 13 and hope to do the same for some of the other globular clusters. It will be interesting and important to study the distribution of these differently colored stars in such clusters. There seems to be a tendency to grouping of these objects in some parts of the cluster.

I give here a brief list of some of the blue stars (those abnormally bright on the Potsdam plate), and also a list of some of the yellow stars (those abnormally bright on the 40-inch plate with yellow color-screen). The numbers are those assigned by Dr. Scheiner in his photographic catalogue of the stars in M 13 ("Der Grosse Sternhaufen im *Hercules*, Messier 13"). The numbers in parentheses are provisional numbers given by me. The places of these last stars are only approximate. Where the Scheiner number is questioned the identification is not certain. The positions refer to Scheiner's zero star No. 382, which is in the north edge of the brightest part of the cluster. A negative sign indicates preceding in α and south in δ.

BLUE STARS

	$\Delta\alpha$	$\Delta\delta$
148	−1′ 22″	−1′ 44″
171 (8)	−1 14	−1 20
175	−1 12	−1 15
179	−1 11	+0 1
188	−1 6	+1 23
216	−0 54	−0 3
234	−0 45	+3 0
382	0 0	0 0
393	+0 4	−0 26
547	+0 45	+1 14
592	+1 1	+0 46
(23)	+1 3	−0 38
630	+1 13	−0 25
667	+1 33	+1 16
749	+2 21	−1 25
766	+2 43	−1 24

The star 148 is *very* blue

YELLOW STARS

	Δα	Δδ
16	−4′ 35″	−1′ 36″
19	−4 27	−1 8
38	−3 46	+0 7
41	−3 37	+0 17
(42)	−3 32	−2 16
44	−3 26	+0 20
(12)	−3 25	−2 1
57	−2 51	+0 47
(35)	−2 47	+0 47
61	−2 45	−2 21
(28)	−2 41	−0 58
64	−2 39	+0 54
66	−2 36	+1 2
(45)	−2 20	−0 11
108	−1 55	−1 36
120	−1 48	−1 35
138	−1 32	−1 41
152 ?	−1 22	−0 58
171 (9)	−1 14	−1 20
203	−1 0	−0 49
(18)	+0 36	+1 45
582	+0 58	−2 15
607	+1 5	+2 40
611 ?	+1 6	+3 54
637	+1 16	+2 12
661	+1 30	−4 18
677	+1 35	−4 20
686	+1 39	−3 25
(43)	+1 52	−0 48
769	+2 48	−0 40

A striking example of this difference of color is shown in (8) and (9) which occupy the place of Scheiner 171 and which with Scheiner 166 form a small close equilateral triangle. On the 40-inch plate these three stars are equal in brightness. The blink microscope, however, shows that (8), which is the following of the two, and (9) are strongly contrasted in color, (9) being strongly yellow and (8) blue while the third star, 166, is normal.

The two known variables in this cluster which were discovered by Professor Bailey, and which I have identified as Scheiner 216 and 630, are both blue stars. At their maxima, at which time comparisons are more readily made, they are relatively brighter with the regular photographic refractors and reflectors than they are visually. The variability of the first one of these (216) was independently discovered visually by me (see *Astrophysical Journal*, **12**, 182–184, 1900, where it was supposed to be new). I have determined the period of 216 to be 5.1 days, and that of 630 to be 6.0 days.

There seem to be several other variables in the cluster. I have not verified their variability yet with certainty, but they are under observation.

I should state that in comparing with other photographs, made with the 13-inch Boyden photographic refractor of the Harvard College Observatory and with the 33-inch correcting lens over the 36-inch object-glass of the Lick refractor, the differences to which I have called attention are not nearly so marked, though they are evident. It would seem, therefore, that the Potsdam telescope is corrected to give a stronger violet image than the other two refractors.

Yerkes Observatory
November 11, 1908

ON THE SEPARATION IN THE MAGNETIC FIELD OF SOME LINES OCCURRING AS DOUBLETS AND TRIPLETS IN SUN-SPOT SPECTRA[1]

BY ARTHUR S. KING

In the "Addendum" to Professor Hale's paper "On the Probable Existence of a Magnetic Field in Sun-Spots,"[2] it was stated that a number of the iron and titanium lines appearing as doublets in the spectra of sun-spots appeared as doublets also in the spectrum of the spark in the magnetic field when observed at right angles to the lines of force, the sun-spot triplets also retaining their character when observed in this manner. This of course referred to the appearance of the lines under the dispersion then used, leaving open the question whether they would show a higher order of separation when studied with a view to determining this. The next step was to observe the polarization of the components, and this has brought out the fact that all of the doublets listed in the "Addendum" are in reality quadruplets, and the list has been much extended, especially on the side of the titanium spectrum, which has been photographed through the range from λ 4400 to λ 6400, not only with the light parallel to the lines of force and also at right angles to them, but in addition a set of photographs has been obtained in which the light at right angles to the lines of force was passed through a Nicol prism in two positions 90° apart. This operation separated the light vibrating parallel to the force lines, producing the middle component of the normal triplet, from the light whose vibrations are in a plane perpendicular to the lines of force, giving the side components of the normal triplet. The great majority of the iron and titanium lines show the regular behavior of the normal triplet, the middle component remaining single; but for the lines appearing as doublets in sun-spots, the Nicol when turned so as to transmit the light with vibrations parallel to the lines of magnetic force gives the middle component, in all but two cases, either as a doubled line or one of such widened and diffuse appearance that evidently only a lack of sufficiently high dispersion stands in the way of a clear resolution of the components. The spot triplets retain in all cases their sharp middle component when examined in the laboratory at right angles to the lines of force.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 34.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 30; *Astrophysical Journal*, 28, 315–343, 1908.

The following table gives the character of each line in the spark with the measurements of its components. The last column, giving the appearance of the line in sun-spots and measurements when these could be made, was kindly contributed by Mr. Adams. Most of the measurements of spark lines were made by Miss Wickham. The magnetic field usually employed with the titanium spark was 12,500 gausses; for iron about 15,000 gausses. When a stronger field was employed for the separation of difficult lines, as in the blue of titanium, the measurements were reduced to the standard field. As the separation of the lines given at right angles to the field by the light vibrating in a plane perpendicular to the lines of force is the same as that given by the light observed parallel to the lines of force, photographs taken in the latter way were sometimes used when better measurements could be thus obtained; though for most of the titanium spectrum good photographs were available for the two positions of the Nicol.

IRON

λ	Spark Observed at Right Angles to Force Lines	$\Delta\lambda$ Spark. Transverse Effect*	$\Delta\lambda$ Spark. Longitudinal Effect*	Character in Sun-Spots
6173.55	Triple	Not separated	1.094	Wide triplet. Central line about $\frac{3}{2}$ intensity of side components
6213.14	Quadruple	0.460	0.840	Wide doublet. $\Delta\lambda = 0.136$
6256.57	Quadruple	0.438	Narrow. Components diffuse	Widened 3.† No evidence of doubling
6301.72	Quadruple	0.287 (scarcely resolved)	0.792	Wide doublet. $\Delta\lambda = 0.138$
6302.71	Triple	Not separated	1.144	Wide triplet. Central line about $\frac{3}{2}$ intensity of side components
6337.05	Quadruple	0.431	0.880	Wide doublet. $\Delta\lambda = 0.172$

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* These columns refer to the spark in the magnetic field when observed across the lines of force and along them respectively.

† The estimates of widening are on an arbitrary scale of 0 to 5.

TITANIUM

λ	Spark Observed at Right Angles to Force Lines	Δλ Spark, Transverse Effect*	Δλ Spark, Longitudinal Effect*	Character in Sun-Spots
4440.49	Quadruple	0.144	0.182	Narrow. Strengthened but scarcely widened
4444.73	Quadruple	0.189	0.202	Narrower than in disk and probably slightly weakened
4450.65	Sextuple	0.189	0.245 (mean for two pairs)	Possibly slightly strengthened; not widened. Enhanced line
4464.62	Quintuple	0.213	0.204 0 0.214	Slightly weakened and narrowed
4471.02	Quadruple	0.398	0.460	Widened 2
4471.40	Septuple	Too diffuse to measure	Inner pair 0.094 Outer pair 0.258	Widened 2. Much strengthened
4439.24	Quadruple	0.134	0.329	Widened 2. Blends with 4489.3
4527.48	Septuple	0.124 0 0.122	Inner pair 0.232 Outer pair 0.510	Widened 3. No evidence of doubling
4529.65	Quadruple	0.181	0.258	Blends with 4529.7. Apparently little affected
4544.83	Septuple	0.124 0 0.127	0.497 (mean for two pairs)	Blends with 4544.7. Much strengthened. Widened 1–2
4590.11	Quadruple	0.207	0.257	Widened 1? Slightly weakened. Enhanced line
617.45	Quadruple	Not clearly resolved	0.311	Widened 2. Much strengthened
4623.24	Quadruple	0.091	0.288	Widened 2. Much strengthened
4629.52	Quintuple	0.136 0 0.132	Too weak to measure	Widened 1. Slightly strengthened
4639.54	Quadruple	0.163	0.457	Widened 3. No evidence of doubling
4639.85	Quadruple	0.138	Too weak to measure	Widened 3. No evidence of doubling
4640.12	Quadruple	0.254	0.592	Widened 4. No evidence of doubling. Fringed to the red
4645.37	Triple	Not separated	0.244	Widened 4. Probably double
4698.94	Quadruple	0.081	0.293	Widened 2. Much strengthened
4710.34	Quadruple	Not clearly resolved	0.375	Much strengthened but narrow. Widened 1?
5016.32	Quadruple	0.204	0.426	Widened 2. No evidence of doubling
5020.17	Quadruple	0.218	0.384	Widened 3. No evidence of doubling
5023.02	Quadruple	0.241	0.374	Widened 3. No evidence of doubling

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* These columns refer to the spark in the magnetic field when observed across the lines of force and along them respectively.

TITANIUM—*Continued*

λ	Spark Observed at Right Angles to Force Lines	Δλ Spark, Transverse Effect*	Δλ Spark, Longitudinal Effect*	Character in Sun-Spots
5025.03	Quadruple	0.412	0.108	Widened 3−4. No evidence of doubling
5066.17	Quadruple	0.283	0.227	Blends with 5056.08 *Cr.* Difficult. Apparently widened 2
5085.51	Quadruple	0.319	Not measurable	Widened 3
5109.60	Quadruple	0.259 (difficult)	Not measurable	Widened 2
5418.98	Quadruple	0.292	0.395	Large shift to red in spot. Widened 2. Maximum toward red side of line. Enhanced line
5426.47	Quadruple	0.408	0.532	Widened 3. Probably double. Measured separation 0.085
5460.72	Quadruple	0.431	0.568	Widened 4. Maximum at about center of line
5481.65	Quadruple	0.429	0.429	Narrow. Fringe to violet making hazy, continuous band as far as λ 5481.45
5712.07	Quadruple	0.313	0.550	Widened 4 but blends with 5712.10 *Fe.* Difficult to judge structure. Possibly double
5716.67	Quadruple	0.417	0.256	Widened 4. Possibly double though line appears of nearly uniform intensity throughout. Apparent components measured 0.105
5720.57	Quintuple	0.630	$\left\{\begin{array}{c}0.338\\0\\0.340\end{array}\right.$	Widened 4. Narrow, sharp maximum in center of line
5903.54	Triple	Not separated	0.601	Widened 4. Probably double. Apparent components measured 0.094
5938.04	Quadruple	Wide and hazy, not clearly resolved	0.612	Widened 4. Double. $\Delta\lambda = 0.093$
5941.98	Quadruple	0.456	0.527	Widened 4. Double. Diffuse center. $\Delta\lambda = 0.113$
6064.85	Triple	Not separated	0.876	Widened 5. Triple. $\Delta\lambda = \left\{\begin{array}{c}0.087\\0\\0.092\end{array}\right.$
6085.47	Quadruple	0.225	0.808	Widened 4. Not clearly doubled
6303.98	Quadruple	0.487	0.493	Widened 3. Double. $\Delta\lambda = 0.087$
6312.46	Quadruple	0.367	0.615	Widened 3. Double. $\Delta\lambda = 0.090$

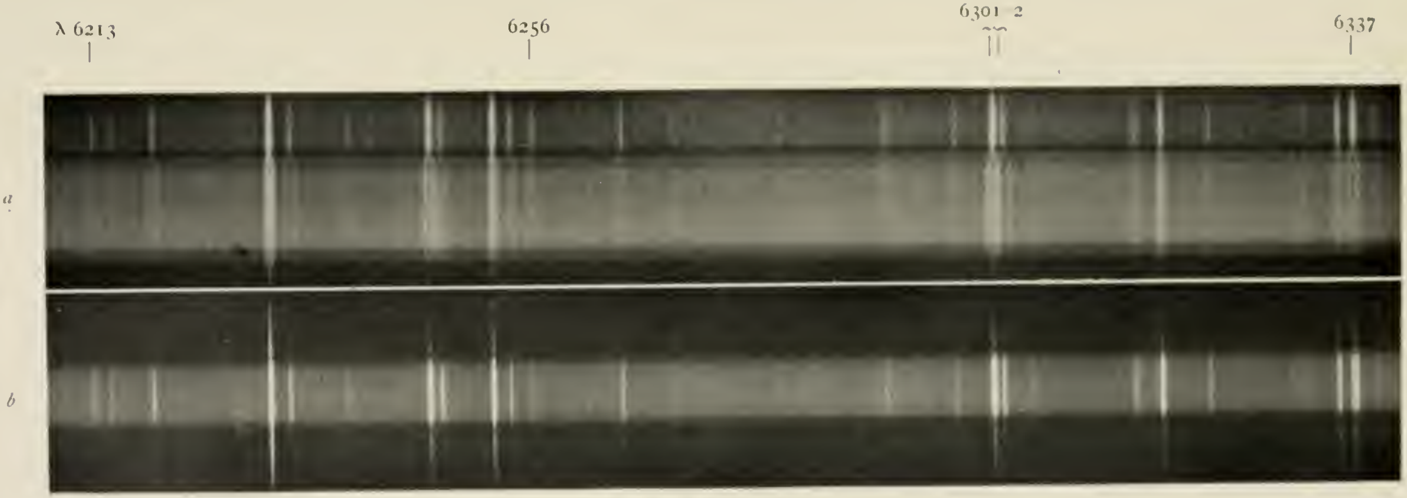
* These columns refer to the spark in the magnetic field when observed across the lines of force and along them respectively.

Plate XI reproduces the iron spectrum from λ 6213 to λ 6337 given by the spark in the magnetic field. The upper spectrum shows the doublets obtained when the spark is observed parallel to the lines of force, the so-called "longitudinal component." The lower was made with the light at right angles to the force lines and passing through a Nicol prism turned so as to transmit the light vibrating parallel to the field, the "transverse component." The quadruplets thus appear as narrow doublets in the latter spectrum, while the triplets show their middle component single.

The purpose of the table is to show to what extent a correspondence exists between the structure of lines given by the spark in the magnetic field and by the sun-spots. This may be discussed best by considering each type of separation in turn.

1. The fairly large group of *quadruplets* in the spark is of special interest when compared with the list of spot doublets. For iron there is practically a complete correspondence for the limited region in the red thus far examined with the Nicol prism. The only apparent exception is λ 6256.57. This is much widened in the spot but does not appear doubled. In the spark it is an unusual type of quadruplet. The transverse component is clearly separated (see Plate XI) while the parts of the longitudinal component are wide and hazy and not clearly resolved. A line of this character, in the weak field of a sun-spot, would have the central portion so filled up that no separation of the parts could be effected. For titanium the correspondence is very close for lines in the yellow, orange and red, beginning at about λ 5700. The most important exception is λ 5903.54, which is much widened in spots and probably double. This line shows in the spark the structure of the normal triplet, the transverse component being narrow. Two extreme types of quadruplets are represented in λ 5941.98 and λ 6085.47. The former has in the spark the character of the iron line λ 6256.57 mentioned above, the transverse component having a wide separation while the longitudinal is separated but little more and the central portion is so filled up by the diffuse and hazy components that a measurement is difficult. The spot line is of an analogous type, having the center filled up, but the maxima at the sides are sufficiently distinct to enable it to be classed as a doublet. λ 6085.47 has its transverse component very

PLATE XI



SPECTRUM OF IRON SPARK IN MAGNETIC FIELD

(*a*) Observed along Lines of Force (Longitudinal Effect); (*b*) Observed across Lines of Force, with Outer Components Removed by Nicol (Transverse Effect)

narrowly separated, with a widely separated longitudinal component. In the spot this line is widened, but not doubled.

In the region of shorter wave-length, the agreement is not nearly so good. λ 5645.37 shows the same relation in spark and spot as λ 5903.54. Several weak and doubtful spot doublets in the blue and green are probably narrow quadruplets in the spark, but are not included in the table. The lines in this region which are clear quadruplets in the spark, however, have no uniform behavior in the spot. Many are considerably widened (1 to 3 on a scale of 5), but do not show doubling, while a few are slightly narrowed in the spot. The change in correspondence begins to show itself in the neighborhood of λ 5000, and the lack of agreement with the region of greater wave-length points to a masking of the real character of the spot lines, presumably by the same agency which causes other influences, such as temperature, to show a much closer accordance between laboratory and spot results in the less refrangible region of the spectrum.

2. The few lines of iron and titanium occurring as *triplets* in sun-spots show a complete agreement with the laboratory results. λλ 6173.55 and 6302.71 of iron and λ 6064.85 of titanium are all wide triplets in the spark, the middle component remaining sharp. The latest measurements of the best photographs show that λ 6302.71 is not asymmetrical, and indicate that the appearance of asymmetry previously noted was due to disturbing influences in both spot and spark spectra.

3. The few titanium lines appearing as *quintuplets* and of still higher separation in the region studied are of two types. λλ 4464.62 and 5720.57 appear as triplets when observed along the lines of force in the laboratory and as quintuplets across the lines of force, two more components being added on the outside when viewed at right angles. Such a line will then have a strong central component at whatever angle to the force lines the light may be taken. λ 5720.57 is favorably located for comparison with the sun-spot line. In the spot the line is very much widened and has a narrow and unusually strong maximum in the center. The structure of the spark line should give this appearance, the quintuplet structure giving a strong widening, while the central maximum is accounted for. In λ 4464.62 the strength of the central component is probably partly responsible

for the apparent narrowing of a rather faint line, the sides being weak and indistinct by contrast with the center.

The other type has one or more pairs of rather widely separated components when viewed along the lines of force; while across the lines of force a close triplet is seen in the middle of the structure. Lines of this type are $\lambda\lambda$ 4471.40, 4527.48, 4544.83, and 4629.52, and by referring to the table these are seen to be both widened and strengthened in sun-spots, in no case doubled. This again accords well with the spark structure, the widely separated components giving a broad line, while the central maximum is not so conspicuous as in the first type.

4. One strong *unseparated* line stands out with great distinctness in the titanium spectrum. This is λ 5714.12, which appears equally narrow in both positions of the Nicol. This line is remarkable for its narrowness in the spot spectrum.

We have thus a good deal of evidence that light coming at an angle to the lines of magnetic force plays a large part in giving the sun-spot lines the structure which is observed. The presence of triplets in the spot spectrum is fully explained. The fact that the spot doublets occur very generally as quadruplets in the laboratory when observed at right angles to the lines of force obviates the necessity of assuming that doublets in the spot are due to the action of the field along the lines of force alone, since in either case the central component would be absent for these lines.

The two inner components of the quadruplets are seldom widely separated, even in the fairly strong laboratory fields; so that what we probably have in the spot lines is a weakening of the center of the line due to a narrow separation of the transverse component, throwing the light into maxima at each side. The transverse effect becomes weaker as the direction of the light becomes more nearly parallel to that of the lines of force, giving a more distinct separation of the components of the doublet. This dependence of the relative intensity of the components upon the angle of the field is unfavorable to quantitative relations between the measured separations of spark and spot lines. It is quite probable that some lines are produced more strongly than others in certain parts of the spot, on account of differences in temperature and in other physical conditions. This would involve a difference in field strength and also of the angle to the lines of force at which the light comes to us, opening the way for many differences in the appearance of the lines in the spot and in the laboratory sources. For example, light coming almost parallel to the force lines of the solar magnetic field would give a strong preponderance of circularly polarized light. Another part of the spot, still in the field of view projected on the slit, but differing either in location or in level or both, and therefore in a different part of the solar magnetic field, might send light at a larger angle to the lines of force, giving to some extent the transverse effect and producing the central line of the triplets. This may seem to require somewhat artificial conditions, but a structure of the spot allowing for this would appear necessary to account for the relative intensity (about $\frac{3}{2}$ to 1) of the middle and side components of spot triplets such as $\lambda\lambda$ 6173 and 6302. This would require a mixture of the longitudinal and transverse effect corresponding to an angle not yet determined exactly, but greater than 30° to the lines of force and much less than 90°.

Many of these doubtful points will, it is hoped, be cleared up as the investigation is extended with improved facilities.

Mount Wilson Solar Observatory
December 5, 1908

THE SPECTRUM OF COMET *c* 1908 (MOREHOUSE)

BY W. W. CAMPBELL AND SEBASTIAN ALBRECHT

The one-prism spectrograph described in Bulletin No. 8 and illustrated in Bulletin No. 62, attached to the 36-inch refractor, was used in observing the spectrum of Comet *c* 1908 (Morehouse).

1908, October 20. Campbell, observer. Although the tail of the comet was easily visible to the naked eye, for at least 3° from the head, the intrinsic brilliancy of all parts was surprisingly low. The image of the head formed by the long-focus telescope was scarcely visible on the reflecting slit-plates of the spectrograph. The feeble nucleus was just visible after the eye had remained in total darkness for several minutes, and could not be used in guiding when the slit was wide enough to give promise of a successful spectrogram.

Visual observations were made with the slit 0.04 inch (1 mm) wide. It was noticed at once that the spectrum had unusual features. The carbon bands whose edges are at λ 4737, λ 5165, and λ 5635 were present in their usual relative intensities. In addition, two bands not hitherto observed at Mount Hamilton in any comet were visible on the more refrangible side of λ 4737. These were probably composed of the two pairs of lines photographed later at λ 4550–70 and λ 4255–76. There appeared to be a slight brightening in the feeble continuous spectrum near $H\beta$. Another band was thought to be visible, far up in the violet, perhaps composed of the lines photographed later at λ 4002 and λ 4022.

A photographic exposure of three hours was made, using the same slit-width. It was necessary to guide with the finding telescope, and no doubt the guiding was inefficient. The spectrogram shows five bright bands just on the limit of vision; so faint, in fact, that they become invisible under the lowest magnification.

1908, Nov. 27, 28, 29. Albrecht, observer. Visual observations were made each evening before beginning the photographs referred to below. The sky was bright with moonlight and twilight. The three usual bands at λ 4737, λ 5165, and λ 5635 were seen. Their positions were estimated closely by means of the iron spectrum, which was thrown into view frequently, and with which the observer is familiar. The green carbon band was much stronger than the blue band, and the blue band was stronger than the yellow band. In addition, a fourth band, quite faint, was visible at a considerable distance to the violet of λ 4737. It was probably the pair at λ 4255–76.

On November 27 a spectrogram was obtained with slit-width of 0.02 inch (0.5 mm) and exposure-time of 75 minutes. The comet spectrum, containing several fairly strong lines or bands, was unfamiliar in appearance. The hydrogen comparison spectrum, underexposed, is visible on only one side of the comet spectrum. As the plate could not, therefore, be accurately oriented on the measuring microscope, a constant correction has been applied to all the micrometer readings to reduce to the same system as for the spectrogram obtained on November 28.

On November 28 the slit-width was 0.008 inch (0.2 mm) and the exposure-time 83 minutes. The recorded spectrum is much fainter than that of November 27, but the comparison spectra are excellent.

On November 29 a 90-minute exposure was made on a plate stained for sensitiveness in the red end of the spectrum. The sky was hazy, and during a part of the exposure thinly clouded. A trace of the band at λ 5165 shows on the plate. The strongest pair of lines recorded on the two earlier spectrograms at λ 4255–76, is easily visible on this plate.

The results of Albrecht's measures of the first two photographs are contained in the table. The brightest line is assigned intensity 10, and a line just visible intensity 0.

It is of interest to note the relative weakness of the edge of the first fluting in the fourth carbon band in the comet, at λ 4737. The band observed in the comet at λ 4690 is probably a blend of the third, fourth, and fifth fluting-edges. The lines at λ 3882 and λ 3869 are doubtless the first and second edges in the third cyanogen band. There is doubt whether the first cyanogen band[1] is represented in the comet, although the presence of two lines certainly and three others possibly within the limits of the band is a point in favor of their cyanogen origin. Attention should be called to the apparent entire absence of the second cyanogen band. We are unable to suggest a probable origin for the

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[1] *Astronomy and Astrophysics*, **13**, 205, 1894.

TABLE I

Bright Lines Photographed

1908 Nov. 27 Slit-Width 0.020 in.	Nov. 28 0.008 in.	Nov. 29 0.012 in.	Mean	Relative Intensities	Carbon and Cyanogen Spectra
3836±	......	....	384–	1	
3869	......	....	3869	1	3871.5, 2d edge, 3d Cyanogen Band
3882.8	3881.4	....	3882.1	2	3883.6, 1st edge, 3d Cyanogen Band
3913	3915	....	3914	2	
4002.8	4002.2	....	4002.5	6	
4023.0	4021.3	....	4022.2	8	
4255.1	4254.7	4255	4254.9	8	
4276.1	4275.1	4276	4275.6	10	
452–?	453–?	....	452–?	..	4502 4515 4532
4548.8	4550.0	....	4549.4	4	4553 1st Cyanogen Band?
4571.2	4569.3	....	4570.2	3	4578
4598?	......	....	460–?	..	4606
4611?	......	....	461–?	..	
4688.2	4692.2	....	4690.2	7	4677. , 5th edge, 4th Carbon Band 4684.9, 4th edge, 4th Carbon Band 4697.6, 3d edge, 4th Carbon Band
4714.7	4715.1	....	4714.9	5	4715.3, 2d edge, 4th Carbon Band
4737	4738	....	4738	0	4737.2, 1st edge, 4th Carbon Band
......	......	516–	516–	..	5165.3, 3d Carbon Band

Bands Observed Visually

4002–22± (?) Position not determined visually.
4255–76± Position not determined visually.
4550–70± Position not determined visually.
4737± Carbon—Position estimated, not accurately measured.
5165± Carbon—Position estimated, not accurately measured.
5635± Carbon—Position estimated, not accurately measured.

strong lines at λ 3913, λ 4002, λ 4022, λ 4255, λ 4276, λ 4549, and λ 4570. The last six of these lines are no doubt identical with three pairs of lines observed by Chrétien[1] in the spectrum of Comet *d* 1907, and with three bands observed in the same comet by Deslandres[2] and Evershed,[3] all using objective-prism spectrographs. Radiations having these wave-lengths seem to be relatively strong in the tails of comets, as it is difficult otherwise to account for their essential absence from the spectra of the heads of Comet *d* 1907 and other comets, as observed with slit-spectrographs.

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[1] *Comptes Rendus*, **145**, 549, 1907.

[2] *Ibid*, **145**, 445, 1907.

[3] *Monthly Notices*, **68**, 16, 1907.

The existence of these pairs of lines, with the lines in the pairs separated by about the same intervals, is strongly suggestive of closely related origins. The most promising hypothesis as to their origin seems to us to be that they are related to each other in much the same way as the well-known carbon bands or cyanogen bands are related. Are these six lines the edges of flutings in three bands of some element or compound, not yet observed in the laboratory?

The photographic intensity of the comet's light as compared with its visual brightness has been surprisingly great. The unusual weakness of the continuous spectrum and the presence of the three pairs of relatively strong bright lines in the blue and violet regions, which were observed as very faint lines or not at all in previous comets, no doubt supply the greater part of the explanation. We consider that the presence of these unusually strong lines more than balances the apparent relative weakness of the cyanogen band at λ 3884–72.

Reflected sunlight seems to contribute in very small measure to the comet's brilliancy.

LICK OBSERVATORY
December 10, 1908

REVIEWS

Populäre Astrophysik. By JULIUS SCHEINER. Leipzig and Berlin: B. G. Teubner, 1908. Pp. 706 with 30 plates and 210 figures in the text. Bound. M. 12.

The field for popular works on astronomy for general and serious readers is very well supplied with numerous and valuable books, but no work of similar merit and equal scope has been available for the reader in astrophysics. Miss Clerke's *Problems in Astrophysics* and Professor Hale's *The Study of Stellar Evolution*, excellent and inspiring though they are, do not, and were not designed to cover the whole of the rapidly developing field of astrophysics. The want is supplied, however, in the present volume; and who could be better equipped to prepare the volume than the author, an active and clever investigator in astrophysics, also well known for his standard works, *Die Spectralanalyse der Gestirne*, *Photographie der Gestirne*, *Strahlung und Temperatur der Sonne*, etc.?

Those students of the University of Berlin in the year 1906 who heard Professor Scheiner's courses entitled *Einleitung in die Astrophysik* will recognize in the present volume an elaboration of those lectures. The lectures were divided into two great parts. The first of these, dealing with instrumental equipment and methods of observation, considered the fundamental physical and physiological conceptions, spectrum analysis, photometry, solar radiation, and celestial photography. The second part gave the results of astrophysical investigations, reviewing in turn the sun, the planets, the moon, comets, meteors, zodiacal light, nebulae, and the fixed stars. The published work, which is thoroughly comprehensive, exceedingly well balanced, and straightforward in style, follows the same general scheme.

One or two errata may be noted. In the lists of lines of the chromospheric spectrum on pp. 370 and 395 the strontium line at 4078 is attributed to calcium, perpetuating an old error. Another legacy of error is found on p. 365, where the diurnal motions of solar rotation from Dunér's spectroscopic observations are given uncorrected for the motion of the earth. That these motions are synodic and not sidereal was pointed out by Dunér in the *Astronomische Nachrichten*, **167**, 167, 1905.

Recent and complete as this volume is, astrophysical research is already making advances beyond its contents. One instance will illustrate. We read on p. 148 that until now the Zeeman effect has found no direct application in astrophysical problems, but, he continues clearsightedly, we must not conclude that it may not be of greatest moment here. And now Hale detects the Zeeman effect in the radiation of sun-spots, proving thereby the presence of a magnetic field about them.

The illustrations in the text are well selected. The plates assembled at the end of the volume illustrate a wide range of results of celestial photography, and are for the most part of very good quality. Certain of the plates have suffered in the reproduction, principal among them being the photoheliogram by Lohse, Plate I, which appears much distorted, the solar corona by Schorr, Plate IX, loses its details, and the comet photographs from the Lick Observatory, Plates XV and XVI, for which the contrast is too great. Plate X, the photograph of the solar corona by Barnard and Ritchey, is erroneously accredited to Frost.

The subject-index is unusually complete, but it is to be regretted that no author-index is added to the volume. The insufficient opacity of the paper is objectionable. Readers, other than Germans, will deplore the use of German text in the printing. This may be an additional incentive to the desired translation of the work into English.

This volume admirably fills a vacancy in popular scientific literature and makes a fitting companion to Vogel's *Newcomb-Englemann's Populäre Astronomie* and will probably be seen beside this work on the bookshelves of working astronomers and scientific readers generally.

P. F.

Die Milchstrasse. Von MAX WOLF. Leipzig: J. A. Barth, 1908. 8vo., pp. 48. Figs. 50. M. 4.

This is the issue in book form of a public lecture by Professor Wolf, illustrated with quite a number of photographs of the star clouds of the galaxy, made with his various telescopes, large and small. The text deals in a popular manner with the peculiarities of the Milky Way, and especially with the theories formulated by various astronomers to account for its existence and to explain what it really is. The main interest in the book, however, should center in the photographs that are contained in it, for no matter how erroneous the various theories concerning the constitution of the Milky Way, the photographs are supposed to tell their own story, from which the student can judge for himself how well the theories fit into the actual appearance of this wonderful zone of stars. The reproductions in this work, unfortunately, are not fair representations of Wolf's photographs of the Milky Way: they are really misleading in many cases. This, however, is due entirely to a want of judgment as to how much of the actual photograph should be used. There is always a falling-off of illumination when the field of a lens is not large enough to cover the entire plate. Outside of this circle of illumination the stars abruptly end, for no images are formed there by the lens. If, therefore, a full-sized print is made from such a negative, the area of illumination, where the light has acted, will appear as a bright region, and the cloud forms will extend, if they are large enough, to the edge of the circle of illumination, and will there abruptly terminate in blackness, as no images of the stars will reach the plate. In a case of this kind one should use only that part of the plate which is well covered by the circle of illumination, by matting out or cutting away the bad part. Figs. 3 to 10 in the present work, on pp. 13 to 16, are samples of this neglect of matting. Any one of these plates or Nos. 25 and 34 (with the reflector) gives the impression of a bright mass of stars isolated on a starless sky, while in reality they are but part of larger star areas. Taking these photographs as a guide, one would form quite an erroneous idea of the true forms of and extent of these cloud regions.

Among the illustrations are some excellent photographs of a number of the well-known nebulae, such as those of *Orion* and *Andromeda*, and also some of the spiral nebulae. They were made with the new 28-inch reflector. These pictures have doubtless suffered much in the reproduction. Figs. 25 and 34 are especially bad, from the inclusion of a portion of the plate beyond the circle of illumination. This fault will be strikingly apparent by comparing Fig. 25 with Fig. 24. It is indeed unfortunate that through a mere oversight in the presentation of these pictures without properly matting out the exterior bad portions, the reader should get an inadequate and erroneous impression of the accurate and effective work which Professor Wolf has for years been carrying on with such remarkable success with the admirable facilities he has established on the Königsstuhl.

E. E. B.

The Theory of Optical Instruments. By E. T. WHITTAKER. (Cambridge Tracts in Mathematics and Mathematical Physics, No. 7.) Cambridge University Press, 1907. Pp. 72, with diagrams. *2s. 6d.*, unbound.

This excellent little work was prepared by the Royal Astronomer for Ireland for those students of astronomy, photography, and spectroscopy who desire "a simple theoretical account of those defects of performance of optical instruments to which the names *coma, curvature of field, astigmatism, distortion, secondary spectrum, want of resolving power*, etc., are given." The hope is also expressed that the attention of pure mathematicians may be drawn by it to some attractive theorems. The author also regards this "tract" as a suggestion toward a modification of the customary university course in geometrical optics.

Books and articles on this topic in English are few indeed, and but few translations have appeared of the numerous German contributions, Professor Silvanus P. Thompson's English edition of Lummer's *Beiträge zur Photographische Optik* excepted. This tract compresses in its small space the essentials of the subjects treated, and should be of wide service.

F.

The Moon in Modern Astronomy. By PHILIP FAUTH. London: A. Owen & Co. Bound, 10*s*.

This book is a summary of twenty years' work on the moon. The first sixty pages are taken up with an introduction by J. E. Gore, a historical survey by the author, in which he mildly reproves American astronomers for their neglect of visual selenography; and with a criticism of the results obtained by photography. The remaining one hundred pages discuss the subject proper. Some of the interesting points brought out are: the relation of crater depth to crater diameter, similarity of crater rings and plains, the bleaching of the surface, the lunar rays. Herr Fauth closes by stating that the moon has a thick coat of ice and supports his statement at length. The maps are excellent, showing a wonderful amount of painstaking work, but the detail which is shown is almost incredible when we find that the author has had only a 7½-inch refractor at his command. Herr Fauth explains that this is due to the character and training of his eye. The book is carefully prepared and is well worth consideration.

H. E. BUCHANAN

Two New Worlds. By E. E. FOURNIER D'ALBE. London: Longmans, Green & Co., 1907. Pp. 157.

Ingenious and well composed as this book, "written in Ireland and largely inspired by Irish thoughts and thinkers," is, we are sorry to say that from a scientific standpoint it is best described as an Irish bull. We regret this the more as the theme was grand and if scientifically accurate the book might have given us at least an imaginative glimpse of value into the beyond. In justification of our phrase, meant in all kindliness, we will explain why the book courts the appellative, being ostensibly one thing and in reality another. The claim of the work is the advancement of proof or at least strong presumption that modern science has data for the existence of two new worlds, one infinitely little, the other infinitely big, and the key to their unlocking is given by the author in the following words: "Time and space are, after all, purely relative. If, at midnight tonight, all things, including ourselves and our measuring instruments, were reduced in size 1000 times, we should be left quite unaware of any such change." The monstrosity of this statement, coming originally from the teachings of Kant but spelled now usually with a small *c*, instead of a capital *K*, fairly takes our scientific breath away. Not know it! Why, the veriest child would perceive it at once, however unversed in physics. Every sensation dependent on two dimensions, the angular magnitude of an object for instance, would be altered in one ratio, while those dependent on three, weight for example, would be altered in another and even a baby on tumbling to the floor would wonder why he hurt himself less than yesterday—on that floor seemingly as far off as ever.

Starting with this fallacy it is not surprising that he reaches the to him remarkable premise and really startling conclusion that: "Hence in the infra-world *space and time are reduced in the same proportion*." It is this curious circumstance which justifies the expression "infra-world." The curious circumstance is simply that distance, which he here calls space (p. 210), and time are both of one dimension and that he takes the earth's distance from the sun and not for instance *Neptune's* as the basis of comparison with atoms and electrons.

As he thus sums up his idea, nothing further in exposing its erroneousness need be said, except that it is a pity that he did not look a little more carefully before leaping onto his worlds and asking others to follow; for to persons untrained in, or averse to, mathematics such works do incalculable harm by leading them to believe that to be founded on figures which really founders on fact. P. L.

The World Machine: The Cosmic Mechanism. By CARL SNYDER. London: Longmans, Green & Co., 1907. Pp. 488. $2.50 net.

This is a popular book, the first part of a rather ambitious undertaking to record and philosophize on the universe and its contents, which is to include *The Mechanism of Life*, and *The Social Mechanism.* This part deals almost entirely with astronomy, and is written in a sketchy, vivid style which is likely to compel the attention of the reader, though at times it becomes almost flippant in its effort to be up to date. The author has evidently read widely, though generally not at first hand, and he gives on the whole a pretty accurate as well as interesting picture of what is known and surmised of cosmogony. The book can be recommended to teachers for collateral reading by a class in descriptive astronomy, and to those who read magazine articles on scientific subjects. It is well printed on the light paper now often used in England, which makes its handling a pleasure. It is not clear why American publishers do not follow this excellent example.

F.

Halley's Comet. By H. H. TURNER. Oxford: The Clarendon Press, 1908. Pp. 32. 1*s*. net.

This evening discourse to the British Association at Dublin is written in the author's familiar and entertaining style. It is illustrated with excellent portraits of Halley and Newton, and by a sketch from *Punch*, and the drawing from the Bayeux Tapestry. It will be widely read by both those who had, and those who did not have, the privilege of hearing the lecture.

Annuaire pour l'an 1909. Publié par le Bureau des Longitudes. Paris: Gauthier-Villars. 16mo. Pp. 941. Fr. 1.85.

This indispensable volume makes its annual (113th) return, compressing a vast amount of scientific information into its small compass. Under the scheme of alternation now in effect, this issue gives, in addition to the usual astronomical data (244 pages), an extensive set of geographical, meteorological, and statistical tables. The tables relating to physics and chemistry appear in the even years. The data regarding variable stars are particularly full for 1909, comprising 139 pages, and the principal "Notice," by M. Bigourdan, is a valuable popular statement of our present knowledge on the same topic (116 pages). The second notice is by M. Lallemand, "Mouvements et déformations de la croûte terrestre" (57 pages).

A Short University Course in Electricity, Sound and Light. By R. A. MILLIKAN AND JOHN MILLS. Boston: Ginn & Co., 1908. Pp. 389; Figs. 258.

The authors state that "this book represents primarily an attempt to secure a satisfactory articulation of the laboratory and classroom phases of instruction." An exposition of the principles of a physical topic is followed by directions for conducting an experiment or measurement, and an example is then added. The book is thoroughly modern, is clearly written, and well illustrated. In view of its practical importance, electricity occupies more than half of the volume, while optics receives about 100 pages. The work will be found useful for consultation by many whose work is in allied branches.

A Practical Manual of Tides and Waves. By W. H. WHEELER. London: Longmans, Green & Co., 1906. Pp. 201; Figs. 19.

This is not an "attempt to deal with the difficult subject of the tides in a scientific way," as the author states in the preface. It gives, however, besides the ordinary geometrical theory, a readable account of various observed facts, collected from many sources, about the tides in seas and rivers, and about oceanic waves caused by wind and seismic disturbances.

The Principles of Physics. By ALFRED P. GAGE, revised by ARTHUR W. GOODSPEED. Boston: Ginn & Co., 1907. Pp. 547; Figs. 420, 18×13 cm.

When the first edition of this book appeared, nearly twenty-five years ago, it constituted quite a departure from the elementary textbooks then existing in English. Excellent textbooks for instruction in physics in secondary schools and colleges are now numerous: this one has also improved with successive revisions, and it is well adapted to its purpose.

Die Luftelektrizität. Von ALBERT GOCKEL. Leipzig: S. Hirzel, 1908. Pp. 206, Figs. 28. Paper, 6 Marks; bound, 7 Marks.

A century and a half ago Franklin suspected and later proved the identity of lightning with frictional electricity. Others followed Franklin's lead in the study of the electric phenomena of the atmosphere, but the first marked improvement even in the apparatus used—the water dropper of Thomson (Kelvin)—came one hundred years later, and it is only now, after the lapse of half a century more, so slow has progress been, that the first book on atmospheric electricity alone has appeared.

In this the entire subject exclusive of thunder storms and auroras, is reviewed, but much the greater portion of the numerous papers referred to bear the dates of 1900 and later, and deal mainly with atmospheric conductivity, with ionization, and with the distribution of radioactive substances; while the older papers are concerned principally with the sign and the potential gradient of the earth's electric field. Atmospheric electricity therefore may be regarded as a new subject for investigation, so much more numerous are the present problems in connection with it, and so different the methods of attack, from those of even ten years ago.

Something is known of the electric potential gradient near the ground, and of its changes as determined by time of day, by season, by mountain and plain, by rainfall and by passing clouds; but this knowledge is fragmentary for elevations greater than one hundred meters, and may be said not to exist for regions beyond two kilometers above the earth's surface. However, this is not now the chief problem. More important are the questions—How does the atmosphere become electrified? and how, in spite of its conductivity, is a difference of potential, often amounting to more than one hundred volts per meter, maintained between it and the earth?

To anyone who wishes a summary of our knowledge to date of atmospheric electricity, which recently has become the object of many investigations, the above book is recommended.

Doubtless many will agree with the reviewer in hoping that in future editions this book, now consisting of 206 octavo pages, will be enlarged by the inclusion of chapters on thunder storms and auroras, by a fuller discussion here and there of the theory, and certainly by the addition of an index.

The author gives July 20, 1752, as the date of Franklin's famous letter to Peter Collinson. This probably refers to his kite letter of October 1 of that year, read on December 21, to the Royal Society. An earlier letter of Franklin's,[1] also to Collinson and dated July 29, 1750, proposes the study of lightning and suggests the use of lightning protectors.

W. J. Humphreys

Mount Weather, Va.

Annales de la Faculté des Sciences de Marseille. Tome XVII, Fascicule 3, 1908.

MM. Buisson and Fabry have published in this pamphlet the best map of the iron spectrum now available for spectroscopists. They were led to do this by the need of such an atlas for the ready identification of the iron lines they have used as "repères" in their determinations of wave-lengths by the interference method. Obtained with powerful apparatus and printed direct from the negatives without enlargement, the maps are very clean and the lines beautifully distinct. The scale is 1 tenth-meter = 0.76 mm, or almost exactly one-fourth that of Rowland's solar atlas. The map is printed on seven plates with four strips to the plate.

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[1] Rotch, *Ciel et Terre*, 27, 433.

F.

THE
ASTROPHYSICAL JOURNAL
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THE COMPLETE BALMER SERIES IN THE SODIUM SPECTRUM

By R. W. WOOD

The largest number of lines forming a Balmer series which has been observed in the spectrum of a terrestrial source occurs in the case of hydrogen, thirteen lines having been found by Cornu and by Ames (Kayser's *Spectroscopie*, **2**, 505). In the case of the hydrogen spectrum of the solar chromosphere Evershed (*ibid.*, p. 506) found 29 lines, the last line corresponding to $n=31$ in the formula. In the case of sodium but seven lines have been previously known, belonging to the principal series. As I have shown in a paper recently published (*Phil. Mag.* [6], **16**, 945, December 1908) it is possible to follow the series much farther in the absorption spectrum, and I gave provisional wave-lengths to the 24 lines which had been observed at the time of writing the paper.

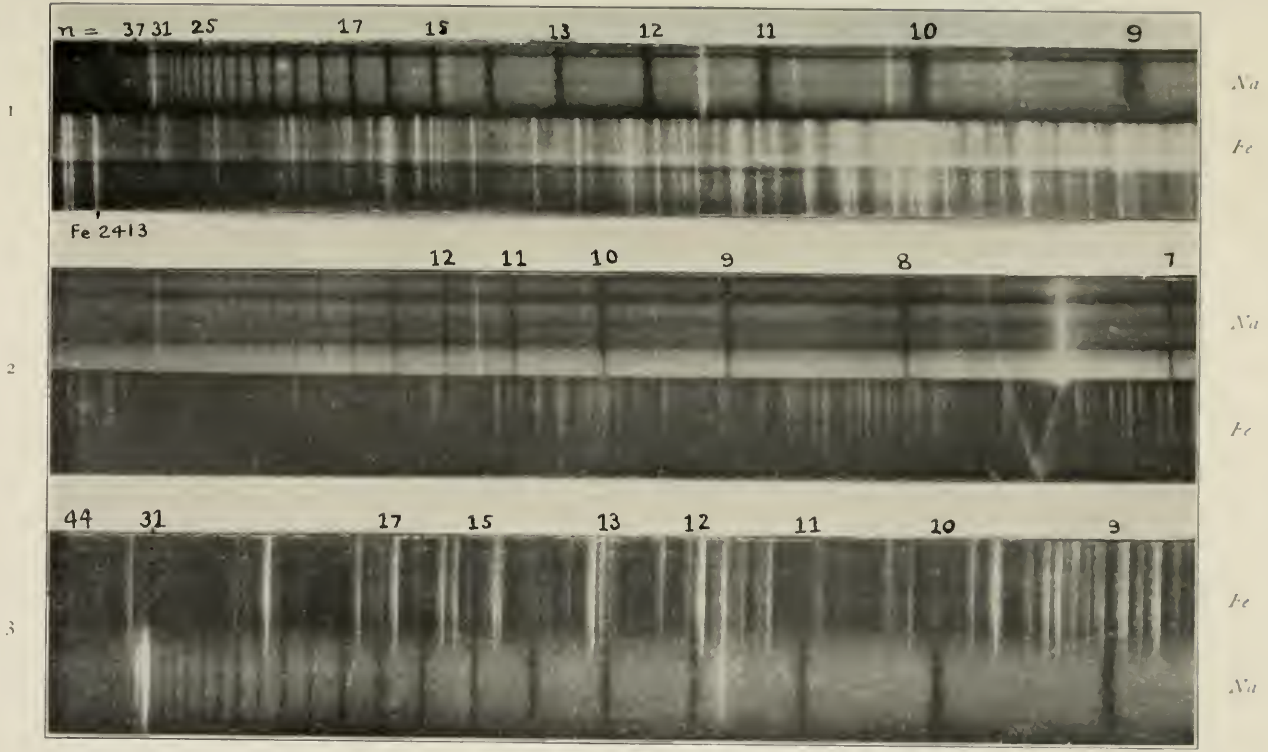
In this paper I ventured the opinion that we were dealing with the complete series, the limit depending merely upon the resolving power of the spectroscope and the vapor-density. This conjecture has been verified by some observations which I have recently made with the large quartz spectrograph at the Bureau of Standards, through the courtesy of Director Stratton. Dr. P. G. Nutting of the bureau very kindly placed his laboratory at my disposal for several days and worked with me upon the problem. The sodium was contained in a seamless tube of thin steel, which was exhausted to a pressure of a millimeter or two, and heated to a full red heat in a combustion furnace. The spectrograph had a focal length of a meter and was furnished with a large Cornu double prism of quartz. The first photograph which we made showed about forty lines, an increase of twelve. We used as a source of light a very powerful cadmium spark obtained from a 10,000-volt transformer with a condenser in the circuit. An exposure of about an hour was sufficient, with a column of sodium vapor 80 cm in length. There were indications, as before, that more lines would come out with a higher resolving power, and we accordingly added two more Cornu prisms, which gave us a dispersion and resolving power comparable with that of a 21-foot grating in the first order. An iron arc comparison spectrum was impressed on each plate, with an exposure of 15 seconds.

It was found that a good deal depended upon getting the sodium vapor at just the right density. The color of the transmitted light was deep violet, but if the vapor became too dense a general absorption appeared in the ultra-violet which spoiled the spectrum. On the best plate I have counted 48 lines, which enables us to test the formula up to the point where $n=50$. This puts solar hydrogen far in the rear with its 29 lines.

Three of the spectra are reproduced on Plate XII. They were taken with different degrees of vapor-density. In the case of the rarer vapor the end of the series does not come into view. It is interesting to note the progressive decrease in the width of the lines with decreasing wave-length. The first five lines of the series are not reproduced, as the scale is too large. With dense vapor they are very broad, and a channeled absorption spectrum was found bordering the first three lines in the ultra-violet, analogous to the one which is found both above and below the D lines. As I have shown in a previous paper published in the *Philosophical Magazine*, it is probable that the mechanism which produces the principal series is identical or connected with that which gives rise to the complicated channeled absorption spectrum, with its thousands of lines, for the absorption of blue-green light by the vapor causes the yellow D lines to appear in the fluorescence spectrum. It is most significant that the other lines of the principal series are bordered by channeled spectra as well as the D lines.

Even on the best plate, on which it is possible to count 48 lines,

PLATE XII



Balmer Series in Absorption Spectrum of Sodium Vapor

there are indications that higher resolving powers would bring more into view, for the head of the band is clearly indicated, though it is not resolvable into lines.

The plates were very carefully measured on the large dividing engine of the university, and the results agreed within about 0.02 of an Ångström unit. They are given in the following table.

WAVE-LENGTHS OF LINES IN BALMER SERIES OF SODIUM VAPOR

n	λ		n	λ	n	λ
3	5896.16	Previous observations	18	2433.85	35	2417.38
	90.19		19	31.43	36	17.10
4	3303.07		20	29.42	37	16.80
	02.47		21	27.72	38	16.56
5	2852.91		22	26.28	39	16.33
6	2680.46		23	25.00	40	16.11
7	2594.05		24	23.88	41	15.89
8	43.82		25	22.90	42	15.70
9	12.15		26	22.04	43	15.52
10	2490.70		27	21.29	44	15.37
11	75.60		28	20.60	45	15.21
12	64.53		29	20.02	46	15.06
13	56.02		30	19.50	47	14.94
14	49.46		31	19.00	48	14.78
15	44.24		32	18.44	49	14.64
16	40.06		33	18.09	50	14.50
17	36.70		34	17.71		

As will be seen from the table, the last 22 lines, i. e., nearly one-half of the whole series thus far observed, fall in a region of the spectrum not wider than the distance between the D lines!

Kayser gives on page 521 the constants of the Balmer formula for the sodium series, computed from the seven lines known at the time.

These seven lines are well represented by the expression

$$10^8\lambda^{-1}=41496.34-127040\,n^{-2}-843841\,n^{-4}.$$

With the complete series now at our disposal it will be necessary to redetermine the constants.

I have calculated the wave-lengths of the 32d and the 50th line from the formula, finding 2417.5 instead of 2418.44 (observed), and 2415.2 instead of 2414.5; that is, the calculated values are separated from the observed by a distance equal to the distance between four of the lines at this point of the spectrum. As will be seen from the table, the distance between the last lines is only 0.16 of an Ångström unit, and we are probably within one unit of the theoretical limit of the series.

I intend to make an attempt to extend the series still farther by photographing the spectrum with the 21-foot grating in the second order, though a very long exposure will be necessary, as a screen will have to be used which will cut out the superposed blue-green of the first order. Such a screen is not difficult to prepare, though its use may increase the necessary time of exposure.

One point of great interest noticed with very dense sodium vapor is the general absorption, which begins exactly at the head of the Balmer series and extends from this point down to the end of the ultra-violet. The vapor is much more transparent to the light between the absorption lines than in the region below the head of the series. The head of the series in the absorption spectrum actually shows much *brighter* on this account than the rest of the spectrum below it, exactly the opposite of what we should expect. Evershed's observation of a faint continuous *emission* spectrum below the head of the hydrogen series (chromosphere) is interesting in this connection. This matter in relation to the density of the vapor and the admixture of small traces of other gases will be further investigated.

Johns Hopkins University
December 9, 1908

ON THE APPARENT DISPERSION OF LIGHT IN SPACE

By PETER LEBEDEW

The discovery, on the one hand, of delays in the time of coincidence of lines, made by Belopolsky[1] in the case of the spectroscopic binary β *Aurigae*, and, on the other hand, the detection of certain photometric peculiarities of variable stars, made almost simultaneously with different apparatus by Nordmann[2] and by Tikhoff,[3] have led these gentlemen to explain the observed phenomena as the result of an appreciable dispersion of light in space. This assumption seemed to me so improbable that I preferred[4] to ascribe the observed phenomena to physical peculiarities of the observed stars, and not to new properties of space; this led to a lively exchange of opinions on the subject.[5]

Since the question of the dispersion of light in space is of significance in principle, it seems to me that a discussion of this matter is not superfluous.

I. DISPERSION OF LIGHT IN SPACE AND THE ELECTROMAGNETIC THEORY OF LIGHT

If we adopt the standpoint of the electromagnetic theory of light, which has been proved many times experimentally and has maintained itself so brilliantly, we can assign a dispersion only to a medium composed of the dispersionless ether and the selectively absorbing molecules imbedded therein. In such a medium the *dispersion is inseparably connected with the absorption of light.*[6] Planck[7] has computed the distance of relaxation, i. e., the thickness of the stratum in which the amplitude of a plane wave is reduced to its e^{th} part for a medium in which the dispersion corresponds to that of hydrogen at 760 mm and 0° C.

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[1] *Bulletin de l'Acad. de St. Pétersbourg*, **21**, 153, 1905 (in Russian).

[2] *Comptes Rendus*, **146**, 266 and 383, 1908.

[3] *Ibid.*, **146**, 570, 1908; *Mitteilungen der Nikolaisternwarte in Pulkowa*, No. 21, 1908.

[4] *Bulletin de l'Acad. de St. Pétersbourg*, **24**, 93, 1906 (in Russian); *Comptes Rendus*, **146**, 1254; **147**, 515, 1908.

[5] A. Belopolsky, *Bulletin de l'Acad. de St. Pétersbourg*, **24**, 97, 1906 (in Russian); C. Nordmann, *Comptes Rendus*, **147**, 24 and 620, 1908; G. A. Tikhoff, *ibid.*, **147**, 170, 1908; J. Stein, *ibid.*, **147**, 228, 1908.

[6] The connection between dispersion and absorption is not limited to solely electromagnetic processes, occurring for all vibrations, both elastic and acoustic.

[7] *Sitzungsberichte der Berliner Acad.*, 1904, 748.

Line	Index	Distance of Relaxation
B	1.00014217	1.8×10^8 cm
D	1.00014294	1.0×10^8 cm
G	1.00014554	2.7×10^7 cm

The delay of light between the lines D and G, which accordi to Nordmann and Tikhoff occurs in space, would have to be from 0.000,000,01 to 0.000,000,34; that is, it would be equal to the dispersion in hydrogen at 0° C. and at from 3 mm to 100 mm pressure. For hydrogen at 3 mm pressure, the relaxation-distance for the D line would amount to about 2.5×10^9 cm. But through such a medium no appreciable fraction of the light of the sun or of the stars could in any way reach the earth. If for hydrogen we substitute any other dispersive gas, and if we make the assumption that this gas is uniformly distributed, or not, in the direction of the ray, the final result is thereby not altered: the sun and the stars would remain invisible. The possibility is therefore excluded that demonstrable dispersion in space can be caused by the presence of ponderable matter subject to known physical laws.

There would be left the hypothesis that the ether itself might produce a dispersion without absorption. This assumption, however, is equivalent to asserting that all our electromagnetic theories must be cast aside at once, since the question as to the dispersion of light in pure ether is by no means one which has been overlooked or has not received attention in scientific investigation. The assumption that there is *absolutely no dispersion* in pure ether forms the *basis of all the electromagnetic theories*, and countless experimental results of the most varied sorts (cathode rays, Röntgen rays, and everything in optics) are in entire harmony with this assumption. It is therefore not permissible to propose that such a dispersion exists in space as a provisional astronomical hypothesis to satisfy observations; it could be proposed only after a mature critical examination of its foundation and with a clear idea of its enormous significance.

In what follows I shall attempt to show that positive evidence of a dispersion of light in space, which Nordmann and Tikhoff believe they have found, gives no occasion for a revision of our electromagnetic theories, inasmuch as the *methods employed are on principle inapplicable* and the positive result announced is *illusory*.

II. THE SPECTROSCOPIC BINARY β *Aurigae*

Tikhoff employed the spectroscopic data as to this star obtained by Belopolsky and himself in the following manner. From spectrograms made for two regions, the "blue" (λ 442 to λ 462), and the "violet" (from λ 393 to λ 410), he computes the epochs of coincidence of the lines, and he finds that for the "violet" rays the epoch is delayed by 0.015 day with respect to the blue rays; from this Tikhoff concludes that there is a demonstrable amount of dispersion of light in space.

In his computation he makes the tacit hypothesis that all "blue" spectral lines and all the "violet" lines, respectively, coincide simultaneously (i. e., differ by less than 0.005 day and are free from systematic errors). This hypothesis is false, however, and consequently Tikhoff's whole computation is illusory, and his publication already cited here gives only mean values from which the error committed can no longer be proved. In order to show this error clearly, I give in Table I the direct measurements,[1] limiting myself to three "violet" lines ($\lambda_1=393$, *Ca;* $\lambda_2=405$, *Fe;* $\lambda_3=408$?), for which the observed data is most extensive in the neighborhood of the second epoch of coincidence, at about 2.00 days. The material is collected for different epochs, E, which are reckoned forward in days from the moment of the first coincidence; under v_1, v_2, v_3, the differences in kilometers per second of the measured velocities of the two components according to the measures of Belopolsky (B) and Tikhoff (T) are given. As Tikhoff states, the velocities (v) are well represented by the formula

$$v=222 \text{ km} \sin \frac{360\times E}{3\cdot9599}.$$

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[1] I extract these original measurements from the hardly accessible paper by Tikhoff, printed in Russian, "Attempt at an Investigation of the Dispersion in Space on the Basis of Observations of the Spectroscopic Binary β *Aurigae*" in the *Publications of the School of Mines* in Jekaterinoslaw, 1905, pp. 32, 33.

With this formula I have computed the epochs C_1, C_2, C_3, of the coincidence of the lines from each separate observation, and I have formed the differences C_1-C_2 and C_2-C_3 for each plate.

TABLE I

E		$\lambda_1=393\ \mu\mu$ v'_1	$\lambda_2=405$ v'_2	$\lambda_3=408$ v'_3	C_1	C_2	C_3	C_1-C_2	C_2-C_3
d		km	km	km	d	d	d	d	d
1.407	T	181.7	....	179.6	2.012	.....	2.001	.....	....
	B	174.5	170.9	185.0	1.977	1.877	2.191	+0.100	−0.314
1.480	T	160.6	150.5	162.4	1.990	1.951	1.977	+0.039	−0.04
	B	163.6	156.7	157.2	2.003	1.973	1.996	+0.030	−0.02
1.585	T	144.2	130.1	136.9	2.031	1.910	2.004	+0.121	−0.094
	B	143.6	125.1	132.2	2.029	1.962	1.987	+0.067	−0.025
2.249	T	− 92.3	− 98.4	.....	1.979	1.960	.....	+0.019	.....
2.366	T	−118.5	−129.5	−103.3	2.011	1.973	2.061	+0.038	−0.088
	B	−116.2	−122.7	−110.0	2.019	1.993	2.039	+0.026	−0.046
2.404	T	−145.1	−132.2	−130.2	1.955	2.002	2.009	−0.047	−0.007
	B	−139.1	−139.9	−132.0	1.977	1.974	2.003	−0.003	−0.029
2.442	T	−146.1	−138.1	−142.3	1.989	2.019	1.989	−0.030	−0.030
2.583	T	−173.0	−171.1	.....	2.020	2.028	.....	−0.008	.....
	B	?	−184.6	−157.3	?	1.969	2.087	.....	−0.118
2.629	T	−184.3	−182.9	.....	2.012	2.017	.....	−0.005	.....
2.632	T	−144.4	−196.7	−178.3	2.186	1.946	2.044	+0.240	−0.098
	B	−157.3	−198.1	−181.5	2.136	1.935	2.029	+0.201	−0.093

How we may now compute the mean values of C_1-C_2 and C_2-C_3, which observations would appear too discordant and uncertain, is of no consequence for the result. There is unquestionably a systematic difference of the epochs of coincidence C_1-C_2 of about +0.02 day, and for C_2-C_3 of about −0.04 day, which applies to the measurements of each observer on exposures made simultaneously on the same plate.

Hence it follows that it is not permissible to combine the results for separate "violet" lines in a mean value, as Tikhoff has done, and comparing it with a similar mean value for the "blue" lines (which were on different plates made with different instruments and at different times), to thence infer a difference of 0.015 day, inasmuch as this difference can have no real physical meaning. It is sufficiently evident from the data cited that the method proposed by

Tikhoff[1] of using the difference of epochs of coincidence[2] for the investigation of the dispersion of light in space is *invalid on principle:* otherwise we should be compelled to assert that there was a "normal" dispersion in the region from λ 393 to λ 405, and an "anomalous" dispersion in the region from λ 405 to λ 408.

It is obvious that the observed difference of the epochs of coincidence is to be sought in the individual peculiarities of the star and not in those of space.

III. THE DISPLACEMENT OF MONOCHROMATIC MINIMA OF VARIABLE STARS

From observations of variable stars with color-filters in approximately monochromatic light, Nordmann by direct photometric measurements, and Tikhoff by estimates of the photographic densities, found in agreement with each other that the epochs of minima in yellow light appreciably anticipate those in violet light.

Thus Nordmann finds for *β Persei* a difference in the time of minima of 16 minutes between λ 680 and λ 430, and of 9 minutes between λ 510 and λ 430. From this a difference of time of 11 minutes may be computed for the interval between λ 560 and λ 430. Tikhoff finds for *RT Persei* a difference in time of 4 minutes for the same spectral interval.

But if we place the results of the determinations of dispersion by the two observers side by side, and note that according to Nordmann the distance of *β Persei* amounts to 60 light-years, while *RT Persei* according to Tikhoff[3] has a distance of 740 light-years, we obtain the ratio:

$$\frac{11}{60}:\frac{4}{740}>30:1.$$

A method which yields such widely different results for the same constant cannot be anything else than incorrect and therefore *wholly inapplicable.*

The results obtained for the other stars observed (*λ Tauri*, *W Ursae majoris*) do not permit of a comparison, since their parallaxes are unknown.

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[1] *Memorie Spett. Ital.*, **27**, 107, 1898.

[2] The comparison of the epochs of coincidence would have a physical basis only for lines of the same spectral series of an element. This is overlooked by Tikhoff.

[3] *Comptes Rendus*, **147**, 171, 1908.

The monochromatic light-curves differ very considerably among themselves. Fig. 1 gives the measurements of Tikhoff for *RT Persei* and *W Ursae majoris*.[1] The fact that the monochromatic curves are *not solely* displaced parallel to themselves, is a proof that this behavior is due to the individual optical properties of the star, or to the imperfection of the methods of observation, and not to the dispersion of light. It is therefore physically impossible without discussing the whole behavior of the curves to extract *solely* the moments of minima and regard their differences as a proof that light is propagated in space with an appreciable dispersion. If we should logically apply this principle further, we should have to regard the assertion as not impossible that the green light of *W Ursae majoris* travels with two different velocities in space.

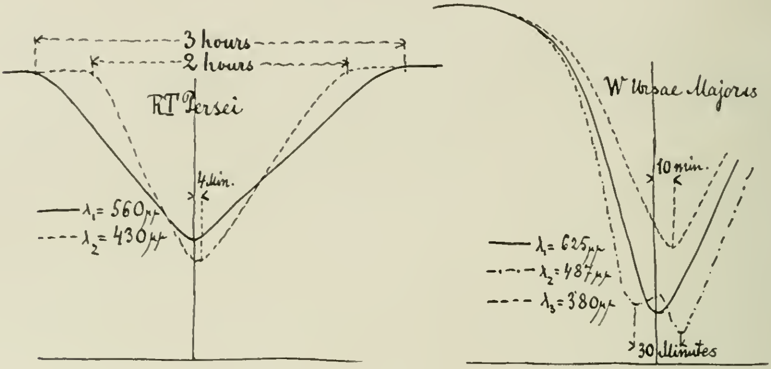

FIG. 1

IV. ON THE PHYSICS OF VARIABLE STARS

The displacement of the epochs of coincidence of the lines for *β Aurigae*, as well as the displacement of the monochromatic minima for *RT Persei* may be attributed in a natural way to physical processes on the variable stars. In what follows I would call attention to two processes which are competent to give to the stars the optical properties already described.

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[1] The form and position of the minima as given by Tikhoff (Fig. 1) can only be accepted with reserve: the photographic method is ill adapted to such investigations, as it furnishes the integral effect of a relatively long exposure time (e. g., for *RT Persei*, 10 to 20 minutes), and the fine details of the minima are concealed. For this case direct photometry as employed by Nordmann is decidedly more appropriate, as this furnishes the instantaneous values of the brightness; unfortunately these observations are not yet published.

The system of β *Aurigae* is according to Vogel[1] composed of two suns which are very close to each other; it is therefore to be assumed that the absorbing masses of gas which surround these suns are under a different pressure on the sides toward each other from that on the sides away from each other. An observer compares the coincidences of lines from two absorbing layers (*a* with *B*, and *A* with *b*, Fig. 2) which are under different degrees of pressure, and are therefore according to Humphreys displaced in the spectrum. The coincidences of lines then will not be observed at the transit of the stars through the line of sight, but at such velocities of the bodies in the line of sight that the Doppler displacement exactly corresponds to the pressure-shift: the epochs of coincidence must

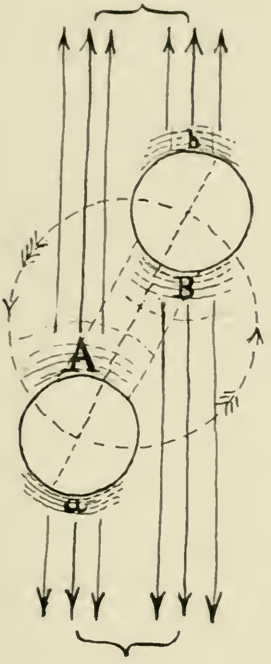

FIG. 2

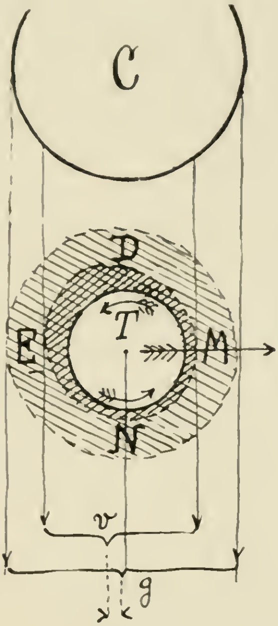

FIG. 3

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[1] *Sitzungsberichte der Berliner Acad.*, 1904, 514.

therefore turn out differently for different lines, which is in agreement with the discovery of Belopolsky as to β *Aurigae*.

He satisfactorily shows from the nature of the monochromatic curves of eclipse that the eclipsing body possesses an atmosphere with a selective absorption, for otherwise we should not be able to explain the conspicuous difference of the amplitudes of these curves.

We must assume in the case of *RT Persei* that the violet-absorbing atmosphere which causes an eclipsing during two hours is wrapped closer about the satellite than the yellow-absorbing atmosphere which causes an eclipsing during three hours. If we make the assumption that these atmospheres are not absolutely concentric and that their optical centers of gravity are displaced *by the thirtieth part* of their diameters with respect to each other in the direction of their orbital velocities, then the displacement of the monochromatic minima of four minutes measured by Tikhoff is quantitatively explained.

Such an asymmetry may be explained by the axial revolution of the satellite. The thermal effect of the central body C (Fig. 3) on the daylight side *D* may produce such alterations in the distribution of the atmosphere about the satellite, that an appreciable difference is caused in the boundaries of the violet-absorbing and the yellow-absorbing elements of the atmosphere on the morning side *M* and on the evening side *E* of the satellite.

Such a case occurs on a small scale in our planetary system. If a person on the moon was to observe a solar eclipse both in the infra-red rays at λ 3.0 μ, where there is a maximum of absorption of water-vapor, and also in the visible rays, at λ 0.5 μ, for which water-vapor is transparent, then he would note a displacement of these monochromatic minima, since the absolute humidity of the earth's atmosphere is greater on the evening side of the earth than on the morning side. This displacement would be measured in fractions of a second for an eclipse by the earth, but in an eclipse by a large satellite moving close to the central body, it might increase to several minutes.

The thermal phenomena are not the only ones which can be here effective: as Nordmann[1] remarks, tidal phenomena in the atmospheres of the central body and of the satellite must similarly produce strong optical asymmetry.

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[1] *Comptes Rendus*, 147, 24, 1908.

The observed data available are too incomplete to permit us to go further into the constitution of the stars β *Aurigae* or *RT Persei*. The ideas presented above as to the structure of these systems are only intended to serve for assigning to already known physical processes the cause of the phenomena regarded by Nordmann and Tikhoff as the dispersion of light in space. However, since such processes are very complicated for each individual star and are quantitatively entirely unknown to us, it is clear that the methods of Tikhoff and of Nordmann are not competent to separate a dispersion in space, even if existing, from these effects, whence *on principle such a dispersion cannot be demonstrated.*

It follows from what has been said above that now as heretofore, in agreement with all of our experience as to electromagnetic processes, we must regard *space as entirely free from any demonstrable dispersion of light.*

Moscow
November 1908

SPECTROSCOPIC INVESTIGATIONS OF THE ROTATION OF THE SUN DURING THE YEAR 1908[1]

By WALTER S. ADAMS

The results of a determination of the rotation of the sun based on photographic observations of the displacements of the spectrum lines were published by the writer in November 1907.[2] Two conclusions derived from this investigation were of special importance as bearing on a continuation of the work. The first was that a considerably longer series of observations was necessary in order to furnish reliable evidence in regard to a variation in the rate of the solar rotation. The second was that the rotation rate differed for different lines, making it particularly desirable to include in later determinations lines of elements showing a wide range in the altitude which they attain in the solar atmosphere. The observations made during the present year, accordingly, divide themselves naturally into two parts. The first is a direct continuation of the earlier work on the rotation of the general reversing layer with a view to detecting a possible variation in the rate of rotation during the interval covered. The second is a study of the lines of certain elements which are known from investigations of the chromospheric spectrum to rise to great altitudes in the sun's atmosphere. Preliminary values for one of these lines, the a line of hydrogen, have already been published.[3] In the present paper is given a more detailed account of the results of the research.

The photographs obtained during 1906–1907 were made with the Snow telescope and the 18-foot (5.5 m) Littrow spectrograph regularly employed with it for general spectroscopic work. During the autumn of 1907 the tower telescope was completed, and the decidedly superior advantages which it offered for an investigation of this sort led to its use for all observations made after January 1908. A detailed description of the tower telescope and the 30-foot (9.1 m) Littrow spectrograph used with it will be found elsewhere.[1] The images of the sun formed by the tower are as a rule considerably superior to those given by the Snow telescope. Not only is the definition of the image better, but there is much greater freedom from changes of focus, and the effect of astigmatism is largely eliminated. The last feature is of especial importance in observations of the rotation, since the effect of astigmatism is to bring upon the slit light from different latitudes on the sun's surface, and consequently to introduce systematic errors into the results. As compared with the Snow telescope the tower has but one serious disadvantage, namely, the color-curve of the objective which is used in place of the concave mirror of the former instrument. With a knowledge of the form of this curve, however, it is merely necessary to make a corresponding allowance in focusing the sun's image upon the slit of the spectrograph, and this has always been done when working in the violet region of the spectrum where the color-curve is comparatively steep. The difference in the size of the visual and the photographic images upon the slit is then readily computed from this difference of focus.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 33.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 20; *Astrophysical Journal*, **26** 203, 1907.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 24; *Astrophysical Journal*, **27**, 213, 1908.

The essential difference between the 30-foot spectrograph and the 18-foot instrument of the Snow telescope is that the former is capable of rotation about a vertical axis. This makes it possible to secure any desired position angle on the sun's surface directly. The spectrograph is provided with a divided circle 30 inches (0.76 m) in diameter upon which the readings of the position angles are made. The spectrograph lens can be focused and the grating rotated by handles which pass up the tube from below, and the frame carrying the plateholder is arranged so that any required tilt may be given to the plate.

The diagonal prism attachment used to bring the opposite limbs of the sun's image upon the slit is considerably more simple than that employed in the first series of observations, owing to the fact that it is fastened directly to the upper end of the spectrograph, and rotates with it, instead of being itself capable of rotation. A small casting about 8 inches high rests on the main plate of the spectrograph, its lower surface coming a short distance above the slit. Four small diagonal prisms are mounted on brass blocks fastened to the under surface of this casting, two immediately above the slit, and the other two below small slots in the casting. The distance between the centers of these slots is equal to the mean diameter of the sun's image. The second pair of prisms are capable of adjustment toward or away from each other, to allow for the variations in the sun's diameter. Each prism is held in a small mounting and provided with adjusting screws. On the top of the casting is an aluminum plate ruled with several concentric circles by means of which the sun's image is centered upon the slit. As soon as the prisms had been accurately adjusted so that the grating was uniformly illuminated, the casting was fastened to the top of the spectrograph, its position being accurately defined by tapered pins. It has been found unnecessary to disturb this adjustment in any way, since it is possible to clean the surfaces of the prisms without removing them from their mountings.

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[1] Hale, *Contributions from the Mount Wilson Solar Observatory*, No. 23; *Astrophysical Journal*, 27, 204, 1908.

It is evident that in a form of instrument in which the diagonal prisms are fixed the danger of errors arising from unequal illumination of the grating surface is much less than in cases where the prisms themselves are movable. I have been careful to test this adjustment frequently, however, usually immediately before the exposure and again at the end when the spectrograph is rotated 90° from its original position. The ratio of aperture to focal length in the case of the objective of the tower telescope is 1 to 60. In a spectrograph of 30 feet focal length, accordingly, a 6-inch grating would be fully illuminated. The one actually used has a ruled surface only 3.25 inches long, and in the higher orders is inclined at such an angle that a beam less than 3 inches in diameter is sufficient to fill it. The factor of safety evidently is considerable.

The grating used in this spectrograph is the same as that employed in the previous determination, with the 18-foot instrument, and described in the earlier paper. The photographs in the violet region of the spectrum have all been made in the third order of the grating; those of the α line of hydrogen in the second order. Both of these orders are exceptionally bright, and the definition is excellent. The linear scale of the plates in the violet of the third order is 1 mm = 0.56 Ångström.

The settings of the position circle of the spectrograph corresponding to the heliographic latitudes desired are made very readily with the aid of a short table of position angles of the sun's axis. In order to apply these, however, a reference line is necessary, and this has been found by observing the transits of the sun's image across the position circle when the coelostat mirror is rotated. The transits of both limbs are observed and the mean value taken as the transit of the sun's center. The readings on the opposite sides of the circle should, of course, differ by 180° if the image has originally been centered upon the slit. The line joining these points gives the true east and west direction, and the calculation of the desired settings becomes very simple. There is a considerable gain of time in obtaining the direction of the reference line in this way, rather than by allowing the image of the sun to drift across the position circle, and the adjustment of the instrument has been found to be so accurate that no appreciable error is introduced.

In selecting the heliographic latitudes to be observed I have followed the same course as in the earlier series of observations. For the reversing layer an exposure is made at every 15° of latitude between 0° and 90°, and so far as possible for at least one intermediate point in each zone. This gives a total of some twelve to fourteen points from which to determine the velocity curve. In the case of the other lines the observations have been limited to every 15° of latitude. The addition of the exposure at the pole of the sun furnishes a most valuable check upon the condition of the instrument, or any possible disturbance of the slit which may lead to a change in the inclination of the spectrum lines. This is an important advantage which the present series of plates possesses over the previous one made with the 18-foot spectrograph. With the latter instrument it was rarely possible to reach the sun's pole on account of the interference of the small diagonal prisms which brought the light to the slit.

It is unnecessary to describe in detail the method of reduction of the plates, since this differs but little from that given in the earlier paper. The calculations of the latitudes have as before been made with the use of De La Rue's reduction tables, and the quantities to be applied to correct the observed velocities for the departure of the sun's pole from its visible edge, as well as for the earth's motion, have been taken from Dunér's recent valuable memoir.[1] Most of the reversing layer plates have been measured by Miss Lasby upon the 150 mm Toepfer measuring machine. In the case of the other plates a small Gaertner comparator has also been used. The periodic errors for both instruments are well below the errors of measurement.

The discussion of the results obtained naturally divides itself into two parts. The first deals with the general reversing layer, by which is understood the region in the solar atmosphere at which the absorption takes place that gives rise to the great majority of the narrow lines in the spectrum. The second part deals with those lines upon which special investigations have been made.

RESULTS FOR THE REVERSING LAYER

In order to facilitate direct comparison with the values obtained during 1906–1907 it is clear that the list of lines upon which the determination is based should be nearly identical with the former list. There are, however, a few lines in the region of the spectrum λ 4190 to λ 4300, not previously included, to which special interest is attached. These have been added to the list, and at the same time two lines in the former list have been omitted. These gave values close to the mean derived from all the lines, and seemed to possess no particular significance. The revised list of lines is as appears in Table I.

In Table I four lines have been added to the list used in the determinations of 1906–1907. Of these λ 4207, like λ 4197 and λ 4216, belongs to the violet cyanogen fluting. Of the remaining three lines, λ 4283 and λ 4289 are due to calcium, and are included for purposes of comparison with λ 4227, while the line λ 4233 is added because of its interesting behavior in the spectrum of the chromosphere. Though assigned in Rowland's table to *Mn* it coincides with a strongly "enhanced" line of *Fe*, and its strength in the chromosphere makes the latter identification the more probable.

The series of plates used in this investigation began in February and continued through October, numbering 33 in all. With the exception of July, at least one plate has been taken during each month,

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[1] *Nova Acta Regiae Societatis Scientiarum Upsaliensis*, Ser. 4, Vol. I. No. 6.

TABLE I

λ	Element	Intensity	Behavior at Limb
4196.699	*La*	2	Much weakened
4197.257	*C*	2	Slightly weakened
4203.730	*Cr*	2	Strengthened and widened
4207.566	*C*	1 N	Weakened
4216.136	*C*	1	Weakened
4220.509	*Fe*	3	Slightly strengthened and widened
4232.887	*Fe*	2	Much strengthened and widened
4233.328	*Mn*	4	Much weakened. This is probably not *Mn*, but "enhanced" line of *Fe*
4257.815	*Mn*	2	Slightly strengthened and widened
4258.477	*Fe*	2	Much strengthened and widened
4265.418	*Fe*	2	Slightly weakened
4266.081	*Mn*	2	Slightly weakened
4268.915	*Fe*	2	Slightly weakened
4276.836	—*Zr*	2	Weakened
4283.169	*Ca*	4	Strengthened and widened
4284.838	*Ni*	1	Slightly weakened
4287.566	*Ti*	1	Slightly strengthened and widened
4288.310	*Ti, Fe*	1	Widened
4289.525	*Ca*	4	Probably slightly strengthened
4290.377	*Ti*	2	Slightly weakened. "Enhanced" line of *Ti*
4290.542	*Fe*	1	Slightly weakened
4291.630	*Fe*	2	Much strengthened

so that the interval may be said to be covered reasonably well. On account, however, of the great variation in the value of the angle the secant of which enters as a factor in correcting the observed velocities for the tilt of the sun's pole, it has been necessary to limit the observations at the highest latitudes to the time when this quantity was smallest. For this reason the plates giving the mean latitude 79°2 cover but a short period of time. In the case of some of the intermediate latitudes, as well, the number of observations has been increased toward the end of the series in order to bring the weights of these points into substantial agreement with the others. The observations at 0° and multiples of 15° from that point have been distributed nearly uniformly throughout the series.

In Table II is given a summary of the individual plates in the same form as that used in the previous determination. The velocities are corrected for the earth's motion, and are derived from a mean for all the lines.

The mean values derived from Table II and grouped about thirteen points of latitude will be found in Table V. Before entering upon a discussion of the results, however, it is desirable to

TABLE II

Number of Plate	Date	Number of Lines	ϕ	v	Number of Plate	Date	Number of Lines	ϕ	v
	1908			km		1908			km
ω 103	Feb. 16	23	0°.2	2.072	ω 120$_2$	June 2	22	50°.3	1.160
			14.7	1.948				65.8	0.697
			29.6	1.639				79.3	0.254
			44.4	1.289	ω 128	June 9	22	0.5	2.049
			59.1	0.776				14.5	1.917
			73.4	0.434				29.5	1.684
ω 105$_1$	Mar. 10	22	0.3	2.068				44.5	1.258
			14.6	1.982				59.5	0.829
			29.4	1.668				74.5	0.379
			44.2	1.343	ω 132	June 10	22	4.4	2.032
			59.3	0.844				19.4	1.896
			74.3	0.408				34.4	1.596
ω 105$_2$	Mar. 10	22	0.4	2.073				49.4	1.169
			15.2	1.960				64.4	0.795
			30.1	1.673				79.4	0.062
			45.1	1.305	ω 134	June 11	22	0.5	2.048
ω 106	Mar. 10	22	60.5	0.794				4.5	2.013
			75.0	0.416				19.5	1.759
			0.4	2.076				34.5	1.562
			15.2	1.971				49.5	1.166
			30.1	1.659				64.5	0.707
			45.1	1.304				79.6	0.262
			60.0	0.799	ω 135$_1$	June 11	22	0.5	2.060
			75.0	0.408				14.5	1.940
ω 113	April 8	22	0 0	2.095				29.5	1.675
			14.9	1.967				44.5	1.245
			29.8	1.682				59.5	0.773
			44.8	1.294				74.5	0.381
			60.7	0.800				79.5	0.263
			75.7	0.363	ω 135$_2$	June 11	22	0.5	2.038
ω 117$_1$	May 26	22	0.6	2.056				14.5	1.943
			14.4	1.900				29.5	1.680
			29.4	1.664				44.5	1.238
			44.6	1.301				59.5	0.772
			60.4	0.827				74.5	0.387
			75.9	0.383				79.5	0.266
ω 117$_2$	May 26	22	0.6	2.060	ω 136	June 11	22	0.5	2.047
			14.4	1.902				4.5	1.982
			29.4	1.664				19.5	1.848
			44.6	1.297				34.5	1.551
			60.4	0.845				49.5	1.149
			75.9	0.388				64.5	0.657
ω 120$_1$	June 2	22	12.5	1.965				79.5	0.254
			2.5	2.052	ω 146	Aug. 5	22	0.5	2.034
			17.5	1.866				14.6	1.934
			32.8	1.654				29.6	1.658
			48.3	1.172				44.9	1.266
			63.8	0.713				60.1	0.755
			77.3	0.335				74.9	0.403
ω 120$_2$	June 2	22	10.5	2.015	ω 147	Aug. 5	22	0.3	2.054
			4.5	2.039				14.6	1.959
			19.5	1.842				29.6	1.687
			34.8	1.609				44.9	1.269

TABLE II—*Continued*

Number of Plate	Date	Number of Lines	φ	v	Number of Plate	Date	Number of Lines	φ	v
	1908			km		1908			km
ω 147	Aug. 5	22	60°.1	0.782	ω 165	Aug. 27	22	49°.4	1.141
			74.9	0.403				64.9	0.694
ω 148	Aug. 5	22	0.3	2.056	ω 166	Aug. 27	22	4.2	2.048
			14.6	1.959				11.6	1.985
			29.6	1.688				19.1	1.875
			44.9	1.265				34.1	1.560
			60.1	0.805				49.4	1.149
			74.9	0.405				64.9	0.697
ω 151	Aug. 6	22	0.1	2.066	ω 179	Sept. 30	22	60.1	0.863
			0.1	2.059				60.1	0.865
			14.6	1.952	ω 180	Sept. 30	22	60.1	0.803
			44.6	1.270				60.1	0.805
			59.8	0.812	ω 182	Oct. 9	22	11.0	1.990
			74.9	0.416				11.0	1.991
ω 161	Aug. 26	22	4.1	2.044				19.0	1.843
			10.8	1.989				33.9	1.483
			19.2	1.886	ω 183	Oct. 9	22	33.9	1.548
			34.2	1.547				19.0	1.864
			49.4	1.081				11.0	1.992
			65.0	0.631				11.0	1.998
ω 162	Aug. 26	22	4.1	2.048				19.0	1.858
			10.8	1.988				33.9	1.549
			19.2	1.880				33.9	1.545
			34.2	1.504				19.0	1.859
			49.4	1.146	ω 184	Oct. 22	22	4.0	2.058
			65.0	0.695				65.5	0.671
ω 163	Aug. 26	22	4.1	2.040				60.3	0.822
			10.8	2.002				50.0	1.115
			19.2	1.890				50.0	1.129
			34.2	1.547				60.3	0.827
			49.4	1.151				65.5	0.667
			65.0	0.691	ω 185	Oct. 22	22	4.0	2.059
ω 164	Aug. 26	22	4.1	2.031				65.5	0.649
			10.8	1.994				50.0	1.104
			19.2	1.872				50.0	1.105
			34.2	1.557				65.5	0.656
		20	49.4	1.147	ω 186	Oct 22	22	4.0	2.056
		20	65.0	0.683				65.5	0.654
ω 165	Aug. 27	22	4.2	2.033				50.0	1.101
			11.6	1.992				65.5	0.633
			19.1	1.880				4.0	2.062
			34.1	1.556					

give the values for the individual lines, using the quantities obtained from all of the plates belonging to the same mean latitude. These are given in Table III. In accordance with the usual notation ξ denotes daily angular motion, and here, of course, corresponds to the sidereal period of rotation.

TABLE III

φ	λ	No. of Plates	v km	ξ
0°.34	4196.699	21	2.034	14°.44
	4197.257	21	2.046	14.52
	4203.730	21	2.061	14.65
	4207.566	21	2.051	14.56
	4216.136	21	2.042	14.50
	4220.509	21	2.058	14.62
	4232.887	21	2.066	14.68
	4233.328	21	2.054	14.59
	4257.815	21	2.076	14.75
	4258.477	21	2.069	14.71
	4265.418	21	2.060	14.63
	4266.081	21	2.074	14.74
	4268.915	21	2.072	14.72
	4276.836	21	2.066	14.67
	4283.169	21	2.070	14.70
	4284.838	21	2.069	14.69
	4287.566	21	2.065	14.66
	4288.310	21	2.066	14.67
	4289.525	21	2.070	14.70
	4290.377	21	2.061	14.64
	4290.542	21	2.070	14.70
	4291.630	21	2.071	14.71
4.08	4196.699	15	2.023	14.40
	4197.257	15	2.026	14.42
	4203.730	15	2.034	14.48
	4207.566	15	2.034	14.48
	4216.136	15	2.028	14.43
	4220.509	15	2.047	14.57
	4232.887	15	2.042	14.54
	4233.328	15	2.049	14.58
	4257.815	15	2.052	14.60
	4258.477	15	2.044	14.55
	4265.418	15	2.041	14.54
	4266.081	15	2.048	14.57
	4268.915	15	2.042	14.53
	4276.836	15	2.039	14.52
	4283.169	15	2.037	14.50
	4284.838	15	2.035	14.48
	4287.566	15	2.032	14.46
	4288.310	15	2.033	14.47
	4289.525	15	2.037	14.49
	4290.377	15	2.032	14.46
	4290.542	15	2.040	14.52
	4291.630	15	2.049	14.58
11.16	4196.699	12	1.986	14.37
	4197.257	12	1.987	14.37
	4203.730	12	2.005	14.48
	4207.566	12	1.997	14.44
	4216.136	12	1.991	14.41
	4220.509	12	2.006	14.49
	4232.887	12	2.001	13.47
	4233.328	12	2.003	14.48
	4257.815	12	2.006	14.49
	4258.477	12	2.006	14.49

TABLE III—*Continued*

φ	λ	No. of Plates	v km	ξ
11°.16	4205.418	12	2.004	14.48
	4266.081	12	2.004	14.48
	4268.915	12	2.001	14.47
	4276.836	12	2.010	14.55
	4283.169	12	1.999	14.45
	4284.838	12	1.995	14.43
	4287.566	12	1.997	14.43
	4288.310	12	1.998	14.44
	4289.525	12	1.999	14.45
	4290.377	12	1.976	14.30
	4290.542	12	2.001	14.43
	4291.630	12	2.000	14.44
14.86	4196.699	18	1.928	14.16
	4197.257	18	1.929	14.17
	4203.730	18	1.946	14.29
	4207.566	18	1.940	14.25
	4216.136	18	1.944	14.29
	4220.509	18	1.946	14.29
	4232.887	18	1.951	14.32
	4233.328	18	1.948	14.30
	4257.815	18	1.957	14.36
	4258.477	18	1.954	14.35
	4265.418	18	1.948	14.30
	4266.081	18	1.949	14.31
	4268.915	18	1.952	14.33
	4276.836	18	1.951	14.33
	4283.169	18	1.951	14.33
	4284.838	18	1.942	14.26
	4287.566	18	1.946	14.29
	4288.310	18	1.954	14.35
	4289.525	18	1.952	14.33
	4290.377	18	1.935	14.21
	4290.542	18	1.956	14.36
	4291.630	18	1.950	14.31
19.21	4196.699	14	1.862	14.00
	4197.257	14	1.862	14.00
	4203.730	14	1.874	14.09
	4207.566	14	1.878	14.12
	4216.136	14	1.865	14.02
	4220.509	14	1.878	14.12
	4232.887	14	1.878	14.12
	4233.328	14	1.881	14.14
	4257.815	14	1.879	14.13
	4258.477	14	1.875	14.09
	4265.418	14	1.865	14.02
	4266.081	14	1.876	14.10
	4268.915	14	1.872	14.06
	4276.836	14	1.866	14.02
	4283.169	14	1.863	14.00
	4284.838	14	1.864	14.01
	4287.566	14	1.865	14.02
	4288.310	14	1.869	14.04
	4289.525	14	1.866	14.03
	4290.377	14	1.862	14.00

TABLE III—*Continued*

φ	λ	No. of Plates	v km	ξ
19°21	4290.542	14	1.872	14°08
	4291.630	14	1.875	14.09
29.66	4196.699	16	1.652	13.50
	4197.257	16	1.655	13.52
	4203.730	16	1.678	13.70
	4207.566	16	1.666	13.61
	4216.136	16	1.647	13.46
	4220.509	16	1.671	13.65
	4232.887	16	1.670	13.64
	4233.328	16	1.663	13.58
	4257.815	16	1.676	13.69
	4258.477	16	1.677	13.69
	4265.418	16	1.666	13.61
	4266.081	16	1.674	13.66
	4268.915	16	1.679	13.71
	4276.836	16	1.673	13.66
	4283.169	16	1.672	13.65
	4284.838	16	1.657	13.53
	4287.566	16	1.672	13.65
	4288.310	16	1.676	13.69
	4289.525	16	1.676	13.69
	4290.377	16	1.666	13.61
	4290.542	16	1.683	13.75
	4291.630	16	1.685	13.77
34.11	4196.699	15	1.551	13.30
	4197.257	15	1.549	13.28
	4203.730	15	1.566	13.42
	4207.566	15	1.565	13.41
	4216.136	15	1.559	13.31
	4220.509	15	1.572	13.48
	4232.887	15	1.564	13.40
	4233.328	15	1.565	13.41
	4257.815	15	1.572	13.48
	4258.477	15	1.565	13.41
	4265.418	15	1.560	13.38
	4266.081	15	1.565	13.41
	4268.915	15	1.562	13.39
	4276.836	15	1.558	13.33
	4283.169	15	1.556	13.31
	4284.838	15	1.560	13.38
	4287.566	15	1.558	13.36
	4288.310	15	1.563	13.40
	4289.525	15	1.561	13.37
	4290.377	15	1.560	13.36
	4290.542	15	1.568	13.41
	4291.630	15	1.572	13.48
44.69	4196.699	17	1.276	12.74
	4197.257	17	1.280	12.78
	4203.730	17	1.289	12.87
	4207.566	17	1.284	12.82
	4216.136	17	1.273	12.71
	4220.509	17	1.290	12.88
	4232.887	17	1.282	12.80
	4233.328	17	1.288	12.86

TABLE III *Continued*

φ	λ	No. of Plates	v km	ξ
44°.69	4257.815	17	1.300	12°.98
	4258.477	17	1.295	12.93
	4265.418	17	1.284	12.82
	4266.081	17	1.298	12.96
	4268.915	17	1.293	12.91
	4276.836	17	1.285	12.83
	4283.169	17	1.288	12.86
	4284.838	17	1.283	12.81
	4287.566	17	1.292	12.90
	4288.310	17	1.286	12.84
	4289.525	17	1.298	12.96
	4290.377	17	1.278	12.76
	4290.542	17	1.295	12.93
	4291.630	17	1.295	12.93
49.56	4196.699	16	1.132	12.39
	4197.257	16	1.131	12.38
	4203.730	16	1.145	12.54
	4207.566	16	1.146	12.54
	4216.136	16	1.136	12.44
	4220.509	16	1.153	12.62
	4232.887	16	1.148	12.56
	4233.328	16	1.148	12.56
	4257.815	15	1.143	12.50
	4258.477	15	1.147	12.55
	4265.418	16	1.142	12.50
	4266.081	16	1.148	12.56
	4268.915	16	1.146	12.53
	4276.836	16	1.145	12.52
	4283.169	16	1.145	12.53
	4284.838	16	1.146	12.54
	4287.566	16	1.143	12.51
	4288.310	16	1.149	12.57
	4289.525	16	1.152	12.61
	4290.377	16	1.140	12.48
	4290.542	16	1.152	12.60
	4291.630	16	1.154	12.63
60.00	4196.699	22	0.796	11.31
	4197.257	22	0.798	11.33
	4203.730	22	0.811	11.57
	4207.566	22	0.808	11.54
	4216.136	22	0.802	11.40
	4220.509	22	0.813	11.52
	4232.887	22	0.807	11.45
	4233.328	22	0.811	11.52
	4257.815	22	0.821	11.66
	4258.477	22	0.811	11.52
	4265.418	22	0.812	11.53
	4266.081	22	0.811	11.52
	4268.915	22	0.818	11.62
	4276.836	22	0.817	11.61
	4283.169	22	0.815	11.60
	4284.838	22	0.812	11.52
	4287.566	22	0.815	11.56
	4288.310	22	0.814	11.55

TABLE III—*Continued*

ϕ	λ	No. of Plates	v km	ξ
60°00........	4289.525	22	0.812	11°53
	4290.377	22	0.805	11.44
	4290.542	22	0.816	11.59
	4291.630	22	0.817	11.60
65.04........	4196.699	17	0.666	11.20
	4197.257	17	0.669	11.25
	4203.730	17	0.678	11.40
	4207.566	17	0.674	11.33
	4216.136	17	0.672	11.27
	4220.509	17	0.684	11.51
	4232.887	17	0.680	11.44
	4233.328	17	0.683	11.49
	4257.815	16	0.683	11.49
	4258.477	16	0.680	11.44
	4265.418	17	0.680	11.44
	4266.081	17	0.685	11.52
	4268.915	17	0.689	11.59
	4276.836	17	0.683	11.49
	4283.169	17	0.680	11.44
	4284.838	17	0.683	11.49
	4287.566	17	0.680	11.44
	4288.310	17	0.682	11.48
	4289.525	17	0.688	11.57
	4290.377	17	0.678	11.40
	4290.542	17	0.684	11.50
	4291.630	17	0.686	11.53
74.90........	4196.699	17	0.382	10.41
	4197.257	17	0.386	10.53
	4203.730	17	0.395	10.76
	4207.566	17	0.389	10.60
	4216.136	17	0.388	10.57
	4220.509	17	0.401	10.93
	4232.887	17	0.399	10.88
	4233.328	17	0.388	10.57
	4257.815	17	0.407	11.09
	4258.477	17	0.401	10.93
	4265.418	17	0.399	10.88
	4266.081	17	0.405	11.04
	4268.915	17	0.407	11.09
	4276.836	17	0.403	10.99
	4283.169	17	0.403	10.99
	4284.838	17	0.409	11.14
	4287.566	17	0.401	10.93
	4288.310	17	0.403	10.98
	4289.525	17	0.399	10.88
	4290.377	17	0.394	10.73
	4290.542	17	0.405	11.04
	4291.630	17	0.402	10.99
79.16........	4196.699	7	0.268	10.10
	4197.257	7	0.270	10.18
	4203.730	7	0.271	10.20
	4207.566	7	0.272	10.25
	4216.136	7	0.263	9.90
	4220.509	7	0.268	10.10

TABLE III—*Continued*

φ	λ	No. of Plates	v km	ξ
79°.16	4232.887	7	0.263	9°.98
	4233.328	7	0.271	10.19
	4257.815	7	0.282	10.62
	4258.477	7	0.269	10.12
	4265.418	7	0.279	10.50
	4266.081	7	0.282	10.60
	4268.915	7	0.278	10.47
	4276.838	7	0.277	10.42
	4283.169	7	0.275	10.38
	4284.838	7	0.272	10.25
	4287.566	7	0.275	10.37
	4288.210	7	0.282	10.63
	4289.525	7	0.281	10.56
	4290.377	7	0.272	10.25
	4290.542	7	0.272	10.24
	4291.630	7	0.270	10.49

The behavior of individual lines will be best shown if we form the difference in the value of ξ between each line and the mean of all the lines.

These differences are found in the short summary which follows:

TABLE IV

λ	0°.3	4°.1	11°.2	14°.9	19°.2	29°.7	34°.1	44°.7	49°.6	60°.0	65°.0	74°.9	79°.2
4196.699	−0°.2	−0°.1	−0°.1	−0°.1	−0°.1	−0°.1	−0°.1	−0°.1	−0°.1	−0°.2	−0°.2	−0°.4	−0°.2
4197.257	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1	−0.2	−0.2	−0.2	−0.3	−0.1
4203.730	0.0	0.0	0.0	0.0	0.0	+0.1	0.0	0.0	0.0	+0.1	0.0	−0.1	−0.1
4207.566	−0.1	0.0	0:0	−0.1	+0.1	0.0	0.0	0.0	0.0	0.0	−0.1	−0.2	−0.1
4216.136	−0.1	−0.1	0.0	0.0	0.0	−0.2	0.0	−0.2	−0.1	−0.1	−0.2	−0.3	−0.4
4220.509	0.0	+0.1	0.0	0.0	+0.1	0.0	+0.1	0.0	+0.1	0.0	+0.1	+0.1	−0.2
4232.887	0.0	0.0	0.0	0.0	+0.1	0.0	0.0	+0.1	0.0	−0.1	0.0	0.0	−0.3
4233.328	−0.1	+0.1	0.0	0.0	+0.1	−0.1	0.0	0.0	0.0	0.0	0.0	−0.1	−0.1
4257.815	+0.1	+0.1	0.0	+0.1	+0.1	+0.1	+0.1	+0.1	0.0	+0.1	+0.1	+0.2	+0.3
4258.477	+0.1	0.0	+0.1	0.0	0.0	+0.1	0.0	+0.1	0.0	0.0	0.0	+0.1	−0.2
4265.418	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	+0.2
4266.081	+0.1	+0.1	0.0	0.0	0.0	0.0	0.0	+0.1	0.0	0.0	+0.1	+0.2	+0.3
4268.915	+0.1	0.0	0.0	0.0	0.0	+0.1	0.0	+0.1	0.0	+0.1	+0.2	+0.2	+0.2
4276.836	0.0	0.0	+0.1	0.0	0.0	0.0	0.0	0.0	0.0	+0.1	0.0	+0.1	+0.1
4283.169	+0.1	0.0	0.0	0.0	−0.1	0.0	0.0	0.0	0.0	+0.1	0.0	+0.1	+0.1
4284.838	0.0	0.0	0.0	0.0	0.0	−0.1	0.0	0.0	0.0	0.0	0.0	+0.3	0.0
4287.566	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	+0.1	+0.1
4288.310	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	+0.1	+0.3
4289.525	+0.1	0.0	0.0	0.0	0.0	+0.1	0.0	+0.1	+0.1	0.0	+0.1	0.0	+0.3
4290.377	0.0	−0.1	−0.1	0.0	−0.1	0.0	0.0	−0.1	−0.1	−0.1	0.0	−0.1	−0.1
4290.542	0.0	0.0	0.0	+0.1	0.0	+0.1	+0.1	+0.1	+0.1	+0.1	+0.1	+0.2	−0.1
4291.630	+0.1	+0.1	0.0	0.0	0.0	+0.1	+0.1	+0.1	+0.1	+0.1	+0.1	+0.2	+0.2

In Table IV the values at 79°.2 are based upon only seven plates, and hence are entitled to about one-half the weight of the other determinations.

An inspection of these results shows that the lines λ 4196, λ 4197, λ 4216, and λ 4290.38 give systematically low values of ξ, while λ 4257, λ 4266, λ 4291, and possibly λ 4268 give high values. A similar result was found for the first six of these lines in 1906–1907, and the general agreement of the two sets of determinations is sufficient to make the reality of these differences very probable, particularly in the case of the first three lines for which the differences are largest. Both series of observations give in the lower latitudes a value of −0°.12 for the lanthanum line λ 4196, and −0°.10 for the mean of the two cyanogen lines λ 4197 and λ 4216. Since these lines originate at a low level in the solar atmosphere they furnish additional evidence for the conclusion based on the special studies of $H\alpha$ and λ 4227 of calcium that the rate of angular rotation increases with increase of height above the sun's photosphere.

A comparison of the present results with those of 1906–1907 shows, however, that there is no such marked increase toward higher latitudes in the size of the deviations for these lines from the mean as was indicated by the earlier observations, although there is still some tendency in this direction. In the previous paper I ascribed this mainly to the effect of errors of measurement on the value of the angular velocity, an effect which in the higher latitudes becomes very large. At 75°, for example, a difference of 0.01 km in linear velocity corresponds to 0°.27 in the daily angular motion, and at 80° to 0°.41. There is still sufficient evidence, however, to warrant the presumption that the retardation becomes somewhat greater in higher latitudes, and this seems to be in harmony with the conclusions derived from investigations upon $H\alpha$ and λ 4227 of calcium.

Of the other lines which give systematic deviations, λ 4257, λ 4266, and λ 4290.38 gave similar results in the observations of 1906–1907, although the values were as a rule somewhat larger than those found here. The line λ 4290.38 was referred to at that time as of particular interest, on account of being an "enhanced" line of titanium, and the additional evidence confirming its behavior is especially important. The remaining two lines λ 4268 and λ 4291 showed no marked tendency toward large residuals, although giving small values of the same sign as those found above. The line λ 4291 is much strengthened at the limb of the sun.

The cyanogen line λ 4207, though showing a marked tendency toward negative residuals, gives considerably smaller values than do the other two cyanogen lines λ 4197 and λ 4216. This line is the most difficult of measurement of any in the list, and a part of the discrepancy may be due to this cause. In Rowland's general table it is ascribed to *Fe*, but in the table of corrections this is changed to *C*. The appearance of the line seems to favor a double origin, its breadth being considerably greater than that of either λ 4197 or λ 4216. If it is due in part to *Fe* the discrepancy would be fully explained.

The "enhanced" line of *Fe* λ 4233.33 gives a value which in the mean agrees exactly with the average from all of the lines. Like the "enhanced" line of *Ti* λ 4290.38, this line shows a considerable shift in position at the limb of the sun as compared with the center, which points to a relatively low-level origin for its "center of gravity." This result is in agreement with the rotation values. As opposed to this, however, is the occurrence of both of these lines in the chromosphere, the line λ 4233 being one of the most prominent in this region of the spectrum. The study of "enhanced" lines under pressure is likely to throw some light upon the behavior of these lines in the sun, but at present the contradiction seems to be marked.

Two other lines in the list deserve especial comment. These are λ 4283 and λ 4289, both due to calcium. The results indicate that slightly larger angular velocities are given by these lines, but the average difference is small, and the agreement with the result for the general reversing layer is close. We know from other considerations that the height attained in the sun's atmosphere by the gas giving rise to λ 4227 is very much greater than by that which produces most of the calcium lines of moderate intensity. Accordingly we should expect a marked difference in velocity. We are obliged, however, to draw a similar conclusion in regard to the two lines in the less refrangible part of the spectrum investigated by M. Perot by interference methods.[1] These lines, λ 5350 and λ 6122, were found to show very much greater angular velocities than the general reversing layer and but slight equatorial acceleration. This result agrees much more closely with what I have found in the study of λ 4227, a discussion of which will be given at a later point in this paper. The line λ 6122 is one of the strong lines belonging to the red triplet in the second sub-series of calcium. The other lines do not belong to known series, although both λ 4283 and λ 4289 have certain definite relations with other lines in their vicinity. The three lines of shorter wave-length do not seem to be present in the chromosphere, at least in any considerable intensity, but λ 6122 is given by Young in the revision of his chromospheric list.[1]

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[1] *Comptes Rendus*, **147**, 340, 1908.

We may now pass on to a consideration of the general results. If mean values are formed from Table II and grouped about thirteen points of latitude we obtain a summary of the following form. The weights are given according to the number of observations.

TABLE V

ϕ	Weight	v km	ξ	Period, Days
0°.3	21	2.063	14°.65	24.57
4.1	15	2.040	14.52	24.79
11.2	12	1.992	14.44	24.98
14.9	18	1.944	14.28	25.21
19.2	14	1.868	14.04	25.64
29.7	16	1.671	13.66	26.35
34.1	15	1.557	13.39	26.97
44.7	17	1.283	12.81	28.10
49.6	16	1.141	12.49	28.82
60.0	22	0.811	11.52	31.25
65.0	17	0.678	11.41	31.55
74.9	17	0.398	10.84	33.21
79.2	7	0.274	10.29	35.26

The velocities given in this table have been plotted graphically, and the full line shown in Fig. 1 indicates the curve to which they correspond most closely. The general agreement of the points is good, the largest deviation from the curve falling at the point of lowest weight 79°.2. Near 50° there is a distinct point of inflection. The curve shown by the broken line in the same figure represents the observations of 1906–1907. For numerical comparison the following table, giving the values from the two series of observations for every 5° of latitude, will be found useful. They are taken from large-scale

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[1] Scheiner's *Astronomical Spectroscopy* (Frost), p. 422.

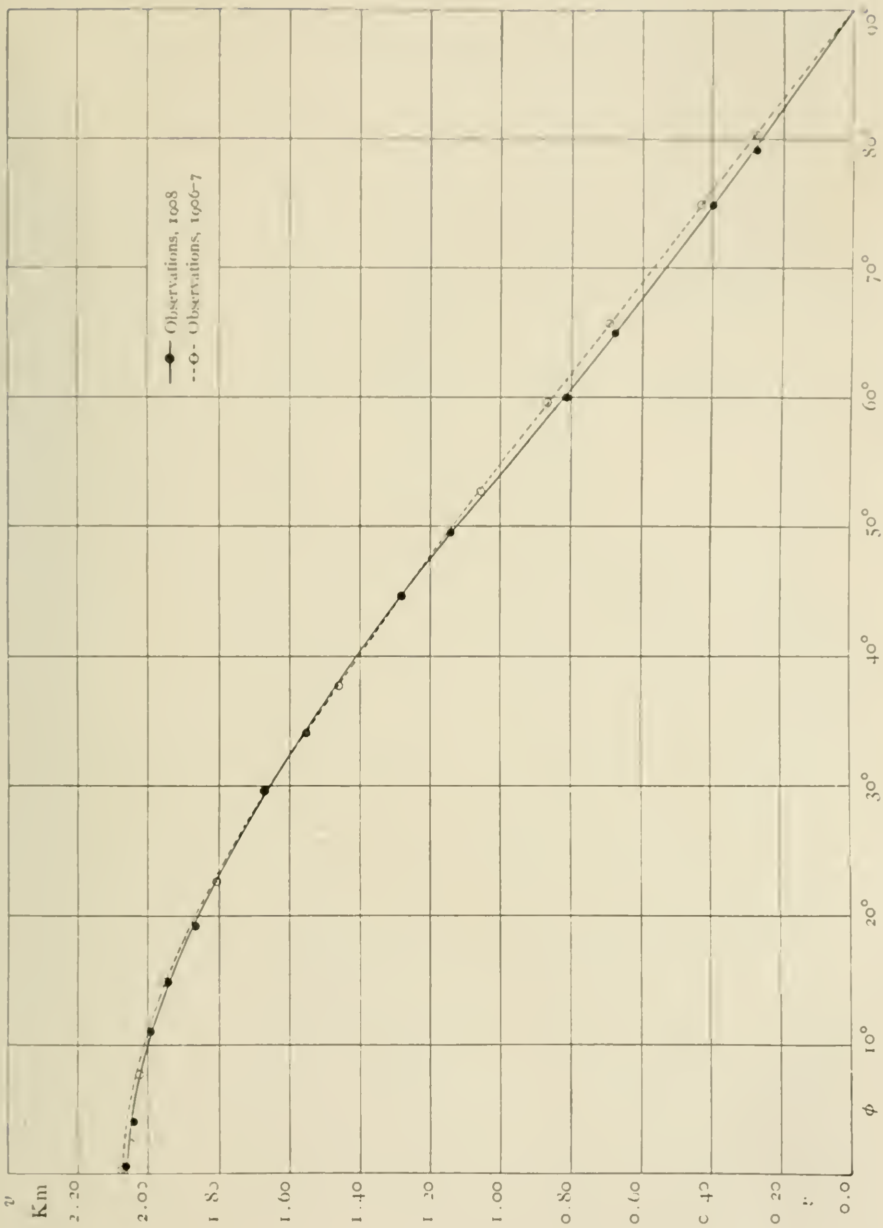

FIG. 1.—Radial Velocity Curves for the Reversing Layer in 1908 and 1906–1907.

drawings of the two curves, and are quite accurate enough for the purpose.

TABLE VI

ϕ	v (1908)	v (1906–7)	ξ (1908)	ξ (1906–7)
	km	km		
0°	2.060	2.077	14°.63	14°.75
5	2.043	2.056	14.56	14.65
10	2.000	2.012	14.42	14.50
15	1.943	1.954	14.28	14.36
20	1.863	1.872	14.08	13.14
25	1.761	1.768	13.80	13.85
30	1.659	1.660	13.60	13.61
35	1.537	1.532	13.32	13.28
40	1.414	1.406	13.10	13.03
45	1.274	1.274	12.79	12.79
50	1.126	1.137	12.44	12.56
55	0.967	0.991	11.97	12.27
60	0.820	0.851	11.64	12.08
65	0.677	0.711	11.37	11.94
70	0.533	0.569	11.06	11.81
75	0.396	0.429	10.86	11.77
80	0.266	0.290	10.88	11.85

A comparison of the two sets of observations shows slightly larger values for the earlier series between latitudes 0° and 25°, practical coincidence between 25° and 50°, and decidedly larger values again for the earlier series above this point. The differences in the lower latitudes reach a maximum of 0.017 km at the equator, and have an average value of about 0.01 km throughout the first 25° of latitude. These quantities, though larger than would be expected from errors of measurement, are still so small as to give little indication of the presence of systematic errors, or to show a possible variation in the sun's rate of rotation.

The matter is different in the higher latitudes, however. At 50° the two series of observations begin to separate from one another, and at 70° a difference of 0.036 km is reached. Though determinations of the probable error have shown that the value is usually somewhat larger in higher latitudes where the measured displacements are small, differences of this amount are evidently systematic, and the cause is to be sought either in a real change of rotation period in the higher latitudes, or in some source of systematic error in the observations, whether instrumental, or in the sun itself. At present there seems to be no definite criterion for settling the question, but there is, I think, sufficient evidence to furnish a presumption against the reality of a change in the sun's rotation period. The most important part of this is the difficulty of conceiving a change of rotation rate which is confined to high latitudes. It would certainly seem probable that any such change of rate would be in some way connected with the sun-spot activity, and would be conspicuous in the zones of greatest spot frequency between 10° and 30° of latitude. This is the conclusion which Halm has drawn from his two series of observations,[1] which show large differences in these zones as well as in higher latitudes. The results given here indicate almost no difference between the values of 1908 and those of 1906–1907 for these zones.

As against this it may be argued that evidence is gradually accumulating to show that the rotation rate at the sun's equator is more nearly constant than at any other latitude, and that the so-called "equatorial acceleration" is more truly a polar retardation. In this direction, for example, point the observations of Halm showing the greatest variations in angular velocity in higher latitudes, as well as the results given here, on the one hand for the low-level lines of lanthanum and cyanogen, and on the other hand for the high-level line λ 4227 of calcium and the α line of hydrogen. All of these show the greatest differences in angular velocity toward the pole. The force of this argument is considerable, and it may perhaps account for the apparent anomaly of a change in rotation rate which is confined to high latitudes on the sun.

A second argument which is entitled to considerable weight is the satisfactory agreement of the results of the 1908 observations with Faye's formula for the rotation period of sun-spots, and the fact that Dunér's observations, as well as those of Halm, are also in excellent accord with the same formula. If we put

$$v=(a+b\cos^2\phi)\cos\phi\ ,$$
$$\xi=a'+b'\cos^2\phi\ ,$$

and deduce the values of the constants by solving the normal equations given by the observations, we find

$$v=(1.506+0.549\cos^2\phi)\cos\phi\ ,$$
$$\xi=10\overset{\circ}{.}62+3\overset{\circ}{.}99\cos^2\phi\ .$$

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[1] *Transactions of the Royal Society of Edinburgh*, **41**, Part I.

In obtaining these results the observations have been assigned their respective weights, but the observation equations have been formed by multiplying with these weights instead of by their square roots.

The residuals given by these equations are as follows:

TABLE VII

ϕ	v km	ξ
0°3	+0.009	+0°04
4.1	−0.006	−0.07
11.2	−0.003	−0.02
14.9	−0.007	−0.07
19.2	−0.016	−0.14
29.7	+0.003	+0.03
34.1	0.000	+0.03
44.7	+0.016	+0.17
49.6	+0.015	+0.19
60.0	−0.010	−0.10
65.0	+0.001	+0.08
74.9	−0.004	−0.05
79.2	−0.013	−0.47

The general agreement of these results is quite as satisfactory as could be expected from a formula of such simple form, the largest residual being the point of low weight and high latitude at 79°2. In the 1906–1907 observations, on the other hand, it was found that Faye's equation gave deviations which, though small, were systematically negative in mean latitudes, and positive in high latitudes, and an equation containing three constants was employed in order to obtain suitable agreement. The fact that the results of Dunér and Halm as well as my own are satisfactorily represented by Faye's formula, while Spörer's formula gives systematic deviations, points to the conclusion that this simple expression represents with sufficient accuracy the motion of the sun's reversing layer to within about 10° of the pole, beyond which observations are lacking at present. The investigation of the zone between 80° and 90° of latitude, undertaken when the position of the sun's pole is most favorable for observations necessarily so difficult, will be of great interest, and should increase materially our knowledge of the law of rotation.

The close absolute agreement of Dunér's results with those given by the 1908 observations is also opposed to the idea of a change in the sun's rate of rotation. A comparison for the six latitudes employed by Dunér gives the following summary:

TABLE VIII

ϕ	Dunér	Adams
	km	km
0°.4	2.07	2.06
15.0	1.97	1.94
30.0	1.69	1.67
45.0	1.27	1.27
60.0	0.80	0.81
74.9	0.40	0.40

Dunér's results are based upon observations extending through six years, from 1887 to 1889, and from 1899 to 1901. Halm in his discussion[1] advocating a variation in the rotation rate of the sun has analyzed these observations, using as a basis Faye's equation in the form

$$v=(a-b\sin^2\phi)\cos\phi\ .$$

In this form the 1908 observations would give

$$v=(2.054-0.549\sin^2\phi)\cos\phi\ .$$

Halm's comparison of the Upsala and the Edinburgh observations on the basis of a three-year period would require a minimum value of the quantity a for the epoch 1908.5, which is about the mean date of these observations. The value found here, however, is practically equal to the largest value obtained by Halm in any one of the years 1901 to 1906, which included two maximum values of a. The value of b is also opposed to the idea of a progressive change in this quantity unless we assume that after decreasing from 1901 to 1905, it has reversed its direction and returned in 1908 to a value almost identical with that of 1901.

The general conclusion from these considerations seems to be that it is probable that the observations of 1906–1907 were affected by slight systematic errors which amounted to as much as 0.03 km in the higher latitudes. Attention has already been called to the fact that the astigmatism of the sun's image may have influenced the values to some extent, and it is possible that changes of focus may have also introduced slight errors. Another possible source of error due to disturbances on the sun itself will be discussed in a succeeding paragraph. In any event, it seems probable that the 1908 series of observations, being comparatively free from such defects of the solar image, and containing as they do a valuable check upon instrumental conditions in the exposures upon the pole of the sun, deserve a considerably higher degree of confidence.

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[1] *Astronomische Nachrichten*, 173, 294, 1907.

Determinations of the probable errors for the observations of 1908 indicate that these are smaller than for the earlier series. From a number of plates taken at random we find at 45° of latitude: for a single line,

$$\epsilon = \pm 0.009 \text{ km },$$

or for the mean value from the plate,

$$\epsilon_0 = \pm 0.002 \text{ km }.$$

The 1906–1907 observations gave

$$\epsilon = \pm 0.015 \text{ km },$$

and

$$\epsilon_0 = \pm 0.004 \text{ km }.$$

In these determinations the lines giving the principal systematically large or small values have been omitted. The 1908 probable errors, accordingly, are based upon sixteen lines, and the 1906–1907 errors upon fourteen lines. The probable errors are slightly larger in the higher latitudes.

THE MOTION OF THE REVERSING LAYER IN THE VICINITY OF A SOLAR VORTEX

On September 15 four plates of the region of the spectrum including λ 4227 were made with the rotation apparatus at latitudes ranging from 0° to 75° in steps of 15° each. On this date two spots of considerable size, one at 6° south latitude, the other at 11° north latitude, were close to the west limb of the sun, the northern spot being but a few hours distant from the visible edge, while the other had just passed beyond it. Plates taken with the spectroheliograph had shown these spots to be surrounded by vortices in which the motion was apparently in opposite directions in the two cases. Observations made by Hale during the passage of the spots across the sun's disk also showed opposite directions of polarization for the components of the double lines in the spot spectra. The region between the two spots, as indicated by photographs taken with the $H\alpha$ line, was in an extremely chaotic state, owing probably to the intermingling of the vortices. One of the settings in the case of the rotation plates fell at latitude 14°9 north, or 4° north of one of the spots; the other at 0°, or 6° north of the second spot, in this case being considerably east as well. When the measurement of the plates was begun it was at once seen that these two latitudes gave extremely discordant values as compared with the normal. Accordingly, the plates were investigated separately and measures made not only on λ 4227, but also on a number of the general reversing layer lines. As in the case of all of the photographs intended for the study of λ 4227, the density was made great in order to facilitate settings upon this very broad line, and for this reason the list of lines regularly used for the reversing layer could not be employed. So a number of stronger lines which appeared suitable for measurement were selected. The individual results for these lines and for λ 4227 are given in the following table:

TABLE IX

Number of Plate	Date 1908	Number of Lines	ϕ	Reversing Layer v km	λ 4227 v km
ω 173	Sept. 15	12	0°0	1.918	1.974
			14.9	1.896	1.960
			29.8	1.662	1.733
			44.7	1.249	1.325
			62.8	0.722	0.813
			75.3	0.387	0.492
ω 174	Sept. 15	12	0.0	1.900	1.963
			14.9	1.882	1.959
			29.8	1.673	1.723
			44.7	1.234	1.325
			62.8	0.731	0.808
			75.3	0.405	0.460
ω 175	Sept. 15	12	0.0	1.954	2.062
			14.9	1.913	1.984
			29.8	1.664	1.722
			44.7	1.257	1.282
			62.8	0.717	0.815
			75.3	0.377	0.460
ω 176	Sept. 15	12	0.0	1.949	2.029
			14.9	1.906	1.982
			29.8	1.667	1.722
			44.7	1.222	1.303
			62.8	0.726	0.859
			75.3	0.287	0.493

If we combine the preceding results we obtain the following summary. The normal value for the reversing layer is given in the third column for comparison.

TABLE X

ϕ	v km	Normal v km	λ 4227 v km
0°0	1.930	2.060	2.007
14.9	1.899	1.945	1.971
29.8	1.666	1.662	1.725
44.7	1.240	1.277	1.309
62.8	0.724	0.720	0.844
75.3	0.389	0.393	0.476

It is seen that in latitudes 30° and more the values derived from these plates agree well with the normal values. At 15°, however, the results are decidedly lower both for the reversing layer and for λ 4227. The same is true of latitude 0°, and the differences here are even greater. The plates taken with the spectroheliograph indicate a direction of motion in the vortex surrounding the northern spot that is counter-clockwise as seen from above. Accordingly, for a position north of the spot the component of motion in the line of sight at the sun's west limb would be toward the observer, and its effect would be to reduce the value of the linear velocity observed on the rotation plates. This is what is found.

In the case of the second spot the vortical motion should be in the opposite direction, judging from polarization and spectroheliograph results, and this is opposed to what is found from the rotation results at latitude 0°, if the latter are affected by this vortex. An examination of the *Ha* photographs shows, however, that the point from which the light is taken into the rotation apparatus falls at a considerable distance from the spot, and over a region of great irregularity where the two vortices seem to mix with each other, and where no well-defined direction of motion seems to exist. In any case the observations should be much more extensive and should include more points of latitude in order to give any definitive evidence as to the question of direction of rotation in such regions. The important fact is that solar vortices of this nature seem to have a marked influence upon the reversing layer, and that rotation results obtained when such regions are at the limb of the sun may be very seriously affected by the proper motion of the reversing layer. Since these vortices usually accompany spots, a natural inference would be that the greatest variations in values of the rotational velocity would be found in the latitudes where spots are most frequent, that is, in the zone 10° to 30°. It is possible that this may to some extent influence the results found by Halm in the two series 1901–1902 and 1903. The fact that vortices frequently exist where there are no spots, and extend to high latitudes, would also account for variations of rotational velocity found beyond the zones of principal spot activity. This may be a partial cause of the differences in latitudes 50° to 80° between the results 1906–1907 and 1908, since neither series contains a sufficiently large number of observations to eliminate the influence of a few cases of such proper motion. The necessary conclusion is that in making spectroscopic observations of the rotation of the sun especial care should be taken to avoid regions of the surface which show evidences of vortical disturbance.

RESULTS FOR λ 4227

The study of at least one of the stronger lines of calcium was made most desirable by the results obtained for the *a* line of hydrogen, as well as the differences found among the various lines used in the investigation of the reversing layer. Unfortunately the lines of greatest interest, namely H and K, are practically excluded by the immense variation in their physical appearance at different parts of the sun's disk, according to the presence or absence of calcium flocculi. They also appear to be especially subject to disturbances arising from motion of the calcium vapor in the line of sight.[1] Excluding these lines, it was evident that the so-called "blue line" of calcium at λ 4226.91 would prove most valuable for investigation. It is known to rise to a great height in the chromosphere, although, of course, much lower than either H or K, and its appearance in the spectrum is such as to admit of measures of considerable accuracy. There is also a decided practical advantage in having the line fall within the region observed for the motion of the reversing layer.

It at first seemed probable that the same plates which were used for the study of the reversing layer could be employed for λ 4227 as well. A few measures, however, showed that the line is of such great intensity, and the wings so broad under high dispersion that especially dense negatives would have to be employed, and it is upon a series of plates taken for this line alone that the results given here are based. An interesting by-product of the study of the line is the discovery that it is double. Several plates show this fact clearly, although the actual separation is extremely small.

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[1] *Contributions from the Solar Observatory*, No. 6; *Astrophysical Journal*, **23**, 45–53, 1906.

The results for the individual plates are given in the table below. In each case the value is the mean of two measures, one by Miss Lasby, and the other by myself.

TABLE XI

Plate	ϕ	v km	ϕ	v km	ϕ	v km	ϕ	v km	ϕ	v km	ϕ	v km
ω 149	0°3	2.10	14°6	2.01	29°6	1.75	44°9	1.32	60°1	0.84	74°9	0.47
ω 150	0.3	2.09	14.6	1.99	29.6	1.72	44.9	1.32	60.1	0.83	74.9	0.46
ω 152	0.1	2.14	14.6	1.99	29.7	1.72	44.6	1.32	59.8	0.89	74.9	0.47
ω 153	0.1	2.12	14.6	1.99	29.7	1.74	44.6	1.33	59.8	0.85	74.9	0.43
ω 154	0.1	2.10	14.6	2.02	29.7	1.74	44.6	1.34	59.8	0.81	74.9	0.43
ω 157	0.3	2.11	14.6	2.04	29.5	1.71	44.1	1.36	58.4	0.88	72.0	0.50
ω 158	0.3	2.10	14.6	2.04	29.5	1.70	44.1	1.34	58.4	0.88	72.0	0.48
ω 167	0.4	2.15	14.5	2.05	29.4	1.78	44.3	1.40	59.5	0.92	74.4	0.52
ω 168	0.4	2.12	14.5	2.05	29.4	1.74	44.3	1.39	59.5	0.92	74.4	0.57
ω 169	0.4	2.15	14.5	2.06	29.4	1.74	44.3	1.39	59.5	0.89	74.4	0.57
ω 170	0.4	2.16	14.5	2.05	29.4	1.75	44.3	1.42	59.5	0.97	74.4	0.54
ω 188	0.1	2.10	14.8	2.09	29.7	1.78	44.9	1.36	60.2	0.90	75.9	0.48
ω 189	0.1	2.12	14.8	2.02	29.7	1.78	44.9	1.39	60.2	0.94	75.9	0.48

These values give the following summary:

TABLE XII

ϕ	λ 4227			REVERSING LAYER		
	v km	ξ	Period	v km	ξ	Period
0°2	2.12	15°0	23.9	2.06	14°7	24.6
14.6	2.03	14.9	23.2	1.95	14.3	25.2
29.6	1.74	14.2	25.3	1.67	13.7	26.4
44.6	1.36	13.6	26.5	1.28	12.8	28.1
59.6	0.89	12.5	28.8	0.82	11.5	31.2
74.6	0.49	13.1	27.4	0.40	10.8	33.2

A comparison of the results for λ 4227 with those for the reversing layer shows: first, that the absolute velocity values are larger for λ 4227; and second, that the decrease of angular velocity is much less marked toward higher latitudes. At the equator the difference in angular velocity is 0°4, while at 45° of latitude it is 0°8. The sudden increase in the value of ξ at 75° will be discussed more fully in connection with the results for *Ha*. If we assume, however, that the increase is due to accidental errors, and take values from the curve at 60° and 75°, we find at 60° a difference in the value of ξ from that for the reversing layer of 1°4, and at 75° of 1°5. As will be seen later, similar results are found for *Hα*.

Reference has already been made to the results obtained by M. Perot for two of the less refrangible lines of calcium by the aid of interference methods. Preliminary values of the angular velocity given by him are as follows:

TABLE XIII

λ	$\phi=0°$	$\phi=45°7$
5349.6	15°1	14°2
6122.4	14.7	14.2

At the equator these results agree closely with those for λ 4227. At 45°, however, they give a considerably larger value, indicating less equatorial acceleration than in the case of λ 4227. This difference may be much modified by more complete results, but if it still remains it will indicate a higher effective level for these lines at the limb than for the more refrangible line. At first sight, in view of the great strength of λ 4227 in the chromosphere, this would seem improbable, but the width and the intensity of its wings and the comparative narrowness of the central line indicate that a large part of the line is produced in a region of relatively dense calcium vapor, and so at a moderately low level. This level must, nevertheless, be above that in which the wings of the stronger iron lines in the violet are formed, since plates of the center and limb spectrum show but a slight effect for λ 4227 when the wings of the iron lines are almost completely obliterated. This is in agreement with the rotation values.

RESULTS FOR THE *α* LINE OF HYDROGEN

The special investigation of the *Hα* line was begun, as stated in a previous paper,[1] in consequence of the remarkable behavior of this line at the limb of the sun. Its marked increase in width, and the absence of any displacement such as is found for the great majority of lines between limb and center, pointed to a very high-level origin, a fact, of course, well known from observations of the intensity of the line in the upper chromosphere. Measures of the center and limb plates also indicated a considerably higher rotational velocity, and this result was confirmed by the earliest rotation plates. The preliminary values for a few of these plates were published in the paper already cited. At the same time at which these determinations were being made, measures were obtained of the motion of rotation given by the hydrogen flocculi upon plates taken with the spectroheliograph in the δ line of hydrogen.[1] These showed a marked tendency toward a uniform rate of rotation, but the absolute rate was decidedly smaller than that derived from the spectroscopic measures. Two possible explanations of this behavior at once presented themselves. The first was that $H\delta$ might give a different rate from $H\alpha$, a conclusion by no means improbable, in view of the different way in which they are affected at the limb of the sun. The second was that the marked increase of intensity of $H\alpha$ near the limb might indicate quite a different level for the spectroscopic determinations from that of the hydrogen flocculi. Photographs of the $H\alpha$ flocculi were essential to test either hypothesis, and experiments were begun by Mr. Hale and myself with the 30-foot spectroheliograph. These when continued by Hale and Ellerman with the 5-foot spectroheliograph led to the discovery of the solar vortices.[2]

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 24; *Astrophysical Journal*, **27**, 213–218, 1908.

The earliest photographs of the $H\alpha$ line were taken on rapid plates sensitized to the red with pinacyanol. After the publication of Wallace's sensitizing process for "Pan-iso" plates[3] this was employed to great advantage. It was found, moreover, that the exposure times on rapid plates sensitized in this way were very moderate. Accordingly it occurred to me to endeavor to obtain the advantage of finer silver grain by the use of slower plates, and for this purpose the Seed "Process" plates were tried. These have proved very successful, giving fine grain and excellent contrast, while the ratio of sensitiveness in the red to that of similarly treated rapid plates seems to be greater than the corresponding ratio of the unbathed plates in the blue. The advantages in the use of these plates for such a line as

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 25; *Astrophysical Journal*, **27**, 219–229, 1908.

[2] Hale, *Contributions from the Mount Wilson Solar Observatory*, No. 26; *Astrophysical Journal*, **28**, 100, 1908.

[3] *Astrophysical Journal*, **26**, 299–325, 1907.

Hα are considerable, since the edges of the line are at best poorly defined, and the finer grain and the superior contrast assist materially in the measurement of the line. It is probable that the later plates show a considerably higher degree of accuracy for this reason, and in the summaries of the results which follow, each series begins with the first photographs taken on the slower plates.

It was found early in the study of the *Hα* line that its width changes very rapidly within a small distance of the sun's limb. In fact, the entire variation in appearance between limb and center seems to take place within a distance of little more than one-thirtieth of the solar radius. It seemed probable that this effect might be due to the relative level in the two cases, the effective level of the line at the limb being higher than that inside the limb. If this is the case, we should expect the effect to show in the rotation values given by the line in the two positions. Accordingly two series of observations have been made on *Hα*; the first at points close to the limb; the second at points averaging about 3 mm inside the limb. The values obtained from the latter have, of course, been reduced to the limb by correcting for this distance.

The results for the individual plates of *Hα* near the limb follow. As in the case of λ 4227 each result is based on two measures.

TABLE XIV

Plate	ϕ	v km	ϕ	v km	ϕ	v km	ϕ	v km	ϕ	v km	ϕ	v km
ω 110	0°.2	2.12	15°.1	2.04	29°.9	1.75	44°.3	1.45	60°.4	0.97	75°.2	0.53
ω 115	0.4	2.13	14.6	2.03	29.6	1.72	44.9	1.38	60.5	0.94	75.9	0.49
ω 118_1	0.6	2.15	14.4	2.05	29.4	1.77	44.9	1.43	60.4	0.98	75.9	0.49
ω 118_2	0.6	2.17	14.4	2.06	29.4	1.74	44.9	1.44	60.4	0.99	75.9	0.52
ω 122	0.5	2.13	14.5	1.99	29.5	1.79	44.5	1.35	59.5	0.93	75.1	0.49
ω 123	0.5	2.16	14.5	2.03	29.5	1.78	44.5	1.41	59.5	0.96	75.1	0.48
ω 124_1	0.5	2.10	14.5	2.09	29.5	1.78	44.5	1.41	59.5	0.99	75.1	0.53
ω 124_2	0.5	2.14	14.5	2.05	29.5	1.78	44.5	1.39	59.5	0.97	75.1	0.52
ω 125_1	0.5	2.17	14.5	2.05	29.5	1.82	44.5	1.42	59.5	0.97	75.0	0.50
ω 125_2	0.5	2.14	14.5	2.02	29.5	1.80	44.5	1.43	59.5	0.96	75.0	0.40
ω 126_1	0.5	2.15	14.5	2.04	29.5	1.81	44.5	1.41	59.5	0.95	75.0	0.50
ω 126_2	0.5	2.16	14.5	2.02	29.5	1.84	44.5	1.42	59.5	0.98	75.0	0.53
ω 127_1	0.5	2.16	14.5	2.04	29.5	1.80	44.5	1.38	59.5	0.95	74.5	0.52
ω 127_2	0.5	2.17	14.5	2.02	29.5	1.83	44.5	1.40	59.5	0.98	74.5	0.50
ω 129	0.5	2.15	14.5	2.04	29.5	1.82	44.5	1.41	59.5	0.96	74.5	0.52
ω 130_1	0.5	2.15	14.5	2.03	29.5	1.80	44.5	1.40	59.5	1.00	74.5	0.53
ω 130_2	0.5	2.14	14.5	2.03	29.5	1.79	44.5	1.40	59.5	0.95	74.5	0.55
ω 131	0.5	2.14	14.5	2.04	29.5	1.81	44.5	1.41	59.5	0.98	74.5	0.52
ω 141	0.1	2.14	14.8	2.04	29.7	1.79	44.9	1.41	60.1	0.99	75.1	0.57
ω 144	0.1	2.15	14.8	2.04	29.7	1.79	44.9	1.44	60.1	1.00	75.1	0.59
ω 171	0.3	2.12	15.2	2.04	30.0	1.76	45.0	1.42	60.2	0.98	75.0	0.60

If we form means from the above results we obtain the following summary. The corresponding results for the reversing layer are given in the last three columns.

TABLE XV

ϕ	$H\alpha$			Reversing Layer		
	v km	ξ	Period	v km	ξ	Period
0°.4	2.15	15°.2	23.6	2.06	14°.6	24.6
14.6	2.05	15.0	24.1	1.95	14.3	25.2
29.6	1.79	14.6	24.7	1.67	13.7	26.3
44.6	1.41	14.0	25.7	1.28	12.8	28.1
59.8	0.97	13.7	26.2	0.81	11.5	31.2
75.0	0.52	14.3	25.2	0.40	10.8	33.2

Two important conclusions are evident from an inspection of these results. The first is that the hydrogen gas giving rise to $H\alpha$ moves at a much higher velocity than the reversing layer. The second is that the law of change of velocity with latitude is very different, the equatorial acceleration being comparatively slight. Both of these conclusions were derived from the preliminary values given in the paper previously referred to, but the absolute values differ considerably. In the preliminary results, for example, no equatorial acceleration whatever was found, while these results give a difference of over 1° in the value of ξ between the equator and high latitudes. The absolute velocity at the equator was also found to be about 0°.3 larger in the preliminary results. The greater part of this difference is no doubt due to the small number of plates upon which the early measures were based, and also to the fact that these plates were comparatively inferior in quality, being obtained previous to the use of the finer-grained "Process" plates. There is, however, another cause which may contribute to a systematic difference in the two cases. This is the fact that the earlier plates were taken at points closer to the edge of the sun than were the later plates, so close, indeed, that on two out of the three $H\alpha$ plates used in the early measures, the bright chromospheric line appears in addition to the dark line. Accordingly it is altogether probable, judging from results found for $H\alpha$ inside the limb, that in the plates used in the preliminary determination a higher average level was in question than in the case of the plates discussed here, and that this higher level would give larger velocities and less equatorial acceleration.

The results obtained from plates of *Ha* taken at points within the limb are given below.

TABLE XVI

Plate	ϕ	v km	ϕ	v km	ϕ	v km	ϕ	v km	ϕ	v km	ϕ	v km
ω 116	0°.1	2.14	14°.9	2.03	29°.9	1.72	45°.1	1.38	60°.0	0.91	76°.2	0.44
ω 119_1	0.6	2.12	14.4	2.01	29.4	1.71	44.9	1.36	60.4	0.91	75.9	0.46
ω 119_2	0.6	2.13	14.4	2.01	29.4	1.74	44.9	1.32	60.4	0.88	75.9	0.42
ω 131	0.8	2.13	14.4	2.01	29.4	1.76	44.9	1.36	59.2	0.96	75.9	0.50
ω 137	0.5	2.10	14.5	1.99	29.4	1.73	44.6	1.39	59.8	0.88	74.9	0.49
ω 138	0.4	2.08	14.5	2.00	29.4	1.74	44.6	1.36	59.8	0.87	74.9	0.49
ω 142	0.1	2.14	14.8	2.03	29.7	1.75	44.9	1.41	60.1	0.94	75.1	0.50
ω 143	0.1	2.13	14.8	1.99	29.7	1.71	44.9	1.37	60.1	0.94	75.1	0.52
ω 172	0.3	2.08	15.2	1.97	30.0	1.71	45.0	1.37	60.2	0.92	75.0	0.53

Summarized, these results give the following. The values for *Ha* at the limb are added for comparison.

TABLE XVII

ϕ	*Ha* (within limb)			*Ha* (limb)		
	v km	ξ	Period	v km	ξ	Period
0°.4	2.12	15°.0	24.0	2.15	15°.2	23.6
14.7	2.00	14.7	24.5	2.04	15.0	24.1
29.6	1.73	14.1	25.5	1.79	14.6	24.7
44.8	1.37	13.7	26.3	1.41	14.0	25.7
60.1	0.91	13.0	27.8	0.96	13.6	26.3
75.4	0.48	13.6	26.4	0.51	14.2	25.4

A comparison of these values shows not only that the velocities for *Ha* at the limb are larger than those at points within the limb, but also that the equatorial acceleration is less, or the rate of rotation more nearly uniform. It is clear, then, that the hydrogen gas producing *Ha* at the limb must lie at a generally higher level, a result which is in agreement with the character of the spectrum line in the two positions. In the case of a gas such as hydrogen, which extends to a great height in the solar atmosphere, it is evident that the thickness of the absorbing layer in the line of sight must increase very rapidly toward the sun's limb. The level of the effective absorption, accordingly, will in general be higher at the limb than on the disk of the sun, and if the upper regions are cooler the effect should show itself in the change of intensity which accompanies decrease of temperature. Kayser has shown[1] that with decreasing temperature there is a strengthening of the red line of hydrogen relative to the more refrangible lines. At the limb of the sun it is found that the intensity of $H\alpha$ is greatly increased as compared with its intensity at the center, $H\beta$ is also slightly increased, while $H\gamma$ and $H\delta$ are somewhat weakened. The agreement with the laboratory results, then, is complete.

It is as yet impossible to compare these results with those obtained from the study of the $H\alpha$ flocculi on spectroheliograph plates, but measures now being made by Miss Ware will furnish the material for such a comparison. It will also be of special interest to discuss the results of the $H\alpha$ and the $H\delta$ flocculi separately, since the general evidence at present indicates a higher level for the $H\alpha$ flocculi as a class. It has not been found possible to use either $H\gamma$ or $H\delta$ to advantage in the spectroscopic study, both on account of the inherent character of the lines and the disturbances produced by the presence of foreign lines.

The results for λ 4227, $H\alpha$ at the limb and $H\alpha$ within the limb, are shown graphically in Fig. 2. The fourth curve, indicated by a dotted line, corresponds to the linear velocity of a body rotating with the uniform angular velocity of 15°2 a day. It is, of course, the usual cosine curve. A marked feature of all of the results is the sudden increase in angular velocity near 75° of latitude. Although at this latitude the angular velocity is especially sensitive to small differences in linear velocity, the persistence of the effect for the three independent series of observations furnishes some presumption of its reality. It is, moreover, the counterpart of the differences from the general reversing layer found among the low-level lines. These, as was seen from the results given earlier in this paper, show exceptionally large deviations in the higher latitudes. At present the question of the reality of this effect must remain an open one.

GENERAL SUMMARY

The main results of this investigation may be summarized as follows:

1. Observations of the rotation of the sun during 1908 give values

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[1] *Festschrift* Ludwig Boltzmann (Leipzig, 1904), p. 38.

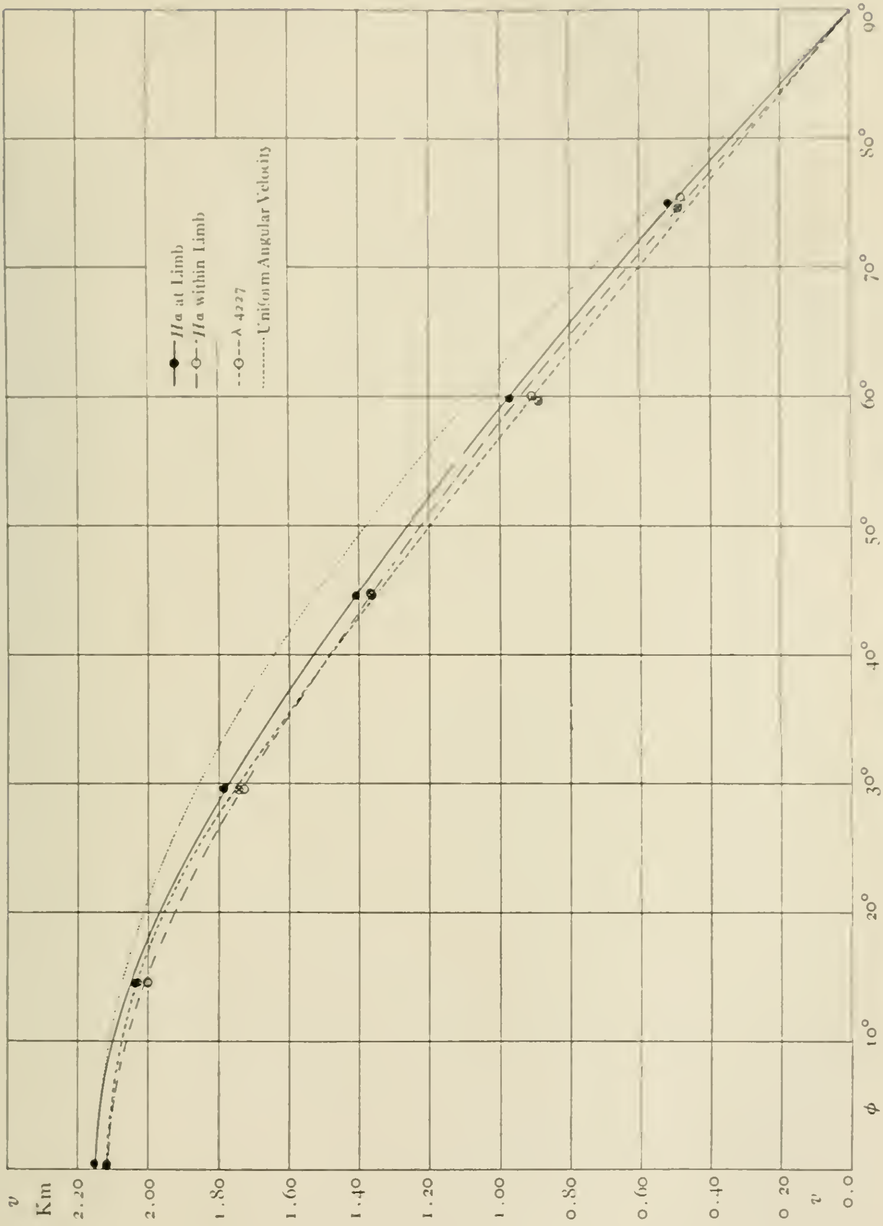

FIG. 2.—Radial Velocity Curves for *Hα* and λ 4227

agreeing closely with those of 1906-1907 between latitudes 0° and 50°. Above 50° they give larger values, the difference in linear velocity reaching at a maximum 0.036 km.

2. The general agreement of the results, and the excellent accord with Dunér's values, are opposed to the existence of a variation in the rotation rate between 1906 and 1908. If any such variation exists it is confined to the higher latitudes, and does not appear in the zones of greatest spot activity. The results are also opposed to a three-year period of variation, such as was obtained by Halm from a comparison of his values with those of Dunér.

3. The observations of 1908 confirm those of 1906-1907 in showing that different lines give different velocities. Lines of lanthanum and cyanogen give low velocities; certain lines of manganese and iron give high velocities. The investigation of two "enhanced" lines indicates a tendency toward low values for lines of this type. In one of the cases this effect is very marked. Lines considerably strengthened at the sun's limb give high values in general.

4. When lines give systematically large or small values for the rotational velocity the differences from the mean become greater toward higher latitudes.

5. The results given by the 1908 observations are satisfied within the limits of accidental error by the equation given by Faye for the motion of the sun-spots observed by Carrington. The fact that the observations of Dunér, Halm, and myself are all satisfied by this equation indicates that this represents the law of rotation of the sun's reversing layer to within at least 10° of the pole.

6. Comparison of the probable errors for the two series of observations indicates a substantial gain in accuracy of measurement for the 1908 series over that of 1906-1907.

7. The motion of the reversing layer in the vicinity of solar vortices may be seriously influenced by the motion of the vortices, and the rotation velocities obtained from such regions are subject to large systematic errors. It is important in taking observations for rotation to avoid all such disturbed areas.

8. A special study of the calcium line λ 4227 shows that the rotational velocity derived from this line is higher than that for the general reversing layer, the difference at the equator amounting to 0°.3. Also the decrease of velocity with increasing latitude is much

less marked than for the reversing layer. At 75° of latitude the angular velocity of λ 4227 is 1°5 greater than for the reversing layer.

9. A special study of the *a* line of hydrogen shows that the rotational velocities which it gives depend upon the distance from the sun's limb. Results obtained from *Ha* at the limb of the sun are considerably larger than those for the reversing layer and show a comparatively small decrease in the value of the angular velocity toward the pole. At the equator the difference from the reversing layer amounts to 0°6, and at 75° of latitude to 3°0.

10. At a distance averaging 35″ inside the sun's limb the results obtained from *Ha* are considerably smaller than the corresponding values at the limb, although still much larger than for the reversing layer. They also average somewhat larger than for λ 4227.

11. The sudden increase in angular velocity at latitude 75° may perhaps be a genuine effect similar to that found among the lines of the reversing layer which give systematic deviations.

12. The large rotational values given by λ 4227 and *Ha*, as well as the differences found for *Ha* at the limb and inside the limb, may all be accounted for upon the basis of differences of level in the solar atmosphere.

It may be of interest, acting on a suggestion from Professor Kapteyn, to call attention to the fact that the difference in the values of the rotation obtained from different spots on the planet *Jupiter* may perhaps be explained on a basis similar to that given for different elements in the sun. A difference of about 6 minutes has been found in the rotation period given by different spots upon the surface of *Jupiter*.[1] The ratio of this difference to the longer rotation period is of much the same order as that obtained by comparing λ 4227 with the reversing layer, or *Ha* with λ 4227. It seems reasonable to conclude, therefore, that the different velocities found in *Jupiter's* atmosphere may be wholly accounted for upon the basis of a difference of level for the various spots observed.

I wish to express my appreciation to Miss Lasby for her careful and accurate measurement of the majority of the plates included in this discussion.

Mount Wilson Solar Observatory
November 1908

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[1] Stanley Williams, *Monthly Notices*, **56**, 143–151, 1896.

STUDIES IN SENSITOMETRY. III

ON THE EVALUATION OF THE RECIPROCITY LAW, BASIC FOG AND PRELIMINARY EXPOSURE

By R. JAMES WALLACE and HARVEY B. LEMON

A cursory examination of past photographic literature is sufficient to show the recurrence of similar ideas and methods which have either been discarded as fallacious, or, because of lack of definite scientific investigation, have never advanced beyond a somewhat nebulous stage of development. The systematic and unbiased investigation of these ideas is of the utmost importance, not only in the upbuilding of a firm foundation upon which may be reared the structure of photographic practice, but such investigation is also of major importance to every scientist who records his results by photography.

The work detailed in the present paper was undertaken in the hope of obtaining definite measurable results and shedding light upon somewhat disputed questions.

Primarily, it was concerned merely with the effect of a supplementary "fogging" exposure upon a plate as influencing the recording power of the sensitive film, but, as the work progressed, it was found necessary to investigate also the influence of varying amounts of "fog" upon recorded densities, and likewise the so-called "reciprocity law."

It is usual in all photographic density measurements to subtract from each reading the total amount due to basic "fog," i. e., the sum of the absorption of the glass, the gelatine, and the general reduction products in the film. As a usual thing this value is comparatively small, amounting in a good, clean-working plate to about 0.05 (Hurter and Driffield), which is practically negligible except on records of small total luminosity. From causes either inherent in the film—induced by subject or apparatus, or intentionally added (as in the case of a preliminary exposure)—this value may increase in an alarming manner and rise as high as 0.45 or 0.5 H. and D. This is equivalent to 20 per cent. of the highest allowable printing

PLATE XIII

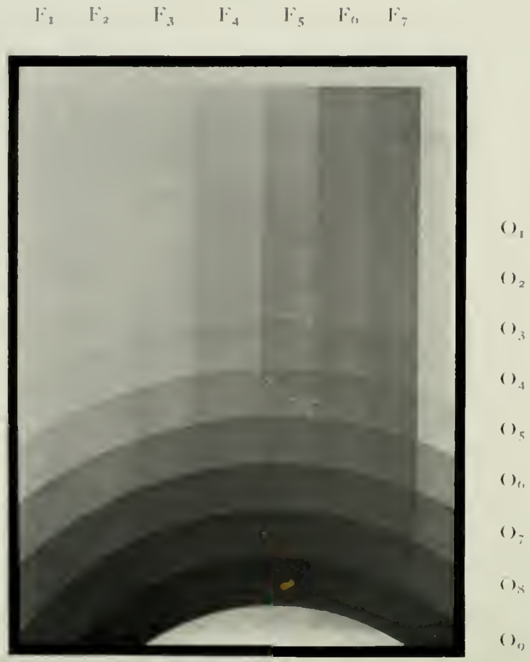

FIG. A

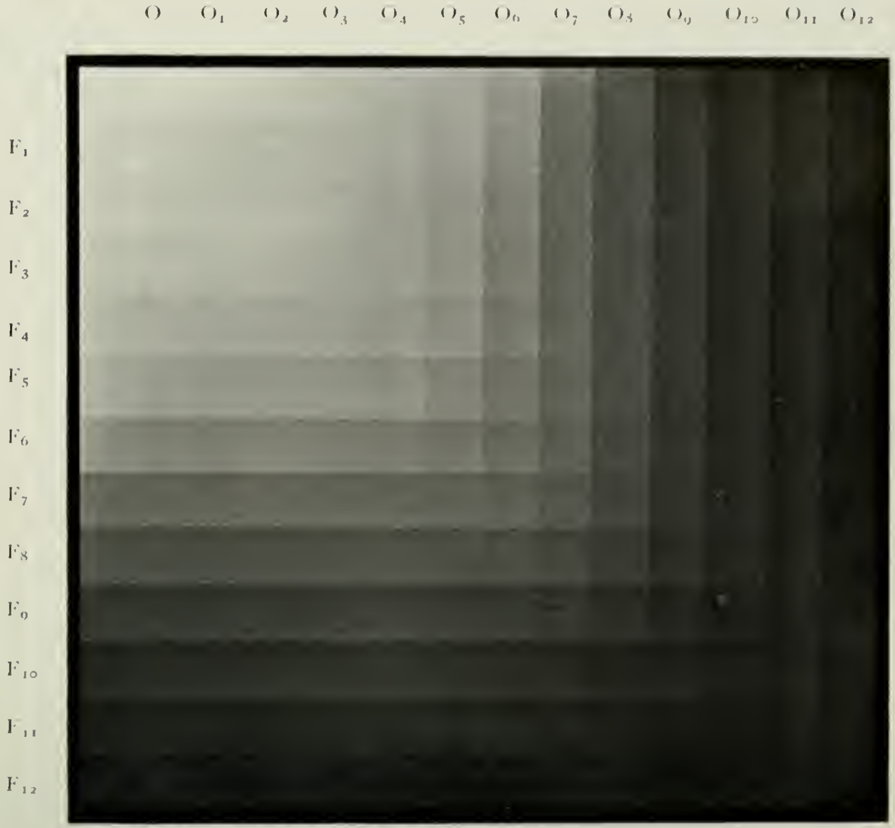

FIG. B

density upon the plate, for when this fog value is subtracted from the various densities, the records are not only in erorr, but result in anomaly.

As an example of such, the following experiment is detailed.

A Seed "G. E. 27" plate was impressed with a series of preliminary exposures in strips A_1, A_2 . . . . A_8 running parallel to the longer edge. The plate was then exposed in the sector-disk machine and received a second series of exposures B_1, B_2 . . . . B_8, and was developed. Measurements of the plate gave the following density values:

TABLE I

No.	Original (Minus Fog)	Original + F_3	Original + F_6
Fog*..........	0.0105	0.2418	0.5278
1.............	.0105	.0216	.0094
2.............	.0105	.0386	.0286
3.............	.0461	.0700	.0486
4.............	.1841	.1662	.1038
5.............	.4327	.3472	.1792
6.............	.7603	.6110	.4082
7.............	1.1669	1.0032	.6988
8.............	1.6443	1.4504	1.1328
9.............	2.2223	1.9130	1.6634

* The value of the "fog" on subscripts 3 and 6 equals the value of the preliminary exposure plus the basic "fog" of the original.

If now these measurements be plotted in the usual manner against log exposure, it will readily be seen (Fig. 1) that all of the curves will cross the original curve at a low density-value, so that in the region of higher density these results are utterly false representations of the plate values; for these, while still in the straight portion of the characteristic curve, are shown as of lower density when the incident light-value is increased. The utter falsity of this result may be appreciated by glancing at Fig. A, Plate XIII, which is a reproduction of a similar negative.

Observation of Fig. A will at once impress the fact that as the increasing supplementary fog value F_n falls upon successively increasing original densities (O_n), the additive effect becomes less and less apparent until it is finally lost; this *extinction point* (so to name it) varying with the relative values of F and O. It is therefore evident that the subtraction of an equal value for this fog (F) from densities

O_n cannot be considered. To determine this variation was therefore obviously the first consideration.

"FOG" VALUE WITH INCREASING DENSITIES

Upon a Seed "27" plate was made a series of exposures to the constant acetylene light, the exposures being so timed that each strip received an exposure increasing by 1.27 times that immediately preceding it, and then developed. When dry the plate was measured in the spectrophotometer at several points along the *length* of the strips in order to determine the evenness of the deposit.[1] This plate was then used as a transparency behind which were exposed a number of other plates for fog value determination.

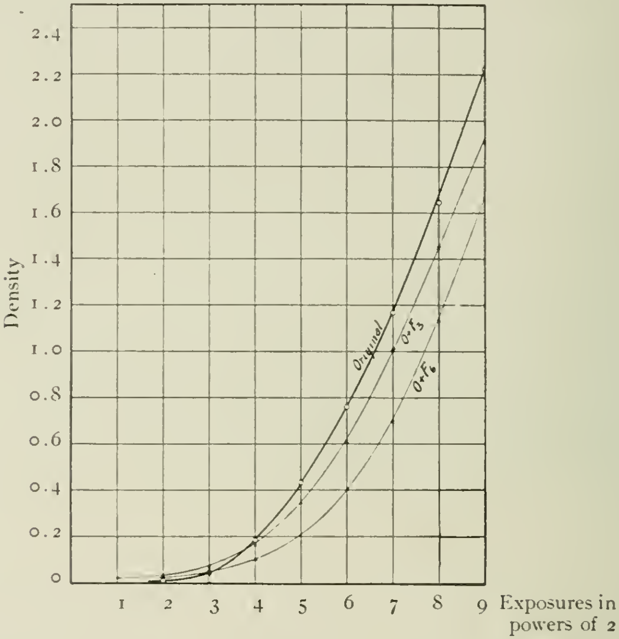

FIG. 1

This transparency plate was set up at a distance of 2.0 meters from the constant acetylene light, and the plate behind and in contact with it received an exposure of 200 seconds. Upon completion of the exposure the plate was rotated 180° and exposed again for another 200 seconds and then developed. A large number of such plates was made in the course of the investigation.

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[1] Owing to the coating inequality of the ordinary commercial plate, it was found necessary to make careful selection before one sufficiently even was found.

Measured densities were then plotted as ordinates against log exposure as abscissae, and as the actinic value of the acetylene burner at a distance of 2.0 meters = 0.15 candle, then

$$\log E = \log I + \log T,$$

where $\log T$ is the value obtained by measurement of the strips on the transparency plate.[1] These curves are shown in Fig. 2.

Referring to Plate XIII and calling one series of exposures the original, its successive strips may be designated $O, O_1, O_2, \ldots O_{20}$, while the other exposures may be termed "fogging strips" and designated as $F_1, F_2, F_3, \ldots F_{20}$; It will of course result that the graphic illustration of $O, O_1, O_2, \ldots O_{20}$ (which has been protected from any additive exposure) will simply represent the characteristic curve of the plate, while F_1 $(O, O_1, O_2, O_3, \ldots O_{20})$ illustrates the change occurring when definite amount of "fog" (F_1) is added to the original series of exposures $O, O_1, O_2, \ldots O_{20}$ (Fig. 2).

In sensitometric and general scientific work where it very often occurs that heavily fogged plates have to be measured, the value of the curves should be obvious: particularly in the measurement of relative brightness in such instances as solar disk phenomena, selective spectral intensity, etc.

In use, one knowing the value of the "fog" under consideration, may at a glance obtain the "minus value" of the same for densities of varying amount. It will be noted that fog increase values appear to become asymptotic to the straight portion of the characteristic curve.

GAIN FROM PRELIMINARY EXPOSURE

To determine the value of preliminary exposure with reference to the photographic visibility of light of feeble intensity, the following method was adopted. Exposure was made to the constant acetylene light through a transparency graduated in strips (whose densities advance in arithmetical progression) for a definite length of time, and at a definite distance. The plate was next turned through 180° and exposed behind the rotating sector disk, the apertures of which advance geometrically. Both exposures were so timed that the developable impression ceased to be visible before exhausting the

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[1] A series of differently timed crossed exposures made directly to the acetylene light without the interposition of the "transparency" gave precisely similar results.

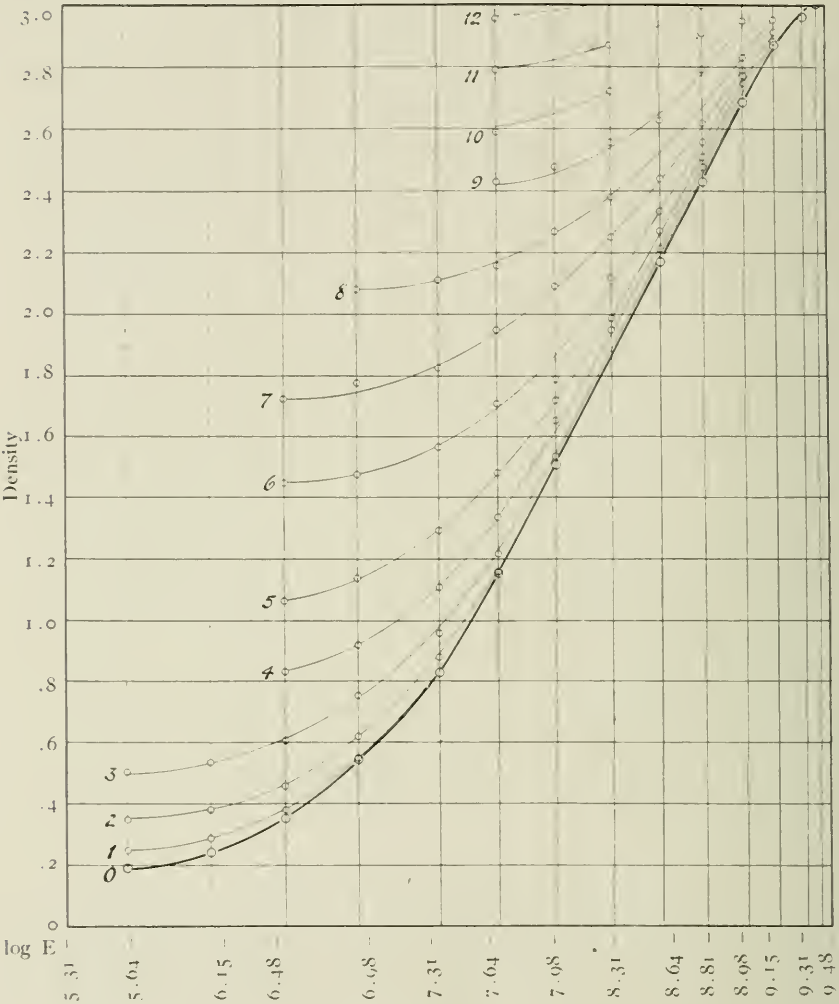

FIG. 2

O.	Original (unfogged)	F_6.	Density = 1.4006
F_1.	Density = 0.2154	F_7.	" = 1.7128
F_2.	" = .3072	F_8.	" = 2.0060
F_3.	" = .4872	F_9.	" = 2.3130
F_4.	" = .7482	F_{10}.	" = 2.5340
F_5.	" = .9970	F_{11}.	" = 2.6866
		F_{12}.	" = 2.9144

number of strips. The plate was then developed as usual under constant conditions.

A very large number of plates (Seed "27") were thus made with the light at different distances and behind variously graduated transparencies, but the results in every case simply verified one another.

Fig. A, Plate XIII, is a reproduction of such a plate as has just been described, and even with the unavoidable loss due to the half tone process it will be seen that there are arcs showing upon some of the "fogged" sections which are totally invisible upon the original unfogged region. This gain is best noticeable at F_4 and the gradual disappearance of the effect of the additive "fog" against increasing density of the original is also clearly shown.

All of the plates thus exposed were assembled and selection for measurement was made of that one which showed the maximum effect of increase. The measurement of such small differences requires the utmost care and is very trying upon the eyes; ample precautions were taken relative to screening off extraneous light.

The density measures of this plate are shown in Table II, and the plotted values in Fig. 3.

TABLE II

No.	Original* (Unfogged)	Original† + Prelim. Exposure
1		0.4938
2		.5232
3	0.1588	.5590
4	.2818	.6512
5	.5324	.7646
6	.8306	.9970
7	1.2574	1.3860
8	1.7128	1.8038
9	2.2108	2.2108

* These values represent the actual density, the value of the density due to glass and gelatine having been subtracted, as is usual in sensitometric measures.

† Density of preliminary fog = 0.4512.

It was at first intended to plot the densities of this plate against actual candle power, but further consideration of the method showed the infeasibility of such a plan, for while it presumably would be true for a certain emulsion evenly coated, yet commercial production does not allow of such a degree of accuracy, and any change in the speed or constitution of the emulsion would necessitate an entirely new determination. We considered it decidedly better, therefore, to plot these densities against exposure ratios in powers of 2 as being the more practical, and also because this method presents uniformity with earlier papers published by one of us.

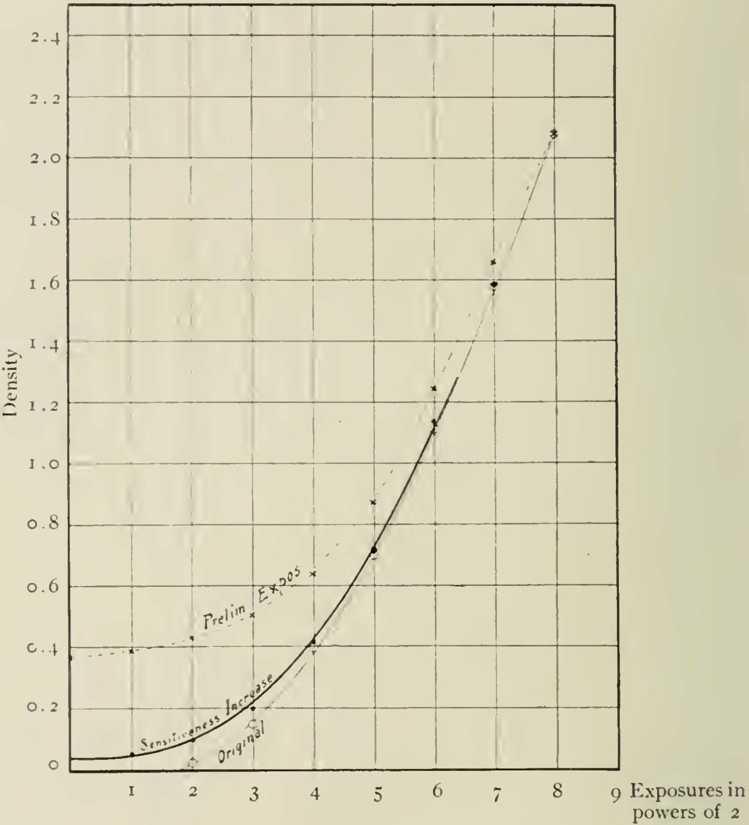

FIG. 3

Considering now these curves shown in Fig. 3, the continuous line represents the original (unfogged), while the dotted line represents the same original exposure but as influenced by an amount of preliminary exposure giving a density value of 0.3218. It is of course understood that this density (0.3218) would appear in practice entirely covering the plate as a "basic fog" and would be representative of the highest transparency thereon. Having a definite measure of this additive fog, we can, by reference to the curves shown in Fig. 2, construct another curve which will represent the true increase or gain in sensitiveness conferred by the preliminary exposure. This curve is shown by the heavy continuous line in Fig. 3.

Just how much has been gained by the method is plainly shown graphically, but is difficult to present in figures. One naturally expects to be able to state in percentages the amount of any change, but in this instance such a method would convey very misleading information; for, while with the most careful examination No. 3 was the first exposure to show any trace of development action on the original (unfogged) scale, yet Nos. 1 and 2 showed perfectly measurable differences on the strip preliminarily fogged. The gain could thus be only represented in percentage by "infinity;" which is ridiculous. We have considered it better therefore to make our deductions from sector exposure No. 3, which gives a density difference of 0.08 (H. and D.). This value corresponds to about 0.2 magnitude on stars near the limit of photographic action, i. e., of a density so low that their impress is but barely discernible on the negative. This gain, as will be seen both from the curve and from Plate XIII (B), is a quantity decreasing with the density: for example, for neighboring stars giving a density value of 0.4 the gain is reduced to 0.08 magnitude.

From these results it therefore appears to us that there is no *practical* advantage in preliminary (or supplementary)[1] exposure in so far as concerns astronomical (stellar) photography. It is, however, perfectly comprehensible that there may be special occasions (more particularly in long spectroscopic exposures) where the exposures are measured by hours or days, when the slight gain would be well worthy of consideration; *ordinarily* the "gain" is a negligible quantity which is in many lines of work offset by the change in gradation curve.[2] The great majority of workers will naturally hesitate to purposely "fog" their plates for so slight a gain.

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[1] Plates made with supplementary exposure resulted in values similar to those preliminarily exposed.

[2] This change is shown by the uniformly lower γ of the "fogged" strip, i. e., by a smaller angle between the straight portion and the log E axis.

RECIPROCITY LAW

The well-known Bunsen-Roscoe "reciprocity law" states that the product $It=E$, where I is the intensity, t the time, and E the exposure. Abney was the first to point out that this law did not hold for photographic plates, and further work has since been undertaken upon the subject by English, Schwarzschild, Mees and Sheppard, and others, from which it results that the formula $It^P=E$ is more in agreement with truth when P is slightly less than unity.

As the matter is of considerable importance from an astronomical standpoint, it was decided to verify these foregoing values upon the plates most generally in use[1] in astronomical and general scientific work, viz., Seed "27" and Cramer "Instantaneous Isochromatic."

A large number of exposures were made upon these respective plates behind the graduated transparency before mentioned, at different distances from the constant acetylene light, and (by means of shutters) for different lengths of time, i. e., different values of P. From the formula $It^P=E$ we obtain:

$$I_1t_1^P=I_2t_2^P ,$$

whence

$$P=\frac{\log \frac{I_1}{I_2}}{\log \frac{t_2}{t_1}} ,$$

and

$$t_2=\text{antilog}\left(\frac{\log I_1-\log I_2+P \log t_1}{P}\right) .$$

In actual practice six separate exposures were made on each plate ("27" and "iso") in the following manner. One portion of the plate was exposed behind the transparency for 40 seconds with the constant light at a distance of 2.0 meters. This "constant" exposure may be represented by m. The light was then removed to a distance of 12.0 meters ($=\frac{1}{36}$ the original intensity) and 4 more exposures were given on the remaining portions. As each exposure time was completed, the movable slides protected that portion from further light-action, which, for example, were in some cases

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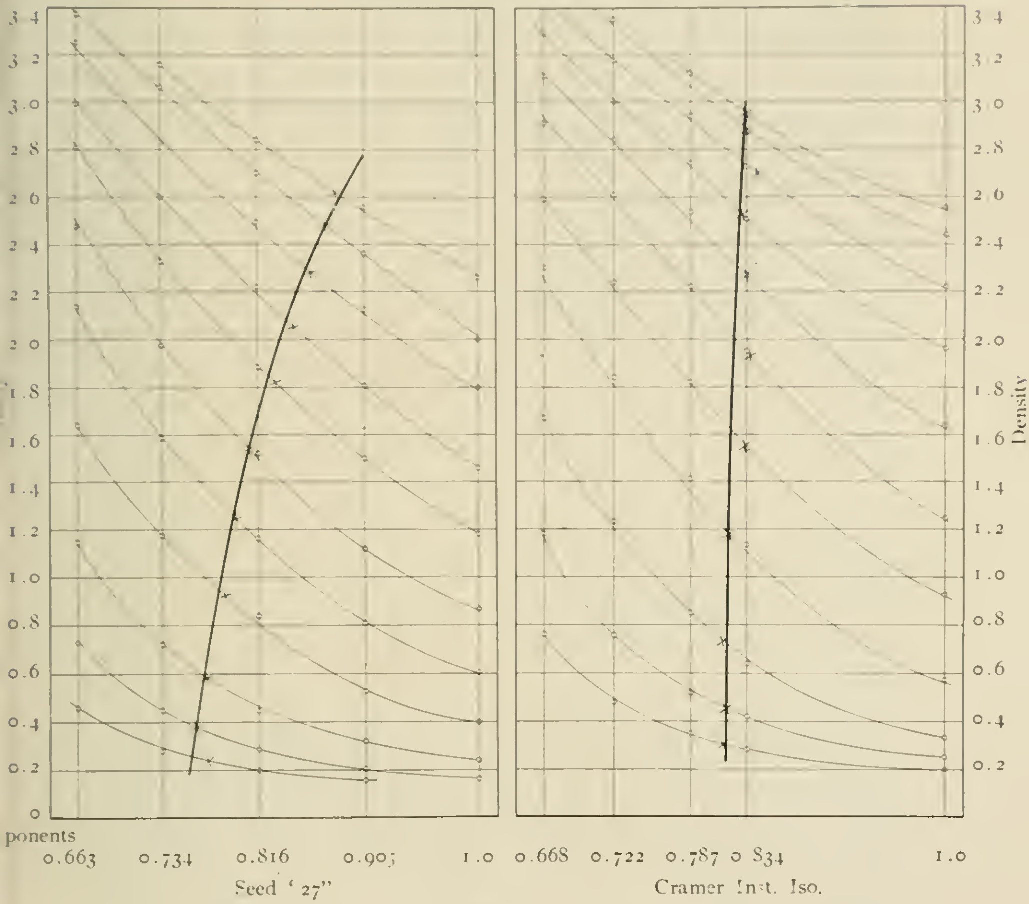
[1] In the United States.

24 minutes $(=m)$ $= P = 1.0$
35 " 4 seconds $= P = 0.905$
53 " 48 " $= P = 0.816$
1 hour 26 " 16 " $= P = 0.734$
2 " 27 " 52 " $= P = 0.663$

although varying values for P were given on different plates.



Seed "27"

Cramer Inst. Iso.

FIG. 4

As was anticipated, examination of the plates after development showed that in every case, with the "27," the exponential value varied with the density. *With the "iso" plate this variance does not obtain,* but remains practically constant even beyond the limit of full printing density. Plotting the measures made from these two types of plate against the true log E, the crossing of the various curves over m is plainly shown.

If, however, these curves be plotted with density against exponents, we obtain a series of smooth curves which present great interest, for upon each unit we may indicate the density value of m, and, connecting the points, obtain a result which gives directly the exponential value required to reproduce a density of any chosen magnitude. Thus it will be noted (Fig. 4) that with a Seed "27" plate for a density value of 2.6 the exponent value required is 0.88, while for $D=0.4$, $P=0.76$.

In astronomical photography, assuming that for certain stars of nth magnitude an exposure of 60 minutes is necessary in order to obtain upon the film a density of 0.4, then the exposure time necessary to obtain images of comparable density from stars 1.0 magnitude fainter would be found as follows:

Let

$$I_1=1$$

$$I_2=\frac{1}{2.5}$$

$$P=0.9$$

$$t_1=60 \text{ minutes}$$

$$t_2=x$$

then

$$\frac{0+0.3979+0.9\times 1.7782}{0.9}=2.22$$

and

$$\log^{-1} 2.22=166 \text{ minutes}=t_2\ .$$

For stars of higher density the time is proportionately shorter.

Regarding the Cramer "inst. iso." plate curves (Fig. 4), it will be seen that when plotted in similar manner the exponential curve is practically a straight line with its mean value at 0.83, which is in practical agreement with Schwarzschild, who found when using Schleussner plates $P=0.86$. It is evident, however, that the value for P will vary with plates of identical "brand" if the emulsion be not coated in one operation at the one time. It is also probable that the P value may vary with the wave-length of the incident light.

YERKES OBSERVATORY
November 30, 1908

THE ARC SPECTRUM OF IRON λ6855 to λ7412

BY E. J. EVANS

Some time ago Professor Fowler suggested to me the investigation and tabulation of the wave-lengths of the titanium lines in the red end of the spectrum.

Since the spectra first studied were prismatic, it was necessary to know the wave-lengths of certain lines in the above region, which were to be employed as standards to calculate the constants λ_0, c, and s_0, in the Cornu-Hartmann formula, $\lambda=\lambda_0+\frac{c}{s-s_0}$.

With this end in view the iron spectrum was investigated in the same region. The instrument employed was a spectrograph of the Littrow pattern having one prism of angle 60° ($\mu_D=1.6467$), and an object glass of 12 ft. focal length. The photographs were taken with Messrs. Wratten's panchromatic plates. The plates were then measured on a stage-micrometer reading to 0.001 mm by estimation, and the reduction to wave-lengths was made by means of the above formula.

Plates of the iron spectrum taken with long exposure showed several lines of greater wave-length than 6855.419, the last iron line tabulated by Rowland. By extrapolation approximate wave-lengths were obtained for these lines. It was immediately noticeable that the wave-lengths fell very near to those of lines in the solar spectrum, which were then assumed to be due to iron vapor. The solar lines 7207.715, 7038.50, and 6855.419 were now taken as standards for the calculation of the constants for interpolation, and the wave-lengths of the other lines determined. The wave-lengths thus calculated were coincident within about 0.04 of a tenth-meter with the solar lines.

It afterward became possible to investigate the iron spectrum with the aid of a Rowland concave grating of 10 ft. radius, and about 14,500 lines to the inch. The temporary mounting of the grating for this work did not give spectra which were strictly normal, but a linear formula could be used for interpolating between standards not far apart. With this instrument a first-order iron spectrum of one hour's exposure was taken, and overlapping it a second-order spectrum of iron. In this way the wave-lengths of some of the lines were obtained, and their coincidence with the previously mentioned solar lines confirmed.

TABLE OF WAVE-LENGTHS OF IRON LINES FROM 6855 TO 7412

Wave-Length	Int.	Nearest Solar Line (Rowland)	Origin (R)	Int. (R)	Remarks
6855.419	10	6855.419	*Fe*	3	Taken as standard
57.50	1	57.515		0	
58.42	5	58.415		2	
62.12	0	62.014 62.210		0000 0	
62.76	1–2	62.76		1	
76.23	0	76.255		000N	
80.90	0–1	80.887		00	
81.82	1	81.720		00	
86.05	4–5	86.000	A(O)	11	
98.56	1–2	98.556		00	
6903.10	1–2	6903.120		00	
11.83	0	11.790	A, –	00N	
15.43	0				
16.96	7	16.948		2	
30.89	0	30.890		00N	
33.33	0	33.305		0N	
33.89	2	33.887		2	
45.46	10	45.477		4	
47.72	1	47.702	A(O)	0	
51.54	5	51.518		1	
60.60	0	60.590		00	
72.17	0	72.205		00	
75.80	1	75.706		0	
76.57	0	76.535		000N	
77.17	0	77.198		00	
77.72	1	77.715	A(WV)	3	
79.12	10	79.120		2	Taken as standard
88.79	2	88.805		0	
7000.16	5	7000.155		1	
00.91	1	00.880		00	
08.26	2	08.225		0	
10.65	1	10.618		000N	
11.67	1–2	11.590	A(WV)	2	
16.32	2	16.330	– A(WV)	1	
16.67	6	16.675	–, A(WV)	3	
23.25	5	23.230		2	
24.32	1	24.340		0	
24.90	2	24.913		1	
38.50	5	38.500		1	

The photographic plates were measured on the stage-micrometer, and in the final reductions the solar wave-lengths as obtained from

Rowland's tables were employed. As far as could be determined the specimen of iron employed was almost pure.

The grating photograph brought out about 90 lines in the spectrum below 6855.419. Some of these were too faint to be accurately measured, and have therefore been omitted in the appended table.

The wave-lengths of the nearest solar lines, with their intensities, have been included in the table. The values taken are those given by Rowland, and the origins of the lines (if any) as given by him have also been included.

TABLE OF WAVE-LENGTHS OF IRON LINES FROM 6855 TO 7412

Wave-Length	Int.	Nearest Solar Line (Rowland)	Origin (R)	Int. (R)	Remarks
7039.08	0	7039.040		0	There is a *Ti* line at 7039.04
68.67	5	68.685		2	
72.13	0	72.130		0N	
83.68	0	83.680		00N	
87.03	1	87.008		00	
90.66	5	90.660		2	
95.68	1	95.690		0	
7107.76	0	7107.740		0	
12.46	1	12.450		0	
31.21	6	31.204		3	
33.29	2–3	33.263		1	
45.64	1	45.615		1	
56.00	0	55.945		0N	
64.76	6	64.725	–, A	2	
76.25	1	76.225	A, –	1	
77.21	0	77.170	A	0	
81.54	2–3	81.500	A	0	
82.27	0	82.260	A ?	00	
87.65	10	87.645	–, A	5N	
7207.715	8	7207.715		1	Taken as standard
20.08	1	19.987		0	
21.57	0	21.492		0	The wave-lengths of these lines were obtained by extrapolation and the lines 7390 and 7412 may be incorrect to about 0.3 tenth-meter
24.02	2	23.930	A	3	
40.39	1				
89.21	0				
93.50	1				
7390.14	2				
7412.05	2				

IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY
LONDON
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THE ARC SPECTRUM OF TITANIUM FROM λ 5866 TO λ 7364

By E. J. EVANS

The arc spectrum of titanium in the region more refrangible than λ 5899 has already been very fully investigated by Hasselberg,[1] but although Rowland and Thalén have dealt with the less refrangible part of the spectrum the list of lines in this region has hitherto been incomplete. In view of the importance of titanium in connection with the spectra of sun-spots, the present investigation was undertaken to supply the missing data, especially in the region 6800 to 7360.

The light obtained from an arc between two carbon or iron poles, the positive of which had been charged with titanium oxide, was first analyzed by the Littrow spectroscope described in the previous paper.[2] The photographs were measured on the stage-micrometer, and the wave-lengths calculated in the usual way.

The iron-pole spectra of titanium gave a very convenient method of obtaining the wave-lengths of the titanium lines without taking a comparison spectrum, as some of the iron lines were photographed simultaneously with those of titanium, and there was no danger of relative displacements. The wave-lengths of iron lines as determined in the previous investigation were used for the calculation of the constants of the Cornu-Hartmann formula, from which the wave-lengths of the titanium lines could be ascertained.

The dispersion given by the prism in the extreme red was comparatively small, and therefore the titanium spectrum in this region was also studied with the previously described Rowland concave grating when it became available. A first-order titanium spectrum with an overlapping second-order iron spectrum was photographed, and the wave-lengths of a few titanium lines calculated. The strong lines were identified with solar lines tabulated by Rowland, and the solar wave-lengths were adopted in making the linear interpolations

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[1] *Köngl. Svenska Vetenskaps-Akadem. Handl.*, Bd. 28, No. 1 (1895).

[2] "The Arc Spectrum of Iron from λ 6855 to λ 7412," p. 157.

TABLE OF TITANIUM LINES (λ 5866 to 7364)

Wave Length	Int.	Nearest Solar Line (Rowland)	Origin (R)	Int. (R)	Remarks
5866.675	6	5866.675	*Ti*	3	Used as standard for Cornu Hartmann formula
77.99	1	78.015		0	
80.48	2	80.490		00	
99.56	5–6	99.518	*Ti*	1	
5903.56	1–2	5903.555		00	
18.79	3–4	18.773	*Ti*	0	
22.35	4–5	22.334	*Ti*	0	
38.03	2–3	38.035		000	
41.97	4	41.985	*Ti*	00	
53.386	5	53.386	*Ti*	1	Taken as standard line
66.08	5	66.055	*Ti*, A ?	2	
78.79	5	78.768	*Ti*	1	
96.11	1	96.164		000N	
99.13	0–1	99.115		0000	
99.90	2–3	99.920	*Ti*, A(WV)	0	
6002.92	0	6002.97		0000N	Probably a fluting line
18.78	0–1	18.76		0000	Probably a fluting line
64.853	4	64.853	*Ti*	00	Taken as standard
85.48	5	85.47	*Ti*, *Fe*	2	
91.40	5	91.395		0	
93.00	2	93.03		000	
98.92	3	98.87		00	
6121.24	2	6121.215		000	
26.435	5	26.435	*Ti*	1	Taken as standard
38.64	0–1	38.725		00N	
46.48	2	46.445		000	
50.02	1–2	49.95		0000N	
86.65	2				
6215.630	3–4	6215.630	*Ti*	000	Taken as standard
20.87	2				
21.75	1–2	21.552	*Fe*—	00Nd ?	
58.60	9	58.322 58.927	*Ti* *Ti*	2 3	6258.29 6258.96 were wave-lengths determined by grating
61.316	6	61.316	*Ti*	1	Taken as standard
6303.97	3	6303.985		000N	Given by Hale as strong *Ti* fluting line
12.47	3	12.456		00N	Given by Hale as strong *Ti* fluting line
18.26	2	18.239	*Fe*	6	This line comes out on carbon poles
36.329	3	36.329	*Ti*	000N	Taken as standard
66.59	4	66.564	*Ti*	000	
6497.92	1–2	6497.840	A(WV)	00Nd ?	
6505.72	0–1	6505.71		0000N	Possibly a fluting line
08.37	1–2	08.380		0000	
36.74	0–1				Possibly a fluting line
6546.479	4	46.479	*Ti*, *Fe*	6	Taken as standard
48.57	1				
54.47	5	54.470	*Ti*	0	
56.31	5	56.308	*Ti*, A(WV)	1	
65.84	2–3	65.783	A ?	000	
75.39	1–2				
99.353	4	99.353	*Ti*	00	Taken as standard
(6651.55)	1				Fluting head. Hale gives 6651.56
66.82	1–2	6666.78		000N	

TABLE OF TITANIUM LINES—*Continued*

Wave-Length	Int.	Nearest Solar Line (Rowland)	Origin (R)	Int. (R)	Remarks
6668.04	1–2				Hale 6668.02. Spot 6668.03
68.68	1–2	6668.640		000N	
77.48	0–1				
6719.90	0–1	6719.880		000N	Hale gives 6719.86 as strong *Ti* fluting line
43.380	5	43.381	*Ti*	1	
64.18	1				
6844.93	1	6844.939		0000	Possibly fluting lines
50.48	1–2				
54.55	1	54.590		000	
61.770	3–4	61.770	*Ti?*	000	
74.24	2				May be a fluting line
6911.43	2–3				Strongest near the pole. *Ti?*
13.51	2	6913.448	A(O)	11	According to Rowland 6913.284 is *Ti?*
26.41	2	26.363		0	
33.42	1–2	33.460		0000	
39.13	3–4	39.050		000	Strongest near the pole. *Ti?*
43.96	2	43.904		0000	
96.91	2–3	96.93		0000	
7004.87	1–2				
06.95	0–1	7006.915	A	000	Fluting line?
08.64	1–2				
11.27	2	11.230	A	000N	
36.09	2–3	36.129		0000	
39.05	4	39.040		0	
39.59	1	39.560	A	0N	May be fluting line
50.92	1–2	50.783	A?	000	
(54.56)	2				Strong flut. head Hale 7054.65 Spot 7054.60
65.41	1	65.40		0000	May be fluting line
69.35	2–3	69.35		0000N	
(87.90)	2				Strong flut. head Hale 7088.00 Spot 7087.90
(99.83)	0–1	99.812		0000N	Fluting line. Hale 7000.77
(7111.70)	0–1	7111.730	A	000N	Fluting line. Hale 7111.73
(25.83)	2				Strong flut. head Hale 7125.88 Spot 7125.89
39.24	2–3	39.215		000	
88.90	0–1	88.875		0000N	Fluting line?
90.22	2	90.159		000	
7209.78	5–6	7209.78	A. —	3	
16.53	3	16.482	A	000	
45.15	4	45.152		00	
52.06	4	52.032		000N	
54.12	0–1				
66.60	0				May be fluting lines
71.83	0				
74.12	0				
7300.05	0				
18.78	1				
45.12	3				These wave-lengths above 7252 were obtained by extrapolation and may be incorrect to 0.2 or 0.3 of a tenth-meter
58.15	2				
7364.50	2				

for other lines. The wave-lengths as determined by the prism were confirmed within about 0.1 of a tenth-meter. In the reduction of the prismatic spectra in the extreme red the Cornu-Hartmann formula was used over quite a long range (λ 6861.77 to 7209.78), and it is interesting to note the close agreement between the wave-lengths obtained in this region with a single prism and those obtained with the grating.

The titanium spectrum was always obtained as a mixture of the line and fluted spectrum, and possibly a few of the tabulated lines, in addition to those doubtful cases indicated, belong to the fluting structure.

The impurity lines identified on the plates were due to calcium, vanadium, barium, and lithium, and they have been eliminated as far as possible in the appended table. The wave-lengths of the lines from 5866 to 6600 have been chiefly obtained from measurements of prismatic spectra, while the wave-lengths from 6600 onward have been derived from the grating photographs.

I am indebted to Professor Fowler for valuable assistance during the course of the investigation.

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London
November 1908

THE MERCURY PARABOLOID AS A REFLECTING TELESCOPE

By R. W. WOOD

The idea of utilizing the principle that the free surface of a liquid, rotating with a uniform velocity, assumes the form of a paraboloid, in the construction of a reflecting telescope is not new. Mercury telescopes have been suggested from time to time for the past half-century, and in the early seventies one was constructed by Mr. R. C. Carrington, on Frensham common. So far as I know, however, no serious attempt has ever been made to devise a method of rotating the fluid without, at the same time, communicating jars to it. The scheme has always been regarded as a joke by astronomers, which is not surprising when one considers the perfection required of an optical surface and the ease with which ripples are set up on a free surface of mercury.

The idea of using a rotating magnetic field to communicate motion to the circular basin of mercury occurred to me last summer, and becoming interested in the problem, more as a mechanical puzzle than anything else, I determined to ascertain if possible whether, after all, the difficulties were insurmountable. A small instrument seven inches in diameter was constructed and arranged to be driven by a revolving ring of small magnets, which pulled a concentric ring of magnets fastened to the dish around with it. The rotor, as we may term the outer ring, was mounted on a support which was completely insulated from the mercury basin, and the jars which it received from the motor along the driving belt were in this way prevented from reaching the liquid. It was of course found impossible to learn much about the instrument in the city, but I convinced myself that the ripples on the surface were no worse when the basin was revolving than when it was at rest.

On removing to East Hampton, L. I., for the summer, where I have fitted up a small laboratory, the experiments were commenced in earnest. The mirror was mounted upon a small brick pier and surrounded by an empty barrel to protect it from air currents. It worked so much better than I had expected, in spite of its poor workmanship, that I at once determined to have a larger instrument constructed of the finest workmanship possible. A week or two was spent in experimenting with the small model, in order to learn as much as possible about its peculiarities and the sources of trouble, after which drawings for a 20-inch instrument were prepared and submitted to Messrs. Warner and Swasey of Cleveland. Mr. Warner very kindly gave his personal attention to the matter, modified the drawings, introducing some new features, and had the work commenced at once. The 7-inch instrument was made from an old casting which I found in the junk heap of the laboratory. It was in the form of a circular disk surmounting a short cylindrical pillar. The disk was turned out into a shallow flat-bottomed basin, and the pillar bored out with a hole about 1 cm in diameter, conical at the end. The whole was supported on a steel cylinder, turned to a cone at the top, with a bearing surface about 5 mm wide at the bottom, the construction adopted in the "turn-tables" made for finishing microscope slides. It was found with this instrument that there were four distinct sources of ripples:

a) Jars from the driving mechanism, i. e., motor, speed-pulleys, etc. These were eliminated by the magnetic clutch previously described.

b) Jars due to the grinding-together of the bearing surfaces of the mercury basin itself. These were always present in the first instrument as the surfaces were not polished. It was hoped that better workmanship would eliminate them, as was found to be the case.

c) Imperfect leveling of the instrument, which sets up a sort of tidal wave; easily overcome provided the axis of rotation is accurately perpendicular to the flat bottom of the dish.

d) Variations in the velocity of rotation, which give rise to very troublesome waves. Of these I shall speak more at length presently. Mr. Warner suggested the use of two slightly tapering conical surfaces for the cylindrical base of the dish and the supporting pillar, with a central shaft of steel, by the elevation of which the dish could be lifted a trifle from the conical supports.

The construction will be readily understood by reference to Fig. 1. A plug of hardened steel *A* was inserted in the iron basin, which was screwed to the top of the conical pillar *K*. This plug rested upon a second plug of hardened steel *B*, which could be raised or lowered by means of the screw *C*, turned by means of the nut *D*. The bearing surfaces, indicated by the arrow *E*, were ground flat and accurately perpendicular to the axis of rotation. The weight of the basin was carried by the steel plug, but there were in addition two conical bearings (dotted in the diagram) which kept the instrument level during rotation. By lowering the screw *C* the weight of the basin could be thrown wholly upon the conical bearings. In this position it could be turned only with some difficulty, owing to the increased friction. The best position was secured by raising the screw by an amount just sufficient to abolish this friction. The driving mechanism or rotor consisted of a large wooden pulley *F*, mounted on ball bearings, carried on a tripod *H*, which was supported independently of the

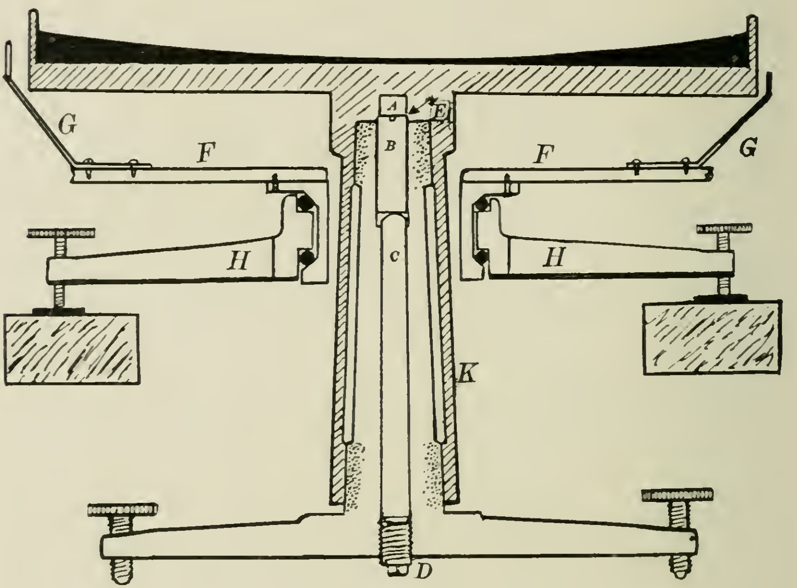


Fig. 1

rotating basin. The original plan was to apply the power of the rotor to the dish by means of a system of magnets, but it was found that fine elastic threads of rubber answered equally well. These were fastened to the brackets *G*, six in number, and attached to the rim of the basin. They applied their force tangentially, and did not appear to transmit the jars of the rotor to the heavy iron basin. A photograph of the instrument, the driving pulleys, and the motor is shown in Fig. 2.

Preliminary experiments were made with the instrument mounted on a brick pier in the laboratory before its installation at the bottom of the cement pit. Ripples were always present here, as it was impossible to secure the complete insulation of the rotor, and the jars from the motor were transmitted to the pier through the floor. A good deal was learned, however, about the adjustments of the instrument in spite of the unfavorable conditions. Accurate leveling is of the greatest importance. The dish was first leveled by means of a spirit level, the final adjustments being made optically.

A Nernst lamp, with its filament vertical, was mounted on the ceiling of the laboratory, at a height of about 12 feet above the mercury surface. At a speed of rotation of about one turn in 3.5 seconds the image was formed at a point where it could be reached easily and examined by standing on a ladder. If a sheet of paper was held a few inches below the focal point, the circular ripples were clearly visible. The bright center of the ripple system did not, however, coincide with the center of the convergent cone of rays. By turning the leveling screws it could, however, be brought accurately to the center which gives us a first approximation to the true level desired. The ripples became much less conspicuous as soon as this operation was performed, and it was speedily discovered that they were in large part due to a sort of tidal wave resulting from imperfect leveling. If the basin is not level the depth of the mercury is slightly greater on the lower side of the basin. This point of maximum depth is carried around to the high side by the rotation of the dish, and the mercury seeks to establish its level again. If the eye is placed in the focus a curious sort of spiral eddy is seen, which is caused by this continual readjustment of level. With the instrument in approximate adjustment, the surface appears perfectly smooth and free from ripples except under especially favorable illumination. The appearance of the extra-focal image as the level is improved is shown in Fig. 3 (upper row). These pictures were made by inclosing a plate in a holder provided with a shutter giving an exposure of about $\frac{1}{25}$ second. In the lower row we have images obtained under the same conditions with the plate almost at the focal point. A star seen under these conditions shows a very marked coma, which can be abolished by turning the leveling screws a proper amount. When the mirror is accurately level, if the image is received on a piece of paper, and one of the leveling screws turned a trifle, the image moves slightly to one side, just as it would do if we tilted slightly a solid mirror. The image develops a coma, of course, but it is interesting that an actual displacement of the point of light can be obtained in this way, a circumstance which one would hardly have anticipated.

Some experiments have been made with two concentric dishes, the inner floating on the liquid contained in the outer. This expedient does not appear to lessen the effect of jars coming from the ground though it improves matters in some other respects. In the end it may be found necessary, though there are certain difficulties attending its use.

One thing which struck me as especially interesting was the slowness with which the velocity of the dish was communicated to the fluid. The mercury picks up its velocity first at the rim, a circular zone of constant velocity and focus crawling in gradually from the edge toward the center. If the eye is placed above the center of the dish a little above a focus, one sees a narrow ring of light of dazzling intensity converge gradually from the rim toward the center. It required over two minutes for all parts of the fluid mass to acquire a constant angular velocity.

The appearance of the room as viewed in the mirror while it is getting up speed is very striking. The rafters of the roof recede to an enormous height and the whole room appears to expand in a most remarkable manner. The appearance of the mercury surface in rotation, and its freedom from ripples are shown in Fig. 4. The distortion of the reflected image is of course due to the astigmatism which results from oblique incidence.

I have had the mirror operating with a focal length of less than three feet, at a velocity of one turn in three seconds, i. e., with a ratio of focus to aperture of 1.7. The dish was not deep enough to make a higher velocity possible, but I have had smaller mirrors running with a focal length of less than one-fourth of the aperture. These very deep paraboloids might prove useful where a great concentration of energy was desirable, as in bolometric determinations of the heat of the stars or the sun.

After completing the preliminary experiments in the laboratory preparations were made for mounting the instrument below ground. An old well-house adjoining the barn was transformed into an observatory, a new floor being laid and a large scuttle cut in the roof. The brick well, which was no longer used, was filled up with large granite blocks and Portland cement, to a height of about two feet above the water level, after which it was lined with cement, forming a water-tight pit about 14 feet deep and 30 inches in diameter. The granite and cement bottom formed a very solid foundation for the mirror to stand upon. A second pit was sunk about six feet from the well, just outside of the well-house, and the two put in communication by means of a tunnel at the bottom. It was the original plan to drive the mirror by means of a pulley-belt running down the side of the cement pit, but as it was found that it was necessary to make very precise adjustments while the mirror was running, the second shaft became necessary. The mirror was mounted upon three blocks of cast iron which rested on the cement floor, and the driving mechanism or rotor was carried upon a wooden *Y*-beam which passed under the floor of the tunnel and was securely anchored to brick piers in the second pit. The electric motor, driven by a 110-volt alternating current, and the speed-pulleys were mounted in the second pit, the belt passing under the floor just above the *Y*-beam and around the wooden pulley of the rotor. The tunnel was lighted by an electric lamp.

The general arrangement of the apparatus is shown in Fig. 5. It will be seen from the diagram that the slight jars from the rotor can reach the revolving mirror only by passing back to the second pit along the *Y*-beam, thence through the brick piers and back through seven or eight feet of sand to the massive granite foundation. It is needless to say that they were quite imperceptible at the end of their journey. Within the well-house, and close to the mouth of the pit

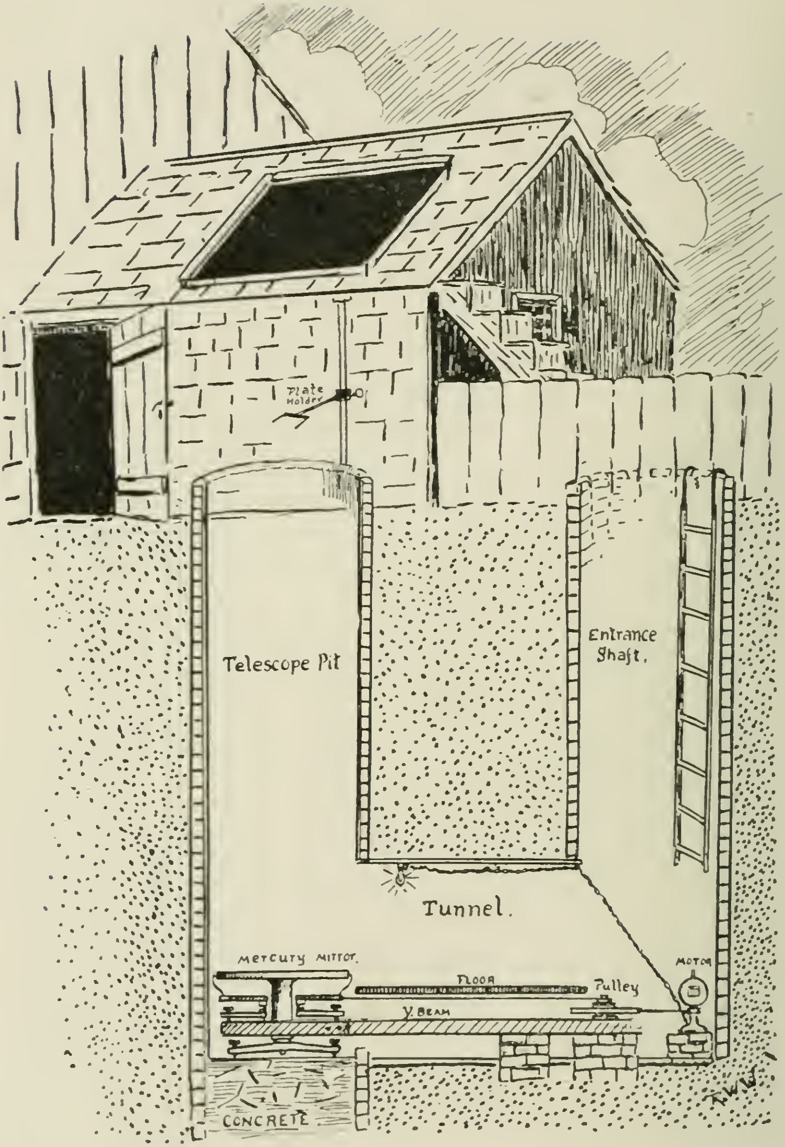


Plate Holder
Telescope Pit
Entrance Shaft.
Tunnel.
Mercury Mirror
Floor
Pulley
Motor
Y Beam
CONCRETE
R.W.W.

Fig. 5

a vertical wooden cylinder was mounted with a horizontal arm carrying a plate-holder. A photographic plate could, by means of this simple device, be held fixed in any position above the pit, for making star trails. Visual observations were made by standing at the mouth of the pit, or upon the roof of the house, according to the focal length used, and holding an eyepiece in the hand.

It was first necessary to determine to what extent vibrations, other than those caused by the motion of the mirror, affected the mercury surface. A five-inch refractor, with an excellent objective figured by Clark, was mounted at the side of the pit, and directed downward at the mercury surface. The star images were perfectly sharp, even when the motor was driving the rotor, showing that no trouble was to be anticipated from tremors running back along the *V*-beam. The approach of a horse and carriage could be detected, however, when it was an eighth of a mile away, and the footsteps of a person running across the lawn fifty yards from the telescope house caused a perceptible vibration of the image. Occasional disturbances were noticed due to live stock in the barn, and after every storm, vibrations were found resulting from the pounding of the surf on the beach a quarter of a mile distant.

At the time when the experiments were commenced the Milky Way was in the zenith at nine o'clock, and its appearance when I observed it from the roof of the house the first time the motor was started amply repaid me for all my trouble. No eyepiece was used, the star images appearing in space about three feet above the mouth of the pit. They appeared perfectly sharp and quite steady, since the cone of light entering the pupil of the eye under these conditions comes from but a small part of the mirror's surface. On examining the images with the eyepiece it was at once seen that the focus changed periodically through a range of perhaps an inch and a half. On viewing the images in space with both eyes they were seen to be rising and falling rhythmically, dancing up and down like will-o'-the-wisps. There were moments when they were quite sharp in the eyepiece, but even then they were not quite stationary, moving about in a sort of Lissajou figure, perhaps a millimeter in diameter. The nature of the disturbance on the mercury surface could be detected by placing the pupil of the eye in coincidence with the star image. The whole surface of the mirror was then seen filled with a blaze of light and the ripples were plainly visible. It was at once apparent that they were not "jar" ripples, for these always appear as narrow concentric circles surrounding the center of the mirror. They often appear nearly stationary owing to interference. The ripples which were causing the trouble appeared to start at definite points on the rim and spread out across the surface, i. e., their centers were on the rim and not at center of the basin. After a good deal of experimenting it was finally found that these ripples were due to slight periodic variations in the velocity, which caused a "slip" between the mercury and the iron rim of the basin. At points where there was any roughness, or where grains of sand happened to lie, the ripples started out.

The periodic change of focus was also found to be due to the variation of the velocity, which occurred once in every revolution of the dish. This change in the velocity is very small, and it is not communicated to the fluid, for, as we have seen, two minutes are required for the establishment of the steady state, starting from rest. When the reduction in speed occurs, though it lasts but a fraction of a second, the liquid at the rim falls a trifle, rising at the center. The propagation of the change of curvature from the rim to the center is almost instantaneous, and not at all like the slow crawling-in of the zone of constant curvature seen when starting the dish. This trouble can probably be overcome to a great extent, if not completely, by making one or two changes in the construction of the instrument. Its presence was detected by fastening a small square of paper to the rim of the basin, and another immediately opposite it on one of the supports to which the rubber bands were fastened. If the dish turned at the same velocity as the rotor, the two pieces of paper should remain in juxtaposition: it was found, however, that once in every revolution, the piece of paper on the basin first lagged behind, and then gained upon, the one fastened to the rotor. A little further experimenting showed that this variation was due to the fact that the friction was a little greater at this point, that is, the force necessary to start the basin varied with the position of the dish. When the dish, in its revolution, reached the position of maximum friction, it lagged a little behind the rotor, the rubber threads stretching a trifle. As soon as the point was passed and the friction diminished, the increased tension on the threads caused a slight acceleration, the basin catching up with, and even running a little ahead of, the rotor. The actual slip between the rotor and the basin only amounted to about a centimeter, but this was quite sufficient to account for all of the disturbances, as was shown by intentionally varying the velocity by small amounts. This trouble was not found with the small model first constructed, which spun on a point instead of a flat surface. The friction is much greater in the latter case, owing to the increased length of the "lever-arm" of the moving surface elements which rub together. I am at the present time taking graphs of the periodic variations in the velocity by means of a pen carried by the rotor, and a small improvised chronograph carried by the basin. These furnish a good deal of information about the effects of raising or lowering the dish, slackening the driving belts, etc. Much better results have been obtained by introducing a small steel ball between the two bearing surfaces. I am of the opinion that a conical point is the best form of support, and as soon as the chronograph experiments are finished, alterations in the construction will be commenced.

A number of star trails were made with the mirror running with a 15-foot focus. One of these (γ *Cygni*) is reproduced in Fig. 3 (lower figure). We obtain a dotted line of bright points, owing to the periodic change in the focus. The points are fairly small, in fact not so very much larger than their theoretical size, and when we consider that they were formed by light-rays concentrated by a rotating mercury surface 20 inches in diameter, at a distance of 15 feet from the photographic plate, they appear to me to be surprisingly good. I feel very confident that the velocity variations can be abolished. It may be necessary in the end to use a motor, the speed of which is controlled by a clock, the plan so successfully worked out by Mr. Gerrish of the Harvard Observatory.

I had no trouble in separating double stars 5″ or 6″ apart, and once I felt pretty sure that I had momentary glimpses of the resolution of both ϵ^1 and ϵ^2 *Lyrae*.

The great nebula of *Andromeda* and the large cluster in *Hercules* both cross the zenith at East Hampton, and will furnish interesting objects next summer when the experiments are taken up again. I have already had some splendid views of the nebula, which appeared almost as satisfactory as in a large refracting telescope.

I have not as yet convinced myself that the mercury telescope will be of use in astronomical observations, even if it is possible to bring it to a high degree of perfection. Its great disadvantage lies in the fact that its observations are confined to a narrow region surrounding the zenith. On the other hand, it has the great advantage of a focus which may be made to vary over as wide a range as we please, and its cost is very small. The 20-inch instrument, with its motor, cement pit, entrance shaft, tunnel, etc., cost only $200, not including the mercury. In the case of small instruments, not over three feet in diameter, it may be feasible to use an auxiliary plane mirror driven by clock work or by hand, for reflecting the light from any part of the sky to the bottom of the pit. An instrument of large size could in this way be placed within the means of any amateur observer, and valuable data might be secured. Whether a gigantic reflector mounted in a suitable latitude would enable us to see more of planetary detail, I do not feel prepared to say. We could in this way combine great focal length and immense light-gathering power, but we should still have the atmospheric disturbances to contend with.

As I stated at the beginning of the paper, the work was undertaken purely as a diversion for the summer months, and if further experimenting shows us that a perfect reflector can eventually be obtained in this way, it will then be time enough to consider whether it is worth while to attempt the construction of a much larger instrument. Then, too, we may be able to discover some substance which can be fused, rotated, and allowed to solidify while revolving with a constant velocity. Refiguring of the surface would very probably be found necessary.

DAMPING OF THE RIPPLES

In the case of a partially viscous substance, there would be much less trouble from ripples. Last summer I made a fairly good mirror of gelatine, and I am at present looking about for a more permanent medium. I shall be very glad to receive any suggestions about easily fusible media which will solidify with an optical surface. The cost of the experiments was defrayed by a grant of $200 from the Elizabeth Thomson fund.

While it has seemed best to bring the instrument to the highest degree of perfection before attempting to devise methods of destroying any residual ripples, some experiments have already been made in this direction. Two methods suggest themselves. We may use a copper vessel, turned to an approximate paraboloid, and use a thin film of mercury (as in the case of the artificial horizon). The rotation would then have the effect of figuring the surface, the thinness of the mercury layer preventing the propagation of ripples. This method has not yet been tried, as the objections to it are obvious.

The ripples can be damped to an appreciable degree by covering the mercury surface with a thin layer of water, which assumes the same curvature as the mercury. Glycerin damps them completely if they are not of too great amplitude to start with. Even if they are of large magnitude, the effect of the glycerin is surprising. With my seven-inch mirror, mounted on a table which was jarred to such an extent by the machinery in the room below that the vibrations could be felt with the hand, I have obtained almost as good definition as at East Hampton with every precaution taken to avoid jars. The action of the glycerin is well illustrated by Figs. 6 and 7, which are photographs of the filament of an incandescent lamp taken with the mirror. The mirror was running at such a speed that the image was formed about a meter above the surface. The belt from the motor drove the dish directly, no rotor being used, and the jars from the city traffic and machinery were so great that the ripples could be seen even in the most unfavorable illumination. Fig. 6 was taken without, Fig. 7 with, the glycerin cover.

Two small dishes of mercury were then placed on the floor, which was violently shaken by shafting attached to the floor beams in the shop below. The jarring was so great as to be unpleasant. The mercury in one dish was covered with glycerin to a depth of about 4 mm. The light from an arc lamp was then reflected from the two dishes directly to a photographic plate, exposed behind a "focal-plane" shutter. The mercury waves can scarcely be seen in the image (Fig. 8) obtained from the glycerin-covered surface, while the other image (Fig. 9) speaks for itself.

It seems quite possible that the glycerin cover may prove useful in making artificial horizons. Inasmuch as my twenty-inch mirror now runs with so little error and with such insignificant ripples that the maximum deviations from perfect focus amount only to a millimeter or less (and this with a 5 meter focal length), I feel very confident that the glycerin cover will make the image practically perfect. So far as I can see now, the glycerin cover will introduce no disturbances, provided its temperature can be kept constant. Even with a variable temperature, the surfaces of equal refractive index would be parallel to the paraboloid, and would probably not give trouble. Stirring the fluid would of course produce striae, if its temperature varied, but stirring only occurs when starting and stopping the mirror. Things would probably come to a steady state in ten or fifteen minutes. I am now making experiments to see whether solid mirrors can be produced by the centrifugal method. Various methods have occurred to me. We may be able to take a cast of the mercury surface by pouring a hot viscous liquid over it, which will solidify on cooling. I have already obtained in this manner pretty good convex paraboloids of resin. These casts could be electrotyped and silvered. If a suitable material can be found, and the electrotypes do not warp, I see no reason why large mirrors cannot be made in this way at very small expense. At all events the method seems well worth trying.

Johns Hopkins University
December 1908

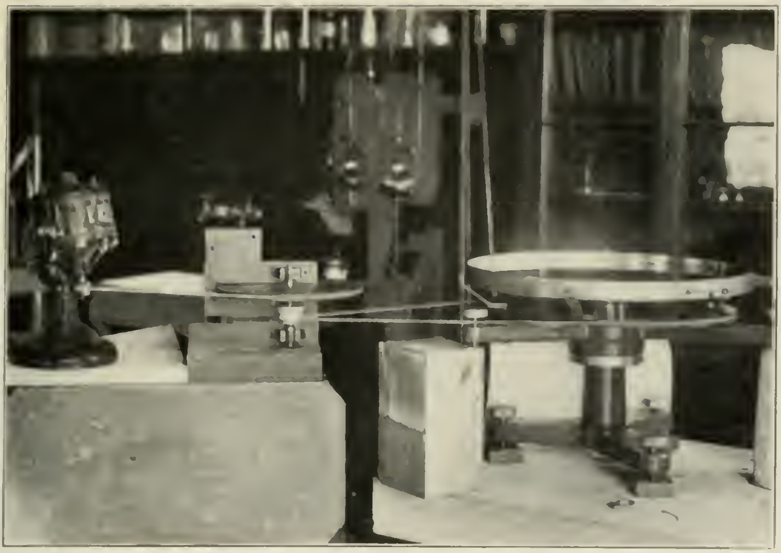


FIG. 2

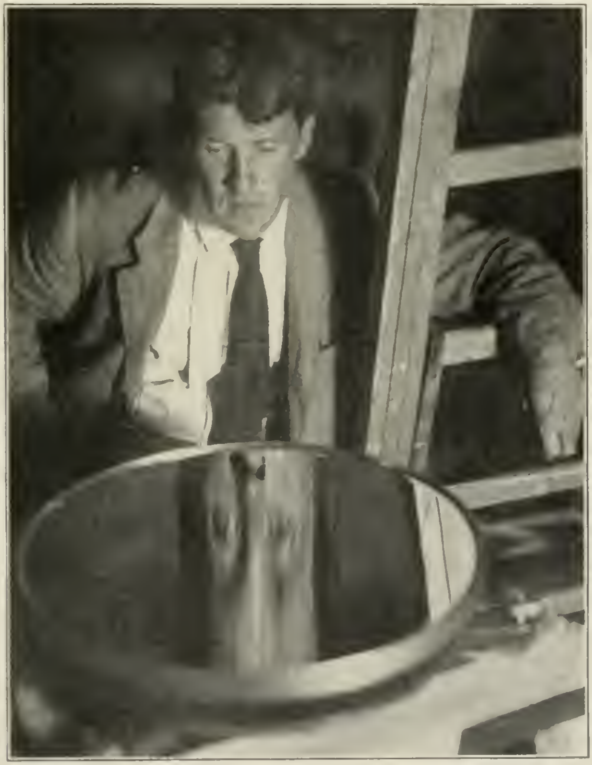


FIG. 4

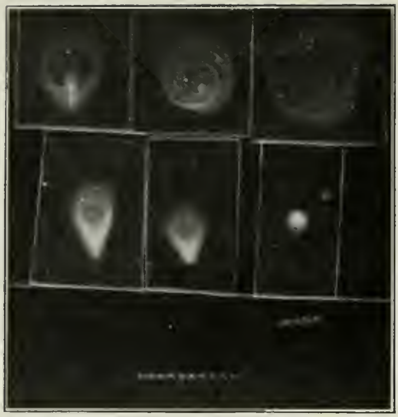


FIG. 3

FIG. 6

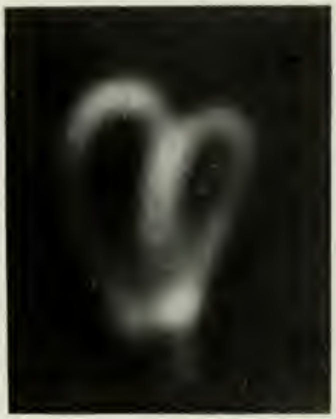
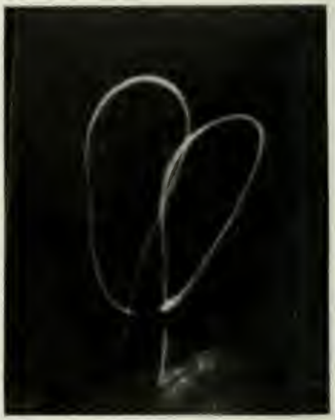


FIG. 7



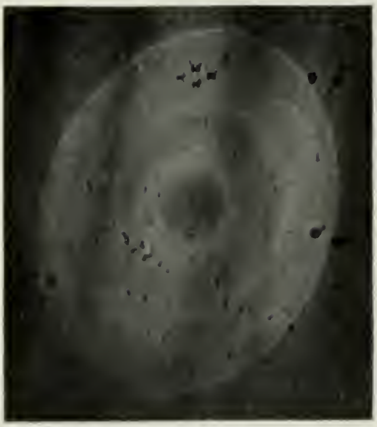


FIG. 8

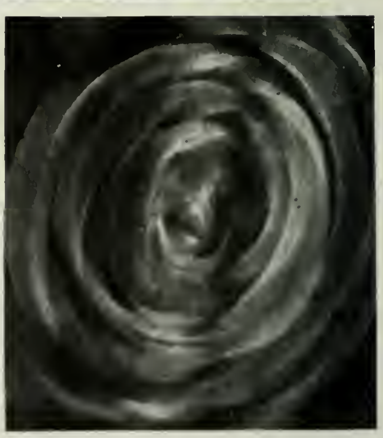


FIG. 9

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THE MAGNESIUM SPECTRUM UNDER REDUCED PRESSURE

By E. E. BROOKS

In a recent paper[1] certain modifications of the magnesium spectrum obtained by using alternating currents in a partial vacuum were described. It is now proposed to give a brief account of the phenomena observed with direct currents, which enabled the spectrum and appearance of each electrode to be studied separately. For this purpose the current from a 2000-volt transformer was rectified by means of a battery of 48 lead-aluminum cells arranged in four groups in order to make use of each half-period. In this way a unidirectional but pulsating discharge was obtained intermediate in properties between the spark and the arc.

The electrodes were always clean magnesium rods rather less than $\frac{1}{4}$ inch (6 mm) in diameter, inclosed in glass tubes except for $\frac{1}{2}$ inch at the end. The distance between them was from $\frac{1}{4}$ to $\frac{1}{2}$ inch, and the working pressure, unless otherwise stated, from 2 to 3 centimeters of mercury. The gases used were air, hydrogen, nitrogen, oxygen, and water vapor, and the primary object of the research was to investigate the mode of occurrence of the "hydride" spectrum.

Two distinct and well-defined types of cathode discharge can be recognized. In one the well-known cathode glow outside the Crookes dark space is the most marked feature. In the other type this glow is absent and in its place several isolated point discharges flicker about on the surface of the otherwise dark electrode. The two forms may occur independently or together, but the second is the most persistent and the most readily obtained and there is often some difficulty in eliminating it when only the first is wanted.

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[1] *Proc. R. S.*, 80, 218, 1908.

The cathode glow, although so conspicuous in vacuum tubes, is by no means so prominent under the present conditions, and the only gas in which it is well developed is hydrogen. Again, in hydrogen there is a definite sequence of phenomena observed in no other gas. The cathode glow, which at first gives only a hydrogen spectrum, becomes gradually permeated by magnesium ions derived from the cathode until the hydrogen spectrum disappears (excepting a trace of the C and F lines), and only that of the metal remains. Then an unstable state is reached and the discharge suddenly brightens up with a copious emission of metallic fumes. In the former paper this was said to occur at the fusing point of the electrode. This is not necessarily the case; the electrode may fuse or it may waste away to a thin shred without losing its stiffness.

The first type of cathode discharge has two prominent characteristics: (1) Whenever it appears, and whatever the gas, the cathode gets hot; (2) the complicated band spectrum attributed to magnesium hydride is completely absent, in spite of the surrounding atmosphere of hydrogen. It gives an ideal sharp-line spectrum, and this simplicity of structure is not merely a special property of magnesium; calcium, which also has a band spectrum, exhibits the same peculiarity, and there is a strong probability that a general law is involved.

A portion of the cathode-glow spectrum in dry hydrogen is shown in Fig. 10, containing also sodium lines derived from the glass tubes.

The second type of cathode discharge (which will be termed for convenience the "jet" discharge) occurs in all gases, and several sub-varieties can be recognized by appearance and spectrum. Some of these depend on the emission of occluded gas, but this does not appear to be an essential factor. Two characteristic features are again apparent: (1) the coolness of the cathode; (2) the great intensity of the "hydride" bands in the spectrum of the jets, if only the merest trace of hydrogen or water vapor be present. These facts also hold good for calcium.

The sequence of changes mentioned above as taking place in the cathode glow in hydrogen cannot be obtained with certainty unless phosphoric anhydride is present in the discharge vessel, and even then the heating of the glass tubes covering the electrodes is exceedingly likely to drive off decomposition products which at once bring about the second or "jet" state. Hence success in obtaining the cathode-glow phenomena alone is a matter of some difficulty. The eruptive jets may be only feebly present, or they may entirely replace the glow, and they are certain to appear unless the gas is well dried.

In hydrogen the "hydride" spectrum was often found to be present at the anode, even when the cathode glow was without a trace of it, and as in other gases it also appeared at the anode when present in small quantity the conviction gradually arose that it was naturally an anode spectrum and that it never occurred at the cathode except in the spectrum of the jets. The presence of phosphoric anhydride always weakened it considerably, but in this gas never completely eliminated it when the rectified current was used. The anode itself is little attacked at first, but it becomes more conspicuous the longer the run and may itself supply ions, occasionally but very seldom fusing. In some experiments on the action of a magnetic field, the anode was frequently found to have received a deposit of metal in one place and to have been eaten away in another.

As no perceptible absorption of hydrogen occurred, it was evident that a solid hydride could only be present in traces, but a gaseous compound might possibly be formed without absorption. This was often looked for without success. The fumes deposited on the sides of the vessel were also examined. As a rule, but not always, they were spontaneously inflammable, taking fire in the globe when air was admitted or in a few seconds after removal. It soon became apparent that the metal dust was ignited by a variable quantity of some foreign substance and that the magnitude of the effect was related to the amount of action on the glass tubes covering the electrodes, chiefly at the cathode. Here the conductivity is reduced by the formation of a well-developed cathode glow, and the discharge then breaks its way through the glass, avoiding the metal end. There is much development of sodium flame and spectrum, and the current increases. It is partly the old "Hittorf effect." Various facts suggested that probably metallic sodium was formed in small quantity, and that this ignited the dust. Consequently it was somewhat difficult to collect fair quantities of the materials for analysis, and such estimations as were possible merely confirmed what was already known, i. e., that if any solid hydride existed it must be exceedingly small in amount. No satisfactory way of avoiding the use of glass tubes was found. Silica tubes were destroyed in a few seconds by intense chemical action; porcelain was also strongly attacked, in each case magnesium silicide being formed.

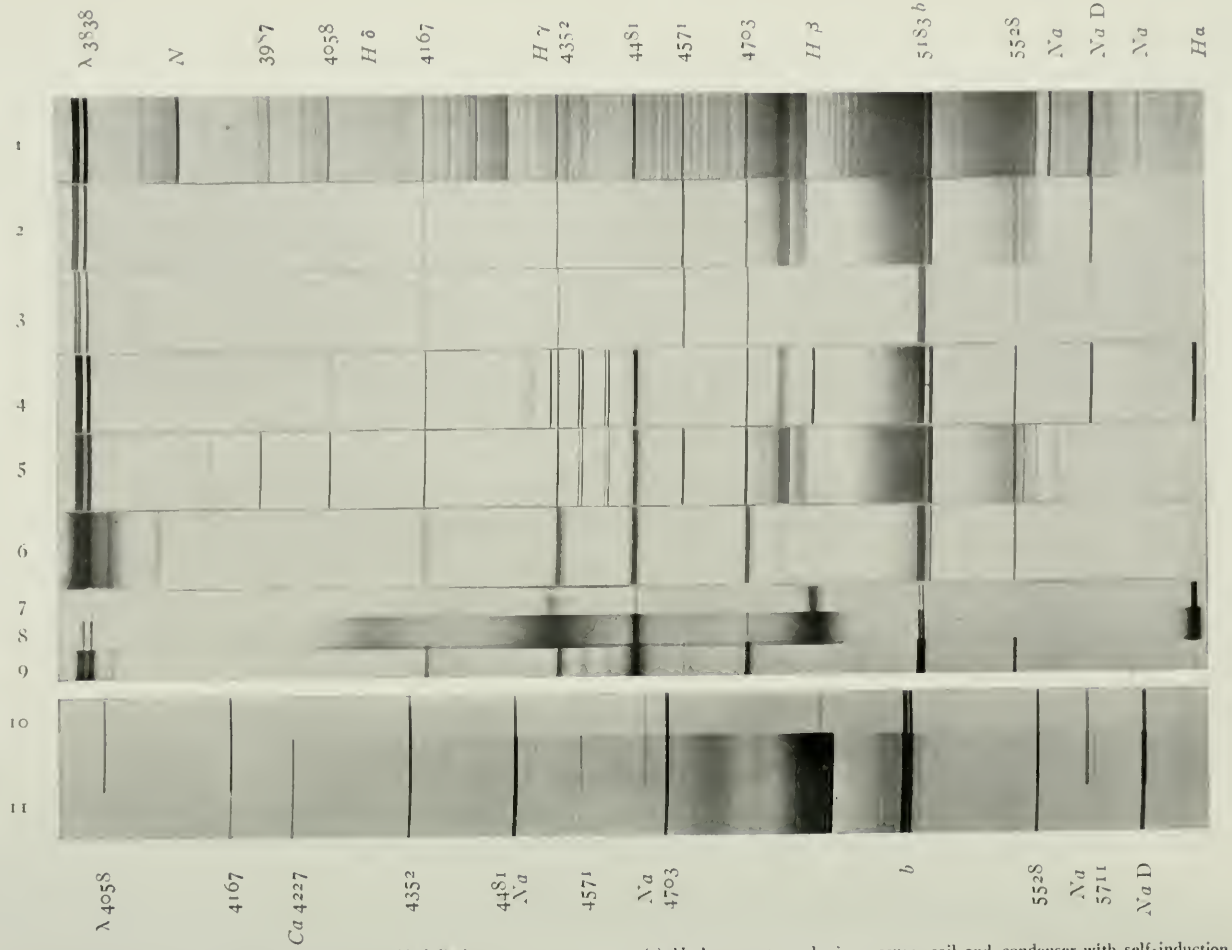
In order to obtain a metallic deposit free from chance impurity it was necessary to abandon the rectified current, and to resort to the ordinary low-tension arc discharge in a hydrogen vacuum, for in this case no inclosing tubes are required. Then the dust was never spontaneously inflammable, and when heated in a vacuum obtained by a mercury pump only such traces of hydrogen and oxides of carbon were obtained as might be expected from almost any materials heated in glass vessels. There was no suggestion of any gaseous compound.

The spectrum of the arc in vacuo has been so exhaustively studied by many observers that only a few observations were made for the sake of comparison. It was at once evident that the "hydride" spectrum was present in very great intensity, apparently occurring more persistently and readily than in any other form of discharge, and further, there was no perceptible difference between the spectra obtained at pressures of from ½ to 2 centimeters in air (dried or not dried), and in dry hydrogen. Attempts were made to get rid of the "hydride" spectrum in dry air, but without much success, and although it was certainly weakened and temporarily eliminated by repeated pumping, it always appeared again after a few minutes' run, the hydrogen lines C and F also becoming visible.

Hence the arc, in vacuo, is especially favorable to the development of the "hydride" spectrum, and the more intense the arc the stronger it becomes, but in consequence of the ready fusion of the electrodes the method offers certain practical difficulties, and it was not found possible to examine the anode and the cathode spectra separately.

The most striking fact about the arc spectrum as a whole is its marked resemblance to the "high-frequency" spectrum obtained under certain critical conditions with induction coil and Leyden jars. They are so nearly identical that it is not always easy to distinguish between them. The most noticeable difference is the absence

PLATE XIV



(1) Nitrogen, not dried, 3cm pressure, cathode, rectified discharge.
(2) Air, not dried, 2cm pressure, anode, rectified discharge.
(3) Nitrogen, dry, 3cm pressure, anode, rectified discharge.
(4) Hydrogen, dry, ½cm pressure, coil and condenser ("Flame" state).
(5) Air, dry, ½cm pressure, arc.

(7) Hydrogen, atmospheric pressure, coil and condenser with self-induction.
(8) Hydrogen, atmospheric pressure, coil and condenser without self-induction.
(9) Air, atmospheric pressure, coil and condenser with self-induction.
(10) Hydrogen, dry, 3cm pressure, cathode glow, rectified discharge.

or weakness of the chief hydrogen lines in the arc spectrum and their sharpness and strength in the coil spectrum. Also the line λ 4481 is more diffuse in the arc and sometimes reversed.

It may be remarked that Liveing and Dewar found the Leyden jar spark unfavorable to development of the hydride spectrum under the conditions of their experiments. I have found that it is completely absent, even as a trace, from the spectrum of the jar spark, in vacuo, unless the discharge assumes the specialized form termed in the former paper "the high-frequency flame" discharge; then it is invariably present in hydrogen and in most other gases. Also, in hydrogen it cannot be eliminated by any process of drying, although that method is effective in other gases.

This spectrum is shown in Fig. 4, Plate XIV. It depends on a very characteristic type of discharge, only obtained when a strong oscillatory current is used with electrodes of very small mass. Fig. 5 is the arc spectrum in dry air, at ½ cm pressure, but it will serve almost equally well to represent that spectrum in air or hydrogen whether dried or not. The only difference is that in air the fluting at λ 5007 is visible at first, rapidly becoming weaker and disappearing as the hydride spectrum increases in strength, while the line λ 4571 is of somewhat variable intensity. In Fig. 4 the carbon fluting at λ 4315 appears as impurity; in the arc spectra the red carbon line at λ 6579 is often seen.

The one common factor connected with the origin of the two spectra shown in Figs. 4 and 5 is high current-density. Whenever this is sufficiently great the hydride spectrum is strong, and most difficult to eliminate by attempting to get rid of hydrogen or water vapor. The intensity of the two pairs of well-marked lines between λ 4481 and λ 4352 (observed originally by Porter and afterward examined by Fowler and Payn)[1] is entirely dependent upon the same factor, but they are not in any way related to the hydride spectrum. They are simply very "short" metallic lines, and are often well seen in the types of spectra shown in Figs. 1, 2, and 3, although seldom strong enough to appear on the negatives. They have not been seen in the cathode-glow spectrum, but are a well-marked feature in the eruptive jets, a fact which seems to indicate that this latter form of cathode discharge is also associated with locally great current-density.

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[1] *Proc. R. S.*, **72**, 257, 1903.

As Crew did not mention the appearance of the hydride spectrum in his work on the hydrogen arc at ordinary pressures, a photograph was taken under these conditions. The result was the spectrum shown in Fig. 6, in which the hydride is distinct enough, but still quite weak. The metallic lines are all broad and diffuse except λ 4571, which is fine and sharp, and a remarkable feature is the very great relative intensity of the violet Mg triplet and the fainter lines close to it, two components of the triplet being reversed. Morse[1] has noticed a similar marked intensity of this triplet in the Wehnelt break spectrum of Mg wire in HCl, and Wilsing[2] also refers to very diffuse absorption lines in this position obtained by spark discharges in water.

The well-known jar-spark spectra obtained in hydrogen and air at atmospheric pressure are appended in Figs. 7, 8, and 9, for purposes of comparison. The spark in hydrogen is quiet and not very luminous, forming a great contrast to the noisy bright spark in air. The metallic lines are feeble and the gaseous lines strong and diffuse, while the hydride spectrum is reduced to a faint trace of its strongest edge at λ 5211, and is best seen with self-induction in the circuit. It will be noticed that the arc spectrum in hydrogen and the spark spectrum in air, both at ordinary pressure, bear a marked resemblance to each other.

The mode of occurrence of the hydride spectrum in gases other than hydrogen was carefully examined, using the rectified current. In all gases a conspicuous effect is the initial heating of the cathode. This is especially noticeable with a clean electrode and rapidly diminishes if the metal becomes oxidized and hence a cathode may almost instantly lose its shape with a small current and then carry a much larger one for an indefinite period without further fusion. It is partly an instance of the Wehnelt effect.

In an air vacuum the cathode glow is absent, the initial heating is soon over, and long runs may be obtained with less danger of fusion than in hydrogen. At first the spectrum is almost entirely gaseous, chiefly nitrogen flutings and the oxide fluting at 5007. As the discharge, somewhat irregular at first, settles down into the eruptive jet form, this spectrum disappears and unless the gas is specially dried the hydride spectrum develops as the 5007 fluting gets weaker, and it may be observed steadily with great intensity for a long time. These statements also apply to nitrogen not completely free from oxygen.

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[1] *Astrophysical Journal*, **19**, 169, 1904.

[2] *Ibid.*, **10**, 121, 1899.

All the best photographs of the hydride spectrum were obtained in this way. No really good negatives of the fainter detail were obtained in hydrogen, chiefly because the use of that gas gives no tangible increase in brilliancy, and greatly adds to the difficulty of obtaining a long run.

The cathode spectrum in air largely deprived of oxygen is shown in Fig. 1, the chief hydride bands being much overexposed in order to bring out the fainter portions. It will be seen that it extends across the whole range of the visible spectrum. Traces of nitrogen flutings are also present. When the discharge vessel contains phosphoric anhydride the hydride spectrum either completely disappears or is reduced to the merest suspicion of its strongest edge.

Meanwhile a gradual change has been taking place in the appearance and spectrum of the anode. This does not begin until the oxygen is used up, as evidenced by the disappearance of the 5007 fluting, and requires a fairly long run and a strong current to bring out properly. The anode at first is not very conspicuous, except for the nitrogen glow. In the end it becomes covered with numerous minute specks of light shining steadily on a bluish ground tint. The gaseous lines disappear, the hydride bands are present but relatively weak, and there remains a sharp metallic spectrum in which the line 4481 is always weak and sometimes absent, while the line 4571, which has been steadily increasing in strength, becomes the brightest line present (excepting the *b* triplet). Exactly the same result is obtained when the air is dried, except that the hydride spectrum is absent, and it is still more readily obtained when the air is first largely deprived of its oxygen.

Figure 2 shows this anode spectrum in air after a long run. It was obtained immediately after one similar to Fig. 1, by merely reversing the direction of the current, and in fact this spectrum was visible at the anode while Fig. 1 was being photographed. Fig. 3

is the anode spectrum in fairly dry and pure nitrogen, and shows the influence of phosphoric anhydride in eliminating the hydride spectrum. It shows traces of cyanogen, which is a common impurity.

Many attempts were made to observe the discharge in pure dry nitrogen, but it was extremely difficult to insure that no oxygen was present. The greater its purity, the greater is the initial heating of the cathode, which almost inevitably fuses, and the more erratic is the discharge at its commencement. The characteristic anode glow and spectrum appear after a time as usual, and when a trace of the hydride is present it is strongest at the anode. Nitride is formed at both electrodes, but never in large quantity, and the absorption of gas goes on very slowly. It does, however, seem probable that the enhancement of the line 4571 is in some way related to the formation of nitride. Professor Hartley[1] photographed the spectrum of magnesium under reduced pressure, using a spark discharge with condenser, and from his observations suggested that it was more than probable that two pairs of lines, one at 3830 and 3837, and the other about 5200 and 5209, were the spectrum of magnesium nitride. The first two are components of, or near to, the violet triplet and these I do not find affected in any way by the presence of nitrogen. The only lines in the second region (apart from the complex shading of the hydride spectrum) are (1) the principal hydride edge at 5211, (2) two oxide lines at 5205 and 5191. Under the conditions of his experiments the hydride edge might easily be present. None of my negatives suggests the existence of any new lines or flutings, and provisionally I regard the spectrum shown in Fig. 3 as characteristic of nitride formation within the region covered by the photograph.

When the rectified discharge is taken in water vapor at low pressures, there is as usual a short initial stage with hot cathode. At first the water vapor or hydrogen spectrum is very strong, but this gradually disappears as time goes on. A small current does not bring out the hydride spectrum; at about 0.5 ampere it appears faintly and at about 1 ampere the anode suddenly swells up and glows red hot throughout, becoming a mass of oxide without fusion and liberating hydrogen in quantity. At the same instant the hydride spectrum flashes out in great intensity, but of course it is only visible for a few seconds in consequence of the great rise of pressure. The cathode is but little attacked. The anode can be brought up to the point of ignition slowly or rapidly by regulating the current, the hydride spectrum is always weak except during the reaction, and, as in all cases, the C and F lines of hydrogen tend to weaken and disappear as that spectrum becomes stronger.

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[1] *Proc. R. Dublin Soc.*, 11, 245, 1907.

It may be remarked that the hydride spectrum is also seen when magnesium is burnt in a current of steam at ordinary pressures—the combustion lasts only a few seconds and there is a strong continuoŭs spectrum, but the brightest hydride fluting is always visible.

When the rectified discharge is taken in pure dried oxygen at a pressure of 3 cm not a trace of the hydride spectrum is seen.

There is some initial heating of the cathode, but it quickly subsides and there is no difficulty on that account. There is no cathode glow and although the cathode discharge is of the eruptive jet type, it exhibits certain peculiarities. There is little absorption of gas with small current, but if it be unduly increased, absorption becomes increasingly rapid, and one of the electrodes, usually the anode, may suddenly ignite and burn to a mass of oxide.

The spectra obtained are highly characteristic. At the anode there is a sharp metallic spectrum, in which the two pairs of lines between 4481 and 4352 are often well seen. The line 4481 is strong and sharp, while 4571 is weak. A noticeable feature is the presence of the calcium lines 4227, 3968, and 3933, which no doubt are derived from the glass, but seldom observed except in oxygen. As might be expected the oxide spectrum is very pronounced, and as there is comparatively little continuous spectrum this method affords a convenient means of examining it. Good negatives are somewhat difficult to obtain; if the current is too weak the fainter detail is not brought out properly, and if only a little stronger, one of the electrodes is liable to burn up. Again, as the electrodes become oxidized the continuous spectrum due to incandescent points of oxide becomes increasingly troublesome.

At the cathode the oxide bands are also seen, but here the prominent feature is the presence of the line spectrum of oxygen in considerable intensity.

A portion of the anode spectrum is shown in Fig. 11. After this exposure on one-half of the slit the apparatus was well washed out with hydrogen, again exhausted, and the same electrode was then used as cathode. In this case the cathode glow appeared free from the jet discharge and Fig. 10 was obtained. At the same time the hydride spectrum was visible at the anode.

The anode spectrum in oxygen exhibits several points of interest. At the red end I find a series of bands with edges sharp toward the red which do not appear to have been previously mentioned. These would be easily photographed were it not that they are first to become lost in the continuous spectrum which gradually increases in intensity as the oxidization of the electrodes proceeds, and hence only short exposures are possible. Visual observations present similar difficulties. Approximately their positions are 6589?, 6314, 6063, 5781, 5477, 5284, 5249, and there are several more edges too faint for measurement. The first of these values is a very rough estimate, the others are probably correct within one or two units. In the neighborhood of the *b* triplet there is a fluting with maxima at 5205, 5191, 5162, 5145, 5123, 5110, for which my measurements agree almost exactly with Eder's, but his line at 5210.7 is almost certainly the principal hydride edge.

Another conspicuous feature is a fluting without sharp edges between the 5007 fluting and the weak *Mg* line at 4730, in the same place as one of the stronger hydride flutings, also without sharp edges. This again does not appear to have been previously mentioned. It cannot be resolved visually, owing to its extreme faintness and it was not photographed until observations were made in oxygen. In the *Denkschriften* of the Vienna Academy (**74**, 54, 1903), there is a plate accompanying a paper by Eder which shows one or two lines faintly resolved, but buried in a strong continuous spectrum which begins just at this point and extends far into the violet.

The wave-lengths of the principal edges are approximately 4826, 4820, 4811, 4802, 4791, 4781, and 4771.[1] The edge at 4781 is rather diffuse, and several faint lines lie between it and 4791. Several other lines, somewhat irregularly spaced, have been measured between 4771 and 4730, which may also belong to the system.

The remaining portions of the oxide spectrum have been frequently observed, and although very strong in these photographs, do not call for special remark. Eder has catalogued a series of bands from 3091 to 3865 which certainly extend as far as 4100, and in fact the whole space up to the *b* triplet appears to be full of exceedingly faint detail. My negatives also show a sharp, well marked line at 4368, and a fainter one at 4381 whose origin has not been satisfactorily determined.

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[1] A first notice of this fluting appeared in *Nature*, July 2, 1908, p. 198.

Returning to the consideration of the hydride spectrum, from the whole series of observations the following conclusions can be drawn: (1) This spectrum represents some transitional and unstable state; (2) its production depends far more upon the nature of the discharge than upon the quantity of hydrogen present, although the presence of this gas in some form appears to be necessary; (3) a trace of water vapor is even more effective than hydrogen, but its presence cannot be regarded as an essential condition ; (4) if really due to a hydride, that body is probably decomposed as fast as it is formed.

It is very suggestive that no increase in the extent and brightness of this spectrum is noticed when a hydrogen atmosphere is used, instead of one in which hydrogen is only due to occluded gas or water vapor and that in the latter case the gas never gets used up, with the disappearance of the corresponding spectrum, as a trace of oxygen does. It is also suggestive that the most important factor appears to be current density at the electrode. Whenever this becomes great enough, the hydride spectrum appears, in spite of efforts to eliminate hydrogen, whereas when it is small, as in ordinary spark discharges, that spectrum is either faint or altogether absent, even in an atmosphere of hydrogen.

The necessity for hydrogen is the only evidence that this spectrum is due to the formation of a compound. The point can scarcely be said to be definitely settled experimentally on account of the practical impossibility of eliminating that gas, but on the whole the facts seem decidedly in its favor. It does not, however, seem necessary to assume the existence of an actual hydride, except for convenience, and unless the term "compound" is to have a very extended meaning. It is perhaps worth remarking that what is generally termed the spectrum of a compound is really that of certain processes which take place during its formation or decomposition, the true nature of the vibrating system being unknown. For instance, MgO, once formed, merely gives a continuous spectrum.

Again, the undoubted formation of nitride appears to show itself only by the strengthening of a single line.

Professor Hartley, as the result of his extensive and long-continued researches on the band spectra of metals in the oxy-hydrogen flame, has recently stated that these are due to the atoms and not to compounds, and his far-reaching conclusions appear to cover the case of magnesium hydride.

The existence of two kinds of cathode discharge, one free from the hydride spectrum, and the other highly favorable to its appearance, seems to be a fact of some importance. In this region the phenomena of discharge depend largely upon the behavior of these positively charged bodies which constitute the canal rays, and it is not impossible that here we may have to reckon with the two kinds of positive entities common to all gases, whose existence has been established by Sir J. J. Thomson. Again, it seems highly probable, as an inference from much experimental data not dealt with in this paper, that the eruptive jet form becomes greatly developed in, and is typical of, the ordinary arc discharge, being dependent on current-density, while the cathode-glow form becomes more prominent in the spark and in vacuum tubes. But in neither case does the preponderance of one type necessarily mean the total exclusion of the other.

Much work as yet incomplete has been done to throw light on the state of affairs at the electrodes. As regards the cathode, whatever the state of Mg particles may be when driven off into the cathode glow in hydrogen, they appear almost immediately to act as if either positively charged or neutral, and in spite of their intimate mixture at first with hydrogen ions in that glow, no trace of the hydride spectrum is seen. On the hypothesis of a compound, this may be explained by supposing the Mg and H ions to be similarly charged and unable to combine, and the fact that the hydride spectrum can in some instances be seen simultaneously at the anode may be regarded as indicating that it is due to a negatively charged aggregate. There are certainly two well-marked stages in the evolution of fumes from the cathode. At first they consist of extremely finely divided metallic dust, which readily rubs up to a bright mirror, but after a long run with rather heavy current there is a somewhat sudden change; much larger metallic particles are projected, and also a number of small metallic spheres, very perfectly formed and with a brilliant metallic luster, which are hollow. They are thin-shelled bubbles averaging about a millimeter in diameter and are interesting as being produced at a surface which as a whole does not necessarily soften. Similar bubbles may be seen in arc residues, but they are much larger and more numerous with the rectified current.

A preliminary study of other metals used as electrodes has brought out certain important differences of behavior; for instance, calcium seems to waste away most rapidly at the anode. It is hoped to deal with these matters more fully in a later paper.

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THE RELATIVE INTENSITIES OF THE YELLOW, ORANGE, AND RED LINES OF CALCIUM IN ELECTRIC FURNACE SPECTRA[1]

By ARTHUR S. KING

The large electric furnace in the Pasadena laboratory[2] has recently been used by the writer to study the effects of different temperatures on the calcium spectrum in the region of greater wave-length, special attention being paid to some twenty lines occurring from λ 5857 to λ 6718, which were affected very differently by changes in the furnace temperatures. All of the prominent arc lines in this region are given by the furnace, and the experiments were arranged to bring out not only the effects of varying the temperature, but also of increasing the quantity of vapor, to observe whether any lines in this region show a behavior similar to that of the "flame line" λ 4227, which was shown in a former investigation[3] to respond but slightly to change of temperature, while enormous widening was brought about by a large increase in the amount of calcium vapor present.

The photographs were made with the vertical Littrow spectrograph with an objective of 13 feet (4 m) focal length, the first order of a 5-inch (13 cm) plane grating being used. The plates were Seed's "27," sensitized by the Wallace three-dye process.[4] The furnace was charged with a few pieces of clean metallic calcium, a large or small amount being used according to the vapor-density desired. The tube was heated in vacuum, first with an e.m.f. of 20 volts which rapidly raised the temperature, reaching in about five minutes a temperature of approximately 2800° C. as measured by a Wanner pyrometer. An exposure was made as the temperature rose, then others of one-half to one minute at the maximum temperature. A vigorous vaporization of the carbon was then taking place. The current was broken and 15 volts substituted, after which one or two low-temperature exposures were made, fifteen to twenty times the length of the exposure now being required to produce a photograph of the same average strength as that at the highest temperature. No close value for this temperature can be given, as it was somewhat variable, the heating of the jacketing running up the temperature so that is was necessary frequently to break the current for a few seconds. The average temperature seemed to be about 2200° C., probably never rising above 2400° C.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 35.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 28; *Astrophysical Journal*, **28**, 300, 1908.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 32; *Astrophysical Journal*, **28**, 389, 1908.

[4] *Astrophysical Journal*, **26**, 209, 1907.

The influence of vapor-density was tested by using very different amounts of calcium in the furnace tube. These were given (1) by the empty tube, the graphite of which contains enough calcium to show the stronger lines distinctly; (2) a small amount (about 0.1 gm) of metallic calcium; (3) a large amount (2 to 3 gms) of the metal.

It may be noted here that chemical action was present in these experiments to a greater degree than is the case when a substance not readily oxidizable is used in the furnace. The bright surfaces of the calcium became dulled while it was being placed in the furnace tube and further time was required to close the chamber and pump out the air. The result was that brilliant red bands appeared during the first minute of the heating. These soon died out for the most part, but some of them persisted through several exposures, especially when a large quantity of calcium was used. These weakened rapidly with time, and the strength of the calcium lines appeared to be quite independent of the existence of the bands, the lines being produced with equal facility whether the bands were strong or barely visible.

The following table includes the lines from λ 5582 to λ 6718, with their intensities in the furnace spectra at high and low temperatures for the three conditions of vapor density given respectively by a large amount of metallic calcium, a small amount, and by the empty tube. The estimates of intensity were made by comparison with a photographic scale in the manner described in a previous paper,[1] modifying the estimates when necessary according to the character of the lines. The spectra given by high vapor-density afforded the most complete material for comparison, on account of the average density of the two photographs being very nearly the same; but some allowance was necessary for the lines at the higher temperature being widened and diffuse when much vapor was present, instead of sharp and black. In the spectra with less calcium present, the spectrum for the lower temperature was as a whole weaker than for the higher temperature, so that a factor of 2 is needed to bring the low-temperature spectra (in the fifth and seventh columns) to an average intensity comparable with that at the higher vapor-density. The intensities are given in the table without reduction.

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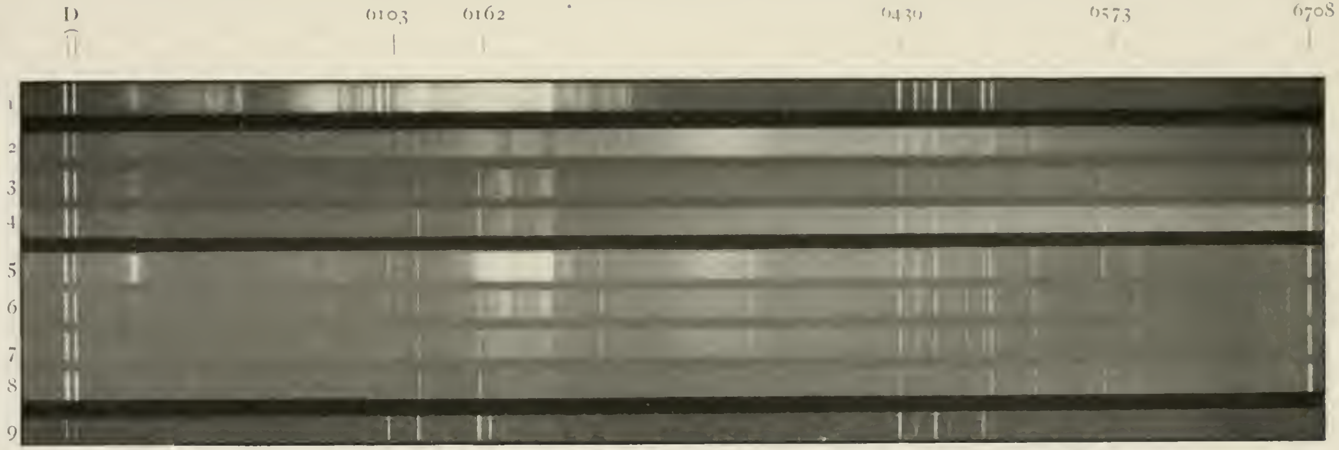
[1] *Contributions from the Mount Wilson Solar Observatory*, No. 28; *Astrophysical Journal*, **28**, 300, 1908.

	HIGH VAPOR-DENSITY		MEDIUM VAPOR-DENSITY		LOW VAPOR-DENSITY	
λ	High Temp.	Low Temp.	High Temp.	Low Temp.	High Temp.	Low Temp.
5582.20	2	1	2	trace	*	1
5588.98	4	3	4	2	4	3
5590.34	2	trace	1	. .	*	trace
5594.69	3	2	3	1	3	2
5598.71	3	2	3	1	*	2
5601.50	2	1	1	. .	*	1
5603.08	2	1	1	. .	*	1
5857.67	5	2	3	trace	5	1
5867.78	2	1	2	1	1	. .
6102.94	9	8	9	6	9	6
6122.43	10	18	12	15	16	10
6161.50	1	. .	trace	. .	. .	. .
6162.39	8	24	14	22	20	15
6163.97	2	1	1	. .	1	. .
6166.65	3	1	2	trace	2	. .
6169.25	4	2	3	trace	4	2
6169.78	6	3	7	1	6	3
6439.29	16	18	18	9	20	10
6450.03	10	6	8	2	6	3
6455.82	4	. .	3	. .	2	. .
6462.78	14	15	16	7	15	7
6471.88	8	4	5	1	4	2
6494.00	12	9	10	3	10	4
6499.88	7	2	3	trace	2	1
6573.03	5	18	5	10	8	11
6708.18	20	80	24	60	30	70
6717.94	3	2	1	. .	2	. .

* Concealed by strong carbon fluting in this photograph.

Description of plate.—Plate XV reproduces the region from λ 5890 to λ 6708 for different temperatures and different quantities of vapor Photograph No. 1 gives the spectrum from calcium in the carbon arc a large quantity of the metal being present. Nos. 2, 3, and 4 are furnace spectra with much calcium in the tube, 20 volts being used on the tube for No. 2 with an exposure of one-half minute when the tube was very hot. Nos. 3 and 4 were taken with 15 volts on the tube and exposures

PLATE XV



The Spectrum of Calcium in the Electric Furnace, with Comparison Arc Spectra

Scale: 1mm = 5 Ångström units

of 5 and 10 minutes respectively after No. 2 had been made, the current being broken several times during No. 4 to keep the temperature down. No. 4 is most comparable in average intensity with No. 2, though No. 3 is useful to show the relative strength of some of the stronger lines at the lower temperature. Nos. 5, 6, 7, and 8 are furnace spectra with a small quantity of calcium in the tube. Nos. 5, 6, and 7 were taken with 20 volts on the tube and successive exposures of four, three-quarters, and one-half minute respectively as the temperature rose, there being no break of the current between exposures. No. 7 best represents the condition for high and nearly constant temperature, while Nos. 5 and 6 are of interest as showing the weakening of the bands as the vaporization progressed in the vacuum, it being possible to keep the lines of nearly the same intensity in the three photographs, by proper timing of exposures. No. 8 is a low-temperature spectrum taken with 15 volts on the tube and an exposure of seven minutes immediately after No. 7. A final arc photograph (No. 9) is added below for comparison, the amount of calcium and exposure time being so adjusted as to give the prominent lines with very weak bands.

Discussion.—Referring to the table, it is obvious at once that the calcium lines in this region differ greatly in their response to the stimulus accompanying increased temperature. The low-temperature lines may be at once selected from the first two columns of the table or by comparing any pair of photographs at high and low temperatures reproduced in Plate XV. These lines strengthen but slowly with rising temperature, while a number of lines barely visible at the lower temperature rapidly attain considerable strength with higher heating of the tube. Several sets of lines may be selected which show this effect in a striking manner, for example, λ 6122 and λ 6162 as compared with λ 6103; λ 6439 and λ 6463 as compared with λ 6450 and λ 6456; λ 6573 and λ 6708 as compared with λ 6718, and a number of other notable contrasts. λ 6103 must, however, be classed among the low-temperature lines.

The object of using different quantities of calcium in the furnace tube was to see how large a part vapor-density may be expected to take in determining the relative intensity of the spectrum lines, calcium being in many ways a favorable substance for a test of this kind. Comparing the first and third columns of the table, these spectra being taken at approximately the same temperature and with very different quantities of vapor, a few of the weaker lines show slightly stronger with higher vapor density, the most decided being λ 6471.88 and λ 6499.88. An interesting difference is seen between λ 6122 and λ 6162, their intensities being in the ratio 10 to 8 at the higher vapor density and 12 to 14 at the lower. It holds in all of the spectra, however, that λ 6122 strengthens with rising temperature more rapidly than λ 6162, so that if the tube were slightly hotter for the spectrum given in the first column the difference between the two lines should be in the direction observed. The same cause would account for the behavior of λ 6471.88 and λ 6499.88, both of these being high-temperature lines and rapidly strengthened as the temperature rises.

The line which we should expect to be most affected by vapor-density is the strong red line λ 6708, which is comparable in many ways to the "flame line" λ 4227, appearing strong at low temperatures with a mere trace of calcium present. It does not readily reverse, however (the widened appearance is due in part to poor focus at the end of the plate), and shows no decided change in appearance or intensity when the quantities of vapor are greatly different. An examination of plates taken in this and the former investigation, covering the spectrum as far as λ 3700, shows no lines except λ 4227 in which change of vapor-density has a decided effect. The conclusion to be drawn from the calcium spectrum is that while increase of vapor-density makes all of the lines wider and more diffuse, it does not appear promising as a general cause in producing changes of relative intensity among lines when temperature and other conditions remain unchanged; and these latter can be held constant for different vapor-densities much more nearly in the furnace than is possible in the arc. It appears probable that λ 4227 and some of the most decided "flame lines" in other spectra are in a class by themselves as regards dependence on vapor-density. However, a study under higher dispersion of the widening produced by increased vapor-density will be necessary for a full discussion of this point.

The *band spectrum* in this region is of interest for the present paper chiefly as an indication of the degree to which chemical action is present. The bands which appear in the furnace photographs occur, with one exception, also when metallic calcium is vaporized in the carbon arc burning in air and are presumably due to the oxide unless possibly they are given by the metal itself. As has been noted, the calcium became oxidized to some degree before the air could be removed from the furnace chamber. Photographs Nos. 5 to 8 of Plate III show the weakening of the band spectrum as the experiment progressed and indicate that this initial state of oxidation gave rise to the the bands. The increase of temperature during exposures 5, 6, and 7 might act to weaken the bands, but in that case No. 8, taken at a a lower temperature comparable to the beginning of No. 5 when the bands were most brilliant, should restore their intensity to some degree, which is not the case. The same argument makes it doubtful as to whether they can belong to the metal; since in that case they should show a behavior similar to that of the low-temperature lines (λ 6708, for example) instead of a steady fading away as time went on.

When a larger amount of calcium was used, the bands were much more persistent, as would be expected from the increased opportunity for chemical action. In No. 2 the stronger bands are seen reversed, while they are bright and fairly distinct in Nos. 3 and 4.

The band referred to, which does not appear in the spectrum of the arc burning in air, is that with prominent heads at λ 6389.3 and λ 6382.2, shaded toward the violet. They show distinctly in Nos. 2 and 5, especially in the latter, and are faintly visible on the negative in the low-temperature photographs. This is the band measured by Olmsted[1] and found especially strong in the calcium arc burning in hydrogen. The band is not usually perceptible in the calcium arc in air or when a plentiful supply of oxygen is present. Hydrogen or water vapor supplied to the arc with partial exclusion of oxygen bring it out strongly. In the furnace spectra it is favored, like the other bands, by the initial conditions when the furnace is heated up. It weakens with time and is not restored by lower temperature, so that it seems unlikely that this band belongs to the metal; while its behavior in the arc, as has been noted, is different from that of the other furnace bands. If the band at λ 6382–6389 is due to a hydrogen compound, this would most probably arise from moisture inside the steel chamber, which would be vaporized at the first heating, and would then be rapidly pumped out, as the pump is usually kept working during the operation of the furnace. It is hoped soon to make further tests as to the origin of the band.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 21; *Astrophysical Journal*, **27**, 66, 1908.

Comparison with sun-spot lines.—The sensitiveness of the calcium lines to temperature changes makes them favorable for use in testing the hypothesis that temperature is one of the chief agencies in causing the differences between the solar spectrum and that of sun-spots. The relation holds, quite as generally as in any spectrum examined, that all of the low-temperature lines are much strengthened in sun-spots, usually in the ratio of about 2 to 1 in photographs where the continuous ground is of the same density for both spot and solar spectra; while the two most conspicuous low-temperature lines, λ 6573 and λ 6708, are intensified in sun-spots in the ratio 15 to 1 and 5 to 0000 (on the Rowland scale) respectively. However, some condition exists on the sun which causes all of the calcium lines to be more or less strengthened in the spot and it frequently happens that some of the high-temperature lines, as for example λλ 5858, 6167, 6456, 6500, are considerably stronger in the spot than in the sun. There is a vital difference, however, between the character of these lines and that of the pronounced low-temperature lines, such as λλ 6103, 6122, 6162, 6439, in that the latter are very much widened and "winged" in the spot, the formation of wings being very slight for the group of high-temperature lines.

So much remains to be settled in regard to the causes of widening in solar lines, the matter being still further complicated by the existence of a magnetic field in sun-spots, which would cause lines to be widened differently according to the type of magnetic separation, that for the purposes of the present paper the matter may best be left with the statement just given of the general relation between the low- and high-temperature lines. An intensive study will soon be made of a few of the calcium lines under higher dispersion, to determine by a comparison of various laboratory conditions what causes are chiefly instrumental in the observed widening of these lines in solar spectra.

It may be well to state what is meant by the constant use in my electric furnace papers of expressions concerning the dependence of spectra upon "temperature," that necessary brevity of statement in the use of this word may not be mistaken for looseness. Nothing in the electric furnace experiments can claim to throw any light on the ultimate mechanism of radiation. We are, however, dealing here with temperature as the primary or rather the *initial* exciting cause; though the experiments leave open the possibility that other agencies, *results* of the high temperature, supply the stimulus finally needed to produce light. With the low voltage employed on the furnace tube, it is scarcely conceivable that the electric current produces any condition that would not be present if the tube were heated by a flame applied to the outside or were imbedded in a mass of hot material, the ease of producing a high temperature being the sole point in favor of the use of the electric current. All of the consequences of the electron theory may hold for the radiation in the furnace, provided we admit that the temperature supplies the necessary energy and controls its amount. This position of temperature as the primary excitation is the essential point of contrast between the furnace on the one hand and the arc and the flame on the other. In the two latter sources, high temperature is the *result*, in the one case of the electrical action, in the other case of the chemical processes of combustion.

Mount Wilson Solar Observatory
January 1909

THE 60-INCH REFLECTOR OF THE MOUNT WILSON SOLAR OBSERVATORY[1]

BY G. W. RITCHEY

The 60-inch reflector of the Mount Wilson Solar Observatory was first tested visually on the stars on the night of December 13, 1908, and the first celestial photograph was secured with it on December 19. A number of accessories of the telescope and its steel dome and building still remain to be completed; the most important of these are the three spectrographs, and the small refrigerating apparatus for holding the large mirror throughout the day at the expected night temperature.

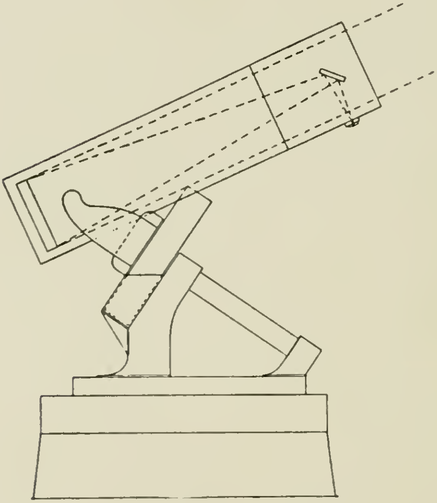

FIG. 1

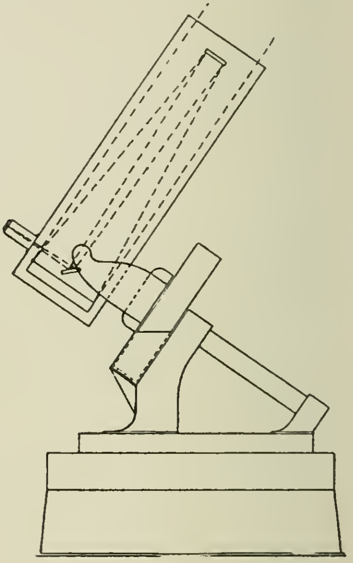

FIG. 2

The 60-inch (152 cm) reflector is designed to be used in four principal ways, as follows: First, as a Newtonian, for direct photography with the double-slide plate-carrier, and for spectroscopic work with a spectrograph carried at the Newtonian focus; in this use the focal length is 299 inches (7.6 m) (see Fig. 1). Second, as a Cassegrainian, for direct photography with the double-slide plate-carrier; in this use the equivalent focal length is approximately 100 feet (30.5 m), and the enlarged image is formed at the north side of the tube, near its lower end (see Fig. 2). Third, as a Cassegrainian for spectroscopic work with a large spectrograph, of the type of the Bruce spectrograph of the Yerkes Observatory, attached to the north side of the strong cast-iron part of the tube, near its lower end; in this use the equivalent focal length is approximately 80 feet (24.4 m) (see Fig. 3). And fourth, as a Cassegrainian-*Coudé* for spectroscopic work with a very large spectrograph mounted on stationary piers in an underground constant-temperature pit; in this use the equivalent focal length is approximately 150 feet (45.5 m) (see Fig. 4.)

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 36.

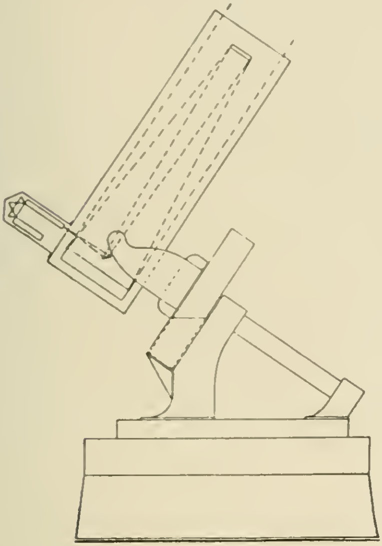


FIG. 3

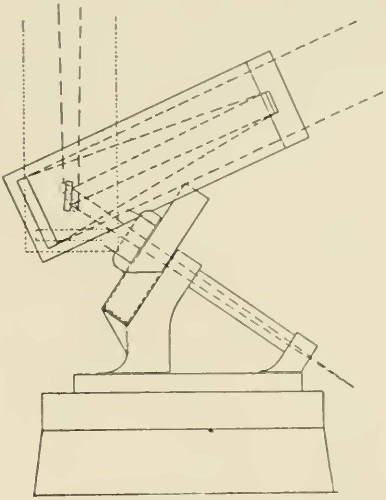


FIG. 4

THE OPTICAL PARTS

For the above uses of the telescope six optical mirrors are required, all of which have been completed in our shop: the 60-inch paraboloidal mirror of 299 inches (7.6 m) focal length, which in its finished condition is 7⅝ inches (19.4 cm) thick at the edge, 6⅞ inches (17.5 cm) thick at the center, and weighs 1900 lbs. (865 kilos);

the Newtonian plane mirror, which is of elliptical outline, is 19¾ inches (50.2 cm) long, 14½ inches (36.8 cm) wide, and 3⅛ inches (7.9 cm) thick; the *Coudé* plane mirror, which is also elliptical in outline, is 22¼ inches (56.5 cm) long, 12½ inches (31.8 cm) wide, and 3⅝ inches (9.2 cm) thick; and three convex hyperboloidal mirrors, with diameters respectively of 16 inches (40.6 cm), 16¾ inches (42.6 cm), and 17½ inches (44.5 cm), and giving equivalent focal lengths, as before stated, of approximately 80 feet (24.4 m), 100 feet (30.5 m), and 150 feet (45.7 m) respectively; each of these mirrors is about 3 inches (7.6 cm) thick. All of these mirrors are polished approximately flat on the back, and when in use in the telescope are silvered on the back as well as on the face, in order that the effect of temperature change may be symmetrical on front and back.

The methods used in grinding, polishing, and testing the mirrors are practically the same as those described in my paper on "The Modern Reflecting Telescope," published by the Smithsonian Institution in 1904. The methods used in testing the optical surfaces are also described in the *Astrophysical Journal*, **19**, 53, 1904. As these methods have now been applied and thoroughly tested in the case of a reflecting telescope of the largest size with satisfactory results, the following remarks upon the optical work may be of interest.

Of much practical importance in its bearing upon the making of very large optical surfaces is the fact that, in the case of the 60-inch glass, grinding and polishing tools of only about one-fourth the area of the glass were used with entire success in excavating the large concave and in fine grinding and polishing; a full-size, flat grinding tool was used only in the preliminary work of securing a perfect surface of revolution. A circular grinding tool of cast-iron, 31½ inches (80 cm) in diameter, was used in all of the fine grinding of the large concave surface. In polishing this concave, and in bringing it to an optically perfect spherical surface preparatory to parabolizing, a 90° sector-shaped polishing tool of exactly one-fourth the area of the large glass was used with the best results. In parabolizing, a circular polishing tool 20 inches (50.8 cm) in diameter was exclusively used in securing the necessary change of curvature from center to edge of the glass; in addition to this the 90° sector tool, used with long diametrical and chordal strokes, was found to be of great value in smoothing out the paraboloidal surface. With these two figuring tools alone, used with the machine, a very close approximation to a true paraboloid was secured. The figuring was completed with much smaller tools used by hand to soften down several slight high zones.

In figuring the large paraboloid, one modification only was found desirable in the polishing machine described in my Smithsonian paper. The two cranks which give the motion to the polishing tools were remade in such a way that their throw or stroke can now be altered at will while the machine is running. The optician is thus enabled to change the position and stroke of the tool with a perfectly smooth progression while parabolizing; these changes are actually made at the end of each revolution of the glass, and a very great improvement in the smoothness of curvature of the paraboloid is at once apparent.

In the early stages of figuring the large paraboloid, testing was done at the center of curvature, by measuring the radius of curvature of the successive zones; in the final stages, however, all tests were made at the focus of the paraboloid, with the aid of a 36-inch (91.4 cm) collimating plane mirror of the finest figure, which was made in our optical shop expressly for the purpose of testing the large paraboloid and the three smaller hyperboloidal mirrors described above. This 36-inch plane mirror was mounted on edge on an iron carriage sliding on massive iron ways carefully finished straight, and could be moved horizontally by means of a long screw; in this manner it could be readily placed so as to show, at the testing knife-edge at the focus of the 60-inch paraboloid, any 36-inch circle of this paraboloid. This test is a most rigorous and satisfactory one, enabling the optician to see, and to determine the character of slight zonal errors which cannot be detected by the test at the center of curvature. The three hyperboloidal mirrors were tested in a similar manner, in conjunction with the 36-inch plane mirror.

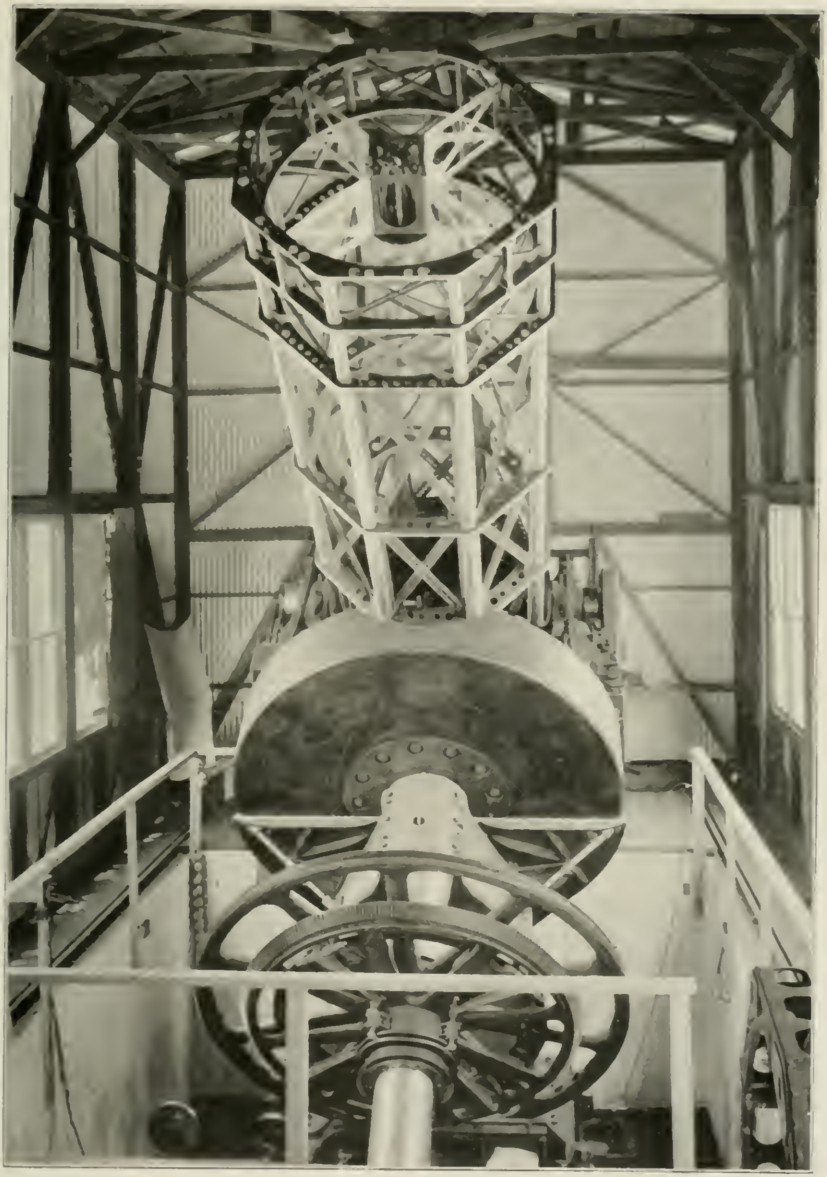
While a collimating plane mirror of the full size of the paraboloid would be desirable and convenient, my experience has shown that a plane mirror with a diameter three-fifths that of the paraboloid, used as above described, gives excellent results. A rough disk of glass has accordingly been ordered for a 60-inch plane mirror for testing the 100-inch (254 cm) Hooker glass.

THE MOUNTING

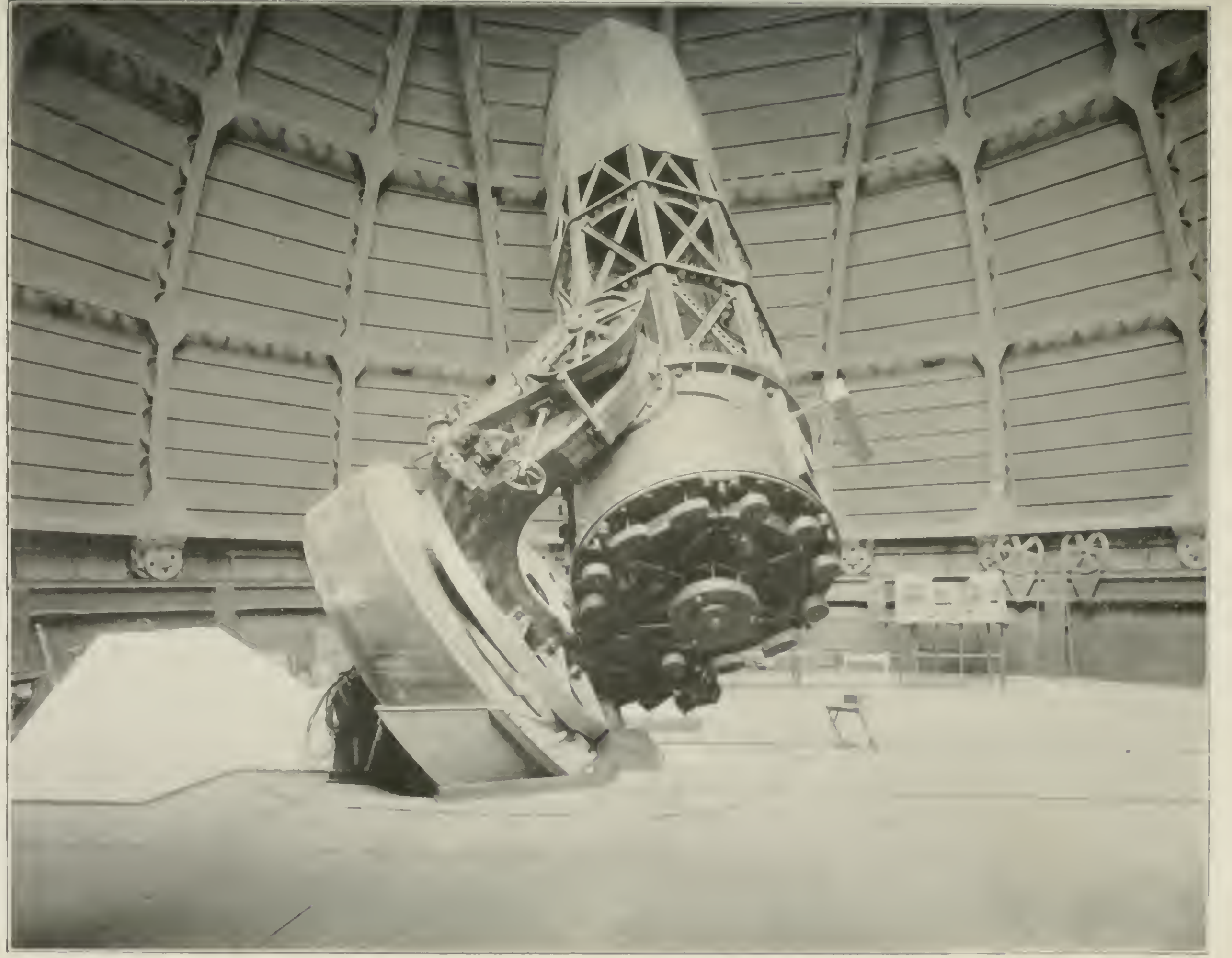
In all essential features the design for a 60-inch mounting described in my Smithsonian paper has been carried out. The general character of the mounting will be seen by reference to Plates XVI and XVII; the first of these shows the mounting as it appeared in our erecting shop in Pasadena, the second as it appears when finally set up in its dome on Mount Wilson.

The very large parts of the mounting, including the base, the polar axis, the mercury float and trough, and the tube, were made by the Union Iron Works Company of San Francisco; a large amount of final machining and finishing of these parts was done by us after their arrival at Pasadena. In addition, the construction of all of the smaller and more refined parts of the mounting, including the driving-clock and its connections, the electric quick- and slow-motion mechanism, the lever-support system of the large mirror, the cells and their supports for the five small mirrors, the automatic-rotation mechanism for the *Coudé*-plane mirror, the large double-slide plate-carrier, the graduated circles, and the cutting and grinding of the 10-foot (3.5 m) worm-gear on the polar axis, was done at our shop in Pasadena.

The cast-iron base is 15 feet (4.57 m) long, 7 feet (2.13 m) wide, 18½ inches (47 cm) deep, and weighs 14,000 lbs. (6350 kilos). The north and south columns, forming the bearings of the polar axis, weigh respectively 9500 lbs. (4275 kilos), and 2000 lbs. (907 kilos). The polar axis is a hollow forging of nickel-steel, hydraulic-forged by the Bethlehem Steel Company, and is turned and ground all over; it is 15 feet (4.6 m) long, varies from 15 to 18 inches (38.1 to 45.7 cm) in diameter, and weighs 9200 lbs. (4140 kilos). At its upper end is a head or flange 4½ feet (1.37 m) in diameter and 6 inches (15.2 cm) thick. To the lower side of this flange is bolted the float, which is a very rigid hollow disk of steel boiler plate, 10 feet (3.05 m) in diameter and 2 feet (61 cm) deep or thick, weighing 8600 lbs. (3900 kilos). To the upper side of the flange of the polar axis is bolted the fork, between the great arms of which the tube swings in



60-INCH REFLECTOR MOUNTING IN ERECTING SHOP



60-Inch Reflector Mounting in Dome

declination on nickel-steel trunnions 7 inches (17.8 cm) in diameter. The fork is of cast-iron, of hollow box-section, is 9 feet (2.7 m) across in extreme width, and weighs 10,400 lbs. (4711 kilos). Twelve nickel-steel bolts, 2½ inches (6.4 cm) in diameter and 3 feet (91.4 cm) long, pass through reamed holes through the base of the fork, through the flange of the polar axis, and through the cast-iron center or hub of the float, thus clamping these massive parts together with extreme strength and rigidity at a region of the mounting where the greatest tendency to flexure occurs.

The float dips in a cast-iron trough which is machined to nearly fit the float, leaving a space of only one-eighth of an inch all around; this space is filled with 650 lbs. (295 kilos) of mercury. The immersed part of the float gives a displacement of about 50 cubic feet (1.4 cu. m) of mercury, thus carrying 21½ tons (19,479 kilos) of the moving parts of the telescope in the fluid, and relieving 95 per cent. of the weight on the large bearings of the polar axis. The mounting is so designed that the center of weight of the moving parts is vertically above the center of flotation.

The large worm-gear for the diurnal rotation of the telescope is 10 feet (3.05 m) in diameter and has 1080 teeth. While being cut, these teeth were spaced with the utmost care with the aid of a 36-inch (91.44 cm) Warner and Swasey graduated circle of the finest quality. The teeth were then hobbed (with a hob of special design by which the accuracy cannot be lost) and were then ground with hone powder of finer and finer grades, with oil, and were finally polished with rouge and oil. This treatment not only eliminates any small irregularities of spacing, but leaves the teeth exquisitely smooth.

The driving-clock is in many respects a copy of the driving-clock of the 40-inch Yerkes refractor, built by Warner and Swasey. I have introduced one important modification, however, as follows: the clock-governor is driven by a weight through a spur-gear train, as usual, but the motion of the governor is communicated to the telescope through the medium of a worm and worm-gear, which are ground with the utmost care to eliminate periodic errors. This worm-gear of 80 teeth, together with the large worm-gear of 1080 teeth on the polar axis, gives the entire reduction from the clock-governor, rotating once in a second, to the polar axis, which rotates once in 24 hours. Furthermore, the two worm-gears named are the only gears used in communicating the rotation of the clock-governor to the polar axis, the driving-clock being so placed that all spur-gears and bevel-gears are dispensed with.

To relieve friction and wear on the polished teeth of the large worm-gear, and on the clock and clock-connections, the following simple expedient is used. A small wire-rope passes over a grooved wheel keyed to the polar axis, runs over two grooved pulleys on the west side of the telescope base, and is loaded with about 100 lbs. of iron weights hanging vertically on the west side of the pier. With this assistance, only about two pounds' pressure on the teeth of the 10-foot worm-gear is required to rotate the moving parts toward the west.

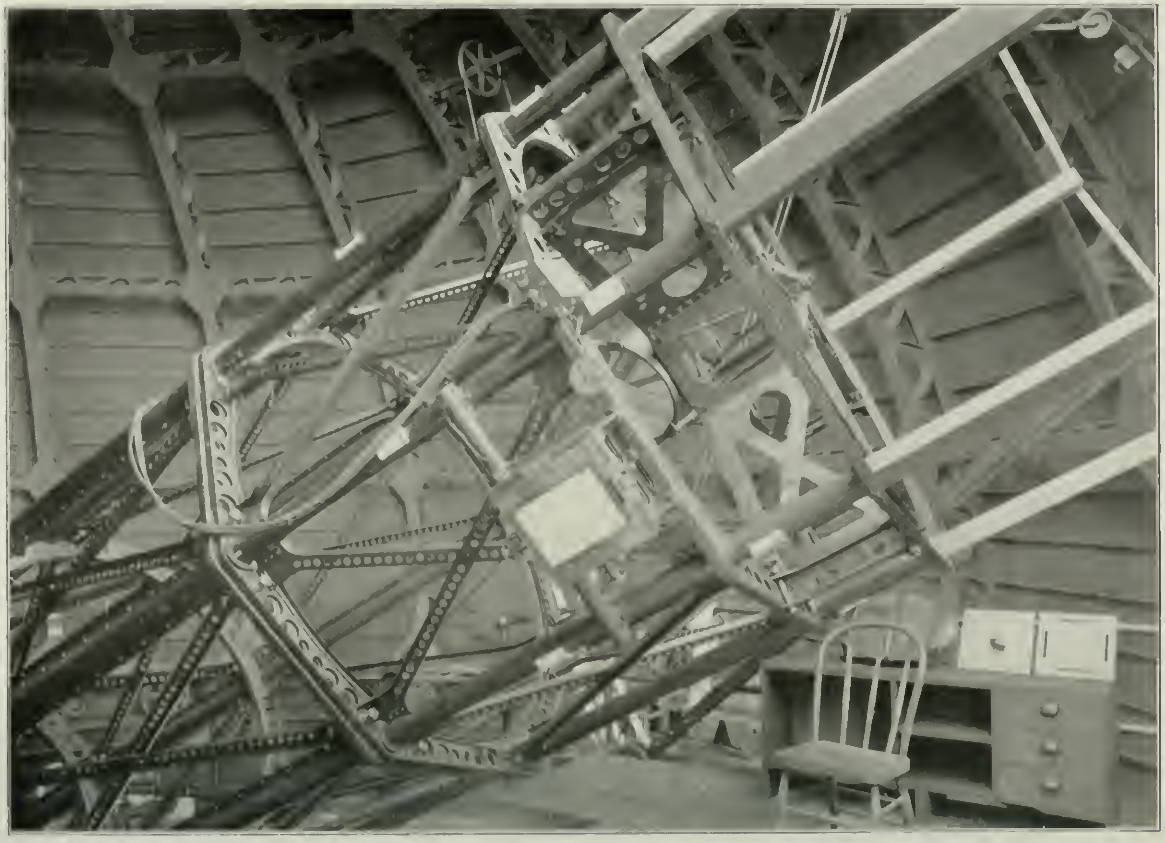
As a further means of keeping the teeth of the large worm-gear continually in the finest condition, a small motor is provided by means of which the gear and its worm can be repolished at any time. In practice this is done several times each week; the worm-shaft is disconnected from the driving-clock by simply removing two small screws, fine graphite and oil are supplied as a lubricant, and the large worm-gear and its worm are run together for an hour.

As a result of the mercury flotation and the care which has been given in finishing the driving-clock, the clock-connections, and the large worm-gear, the great telescope follows the stars with exquisite smoothness and accuracy, despite the fact that the moving parts weigh nearly 23 tons.

The telescope is provided with electric quick and slow motions. The former give a speed of 30 degrees per minute of time, in both right ascension and declination. The latter are arranged to give two speeds, one of six minutes of arc in a minute of time, for ordinary fine setting, and a slower one of one-half minute of arc in a minute of time for guiding with the spectrographs. The electric wiring is so arranged that the slow motions can be operated from several convenient points.

The octagonal skeleton tube is worthy of description because of its extraordinary rigidity. It consists of eight conical tubes of $\frac{1}{8}$ inch (3.2 mm) sheet steel, about 15 feet (4.6 m) long, tapering from 5 inches (12.7 cm) diameter at their lower ends to 3 inches (7.6 cm)

PLATE XVIII



Double-Slide Plate-Carrier in Principal Focus of 60-Inch Reflector diameter at their upper ends; each tube is made of two parts riveted together, with two flanges 135° apart extending their whole length; to these flanges are riveted the diagonal braces; three rigid rings connect the eight tubes together. Any one of four interchangeable extensions of the tube, called "cages," of similar construction, can be connected to the upper end of this permanent part of the skeleton tube; one of these carries the Newtonian plane mirror and its accessories; this is shown in Plate XVIII; each of the others carries one of the small hyperboloidal mirrors, with its cell, cell-support, and small electric motor for focusing. A simple and effective machine, called the "cage-lift," which is suspended from the framework of the dome, enables the observer or assistant to interchange the cages quickly and safely.

The Newtonian "cage" or tube-extension can be connected to the tube in four different positions 90° apart, that is, with the diagonal plane mirror and the double-slide plate-carrier facing either north, east, south, or west. The cage-lift is so designed that the tube-extension can be rotated to the position desired while suspended in it, before being attached to the tube.

The double-slide plate-carrier is most carefully designed, and is much more elaborate than those which I have made and used in the past. It is so planned that it will take either 5×7 inch (12.7×17.8 cm) or 6½×8½ inch (16.5×21.6 cm) photographic plates. This and other features of the design allow a very large range of movement of the guiding eyepiece, for choosing the most suitable guiding-star available—a matter of the utmost importance in using this attachment. Provision is made for altering the plane of the photographic plate when desired, during the exposure, without danger of relative rotation of the field and plate.

Mention should also be made of the lever-support system of the 60-inch mirror. The system fully described in my Smithsonian paper has been used without modification. The mirror is "floated" so that no flexure occurs sufficiently large to be detected by optical tests; and, in addition, the position of the mirror is defined so perfectly with reference to its tube that no wandering or jumping of the star-images, due to the slipping of the mirror in its cell, can be detected in the guiding eyepiece.

THE STEEL BUILDING AND DOME

The building which supports the dome is entirely of light steel construction. Twenty columns, each 22 feet (6.7 m) high, form the corners of the 20-sided equilateral polygon. These columns support twenty horizontal box-girders, which carry the double track upon which the dome revolves.

The building has two sheet-metal walls; the inner one is of $\frac{1}{16}$ inch galvanized sheet steel, and is planned to be air-tight; the outer wall, two feet distant from the other, is of light galvanized sheet steel, and serves merely as a sun-protection. A free circulation of air is allowed between the two; both are painted white. Sixteen sheet-metal windows, closing air-tight against heavy rubber packing, are easily accessible from the lower floor, and assist in ventilating the building and dome quickly when desired.

The ground floor of the building is of cement; on this floor are the dark rooms and the electric machinery for revolving the dome. Nineteen feet above this is the operating floor, of thin checkered steel plate supported by light steel columns and I-beams. From this floor are operated the dome-drive machinery, the dome-shutter, the wind-screen, the quick and slow motions of the telescope, the right ascension and declination clamps, etc.; from this floor also the right ascension and declination circles are read. On this floor are arranged the silvering carriage (which is necessary when removing the large mirror from the tube, and when silvering it) and the interchangeable ends of the tube which are not in use on the telescope. In this floor are twelve large trap-doors, $3 \times 7\frac{1}{2}$ feet (0.9×2.3 m) in size, which can be quickly opened to assist in ventilating the building and dome.

The dome is 58 feet (17.7 m) in diameter, and is of light steel construction with sheet-metal covering, coated inside with granulated cork, to prevent dripping from condensation of moisture, and painted white outside. It revolves on double tracks which are machined true, the dome wheels being all double, conical, and furnished with the best Hess-Bright ball-bearings. The dome moves with great smoothness. Two motors are used in turning it: one of three horse-power, which is so geared that the dome makes one complete revolution in six minutes; and a one-horse-power motor,

PLATE XIX

which drives a variable-speed machine, by which the speed of the dome can be changed at will (by simply turning a hand-wheel) to any point between one revolution in one hour and one revolution in twenty-five hours; this allows the observer, when working on the observing platform attached to the dome, to be moved with exactly the right speed to compensate for the horizontal component of the motion of the telescope.

The dome-shutter is extremely large, having a clear opening 16 feet (4.9 m) wide. Instead of opening horizontally in halves, the shutter runs back over the dome, as shown in Plate XIX; it is opened and closed by a six-horse-power electric motor. A large metal door, below the shutter opening, 17 feet (5.2 m) long by 8 feet (2.4 m) high, turning outward on hinges at its bottom, can be opened when it is necessary to observe objects near the horizon.

A light metal observing platform, 17 feet (5.2 m) long by 9 feet (2.7 m) wide, travels up and down the curve of the shutter opening, by means of a three-horse-power electric motor. This platform can be operated either from the operating floor or from the platform itself, by simply pushing a button; it is so designed that it automatically remains horizontal in all positions. In addition to its vertical movement, and its horizontal movement with the dome, the platform can be moved about 30 inches (76.2 cm) radially with respect to the vertical axis of the dome. This combination of motions enables the observer to reach the upper end of the tube, and to work with the telescope as a Newtonian, with the utmost convenience, in most positions of the telescope.

A wind-screen 17 feet (5.2 m) wide by 35 feet (10.7 m) long is provided, which can be quickly raised and lowered in the shutter opening by suitable mechanism; it is made of heavy black canvas supported on large steel tubes with rollers at their ends. This protects the telescope from wind, and also from lights in the valley.

As will be seen in Plate XIX, the exterior of the dome is covered with a strong frame-work of steel pipe. This will be covered during the spring, summer, and fall with gores of white canvas, laced on, at a distance of about two feet from the sheet-steel covering of the dome; provision is made for ample circulation of air beneath the canvas. This, together with the white outer wall of the building below, affords a complete sun-protection for the building and dome.

Furthermore, the entire building and dome are planned to close air-tight. For this purpose a frictionless water-seal is provided at the junction of the building with the dome; and all outside doors and windows close tightly against heavy rubber packing. The dome-shutter also is lined all around with air-tight cushions which can be pressed tightly in place by means of two levers operating a series of toggle-joints. In the early morning, after a night's work, the dome and building will be closed, not to be opened until after sunset, and thus a great volume of 120,000 cubic feet (3360 cu. m) of cool night air will be shut in air-tight. It is believed that this provision, together with the complete sun-protection of the dome and building, will reduce the rise of temperature within the structure during the day to a very few degrees. This protection from daily temperature changes should be sufficient for the telescope mounting and for the smaller mirrors. To further protect the large mirror during the day, a small refrigerating plant will be installed within a few months, which will supply constant-temperature air, at the expected night-temperature, circulating through a jacket inclosing the entire lower end of the telescope tube. The necessity for this protection of the large mirror from the daily rise and fall of temperature, to preserve the finest optical figure, was fully demonstrated by a long series of experiments in the optical shop with this mirror after its completion.

It will be seen also that a serious effort has been made in the design of the dome and building to eliminate the so-called dome- and building-effect, that is, the local effect upon atmospheric definition caused by heat-radiation from, or air currents in, the dome and building, which is often a most serious detriment to the successful performance of large telescopes. An hour after sunset the 16 ventilating windows near the lower floor of the building, the 12 trap-doors in the operating floor, and the great dome-shutter, the latter having an opening 16×45 feet (4.9×13.7 m) in size, are all opened, and the light metal columns, girders, and walls, inside and outside, quickly assume the temperature of the night air. The ventilating doors and windows are then closed, to prevent draughts. The very fine definition which we have already had on Mount Wilson with the 60-inch aperture, even on winter nights, indicates not only that the provisions just described are highly effective, but that the general atmospheric conditions at night on the mountain will prove sufficiently good for this and even larger apertures.

In this brief description of the new reflector and its accessories, I have called attention chiefly to those refinements and special features which, in designing the instrument, have appeared to me necessary. Mere bigness is no criterion of efficiency; if a great telescope is to yield a gain in results even approximately proportional to its increase in size, the utmost care must be given to meeting all those conditions which experience in the use of large telescopes has shown to militate against their successful performance. It was a most serious question whether it would be possible to give as fine a figure to the 60-inch mirror as was attained in the case of the 24-inch Yerkes mirror; the difficulties were of course incomparably greater, but the final figure of the 60-inch is decidedly better than that of the smaller mirror, and it is confidently expected that the temperature control will enable it to remain so while the telescope is in use. Similarly, it was a serious question whether the moving parts of the 60-inch telescope, weighing 23 tons (20,838 kilos), could be made to follow the stars as smoothly as those of the 24-inch, which weigh one ton (907 kilos); such smoothness of following is of course necessary for the finest results in photography; and it must be remembered that much greater smoothness and accuracy of motion are actually required in the large instrument, on account of its greater focal length and magnifying power. It is therefore a great satisfaction to see the star in the guiding eyepiece of the 60-inch remain perfectly bisected on the spider-lines for several minutes at a time, without perceptible tremor, and in addition, frequently to see the image of the guiding star itself, even with winter conditions, as small and sharp, and with its diffraction pattern as clearly cut, as I have ever seen in the 24-inch at Yerkes.

For the successful performance of the new reflector, special credit is due to Mr. Jacomini, foreman of the instrument shop of the Observatory; to Mr. Barnes, Mr. Schrock, and Mr. Kinney, for their great care and skill in the optical work, and to Mr. Dowd, for the installation of the complicated electrical work of the telescope mounting and dome.

I am indebted also to Director Hale for the opportunity afforded me to carry out in their entirety my plans for the great reflector and its dome, building, and accessories.

JANUARY 7, 1909

THE ULTRA-VIOLET ABSORPTION SPECTRA OF CERTAIN METALLIC VAPORS AND THEIR MIXTURES

By R. W. WOOD and D. V. GUTHRIE

It has been found by one of the present writers that the absorption spectrum of a substance in the state of a vapor may be modified by the presence of another gas or vapor, in such a way as to indicate that there is some interaction between the molecules. In the case of iodine vapor[1] mixed with the vapor of ether or bisulphide of carbon of a given density, if the iodine is not present in excess we have the absorption spectrum which is characteristic of iodine solutions, i. e., a broad band which obliterates the blue-green and violet region. If a little more iodine is added we get at once the absorption spectrum of gaseous iodine, a spectrum containing hundreds of fine lines in the orange and yellow region. In other words, a given quantity of ether vapor of a given density is able to dissolve a given quantity of iodine, any excess of the latter substance existing in the gaseous state, that is, with its molecules unaffected by the other vapor.

A phenomenon of a different nature has been observed in the case of mercury vapor,[2] which has a strong absorption band at wave-length λ 2536. If the vapor is evolved in a vacuum, this band broadens rapidly on the less refrangible side, attaining a width of three or four hundred Angström units. There is little or no broadening in the other direction. If air, hydrogen, helium, nitrogen, or some other inert gas is present, the band broadens symmetrically at first, and, after acquiring a certain width, continues to increase only in the direction of longer wave-lengths.

The marked unilateral broadening of the mercury band is in itself a point of interest, and has been recently discussed by Larmor,[3] who suggests that the unsymmetrical nature of the band may be due to the formation of loose molecular aggregates, which on account of mutual influence tend to vibrate in longer periods, and so give rise to the displaced part of the band. Another interesting case of this kind is described by Lyman,[1] who finds an absorption band for oxygen in the extreme ultra-violet, which exhibits markedly unsymmetrical broadening. He applies Larmor's explanation to this band, observing that the conditions demanded by the theory exist in the case of oxygen. Since absorption bands of this character possess much interest, it is important to find whether they exist in case of other metallic vapors besides that of mercury.

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[1] Wood, "Absorption Spectra of Solutions of Iodine and Bromine above the Critical Temperature," *Phil. Mag.*, **41**, 423, May 1896.

[2] Wood, *Astrophysical Journal*, **26**, 41, 1907.

[3] *Astrophysical Journal*, **26**, 120, 1907.

OBJECTS OF INVESTIGATION

The present investigation was undertaken with the following objects in view.

1. To obtain the absorption spectra of certain metallic vapors, particularly in the ultra-violet region of the spectrum, where they have not been studied heretofore.

2. To observe the manner in which the absorption bands of each metal behave with increasing vapor-density.

3. To determine whether the absorption bands are affected in any way by the presence of a foreign substance.

APPARATUS AND METHODS

A small quantity of the metal to be studied was placed in a quartz bulb of 10 or 15 cc capacity, which was then exhausted and sealed. Quartz furnishes a very efficient material for use in examining the absorption of metals which volatilize at a temperature only moderately high, for, in addition to its transparency to ultra-violet light, it does not soften in the hottest flame of the ordinary blast lamp and so can withstand great pressures of the inclosed vapor. Prolonged heating of the bulbs in the flame of the Bunsen burner causes them to become incrusted on the outer surface with a material which was quite opaque to the shorter wave-lengths, apparently due to a sort of devitrification of the quartz. This could be removed, however, and the transparency of the bulb restored, by heating it to the softening point for a short time in the oxy-hydrogen flame.

As a source of light, a spark discharge between cadmium electrodes was used, a Leyden jar or a condenser being placed in parallel with the secondary circuit. This source furnishes a strong and fairly uniform continuous spectrum in addition to the scattered lines.

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[1] *Astrophysical Journal*, 27, 87, 1908.

Whenever practicable the bulb was placed in a small air bath, formed of a piece of iron tubing not much greater in diameter than the bulb, with small holes cut in the sides to permit the light to pass through it. To determine the temperature in the air bath, a platinum-rhodium thermo-couple connected with a galvanometer reading the temperature directly up to 1400° was used. The highest temperature that could be secured in the air bath, heating it with two large burners with strong air blast, was about 700°, so when higher temperatures were necessary the bulb was heated directly in the flame of the burner or blast lamp.

In part of the work a quartz lens was used to bring the light from the spark, after passing through the quartz bulb, to a focus on the slit of the spectrograph, but usually it was found advisable to pass the light of the spark directly through the bulb into the instrument, the spark being distant 12 or 15 cm from the slit, and the quartz bulb about half-way between the two. The instrument used to photograph the spectra was a small quartz spectrograph. A device attached to the plate-holder enabled it to be raised by successive steps, so that the spectra could be photographed in succession as the temperature was raised and the vapor-density increased. The plates used were Seed's 26. The range of the spectrum which could be investigated extended from about λ 5200 to λ 2150.

OBSERVATIONS AND RESULTS

1. *Cadmium.*—Two absorption lines were found for cadmium, corresponding to the emission lines λ 2288.1 and λ 3261.2, the former being much the more prominent. As the vapor in the bulb increases in density the emission line of the spark λ 2288 appears bisected by a fine dark line, the widening of which finally obliterates the bright line entirely (Fig. 1). The dark band eventually attains a width of nearly 200 Å. U. The other absorption line makes its appearance in a different manner. There are two bright lines in the *Cd* spark spectrum at wave-lengths 3251 and 3261. As the vapor in the bulb increases in density the latter bright line gradually fades away, disappearing entirely at one stage. Its place is then taken by a dark line, which, however, never becomes very broad with the vapor-densities used in the present work. This apparent difference in the manner in which

the absorption line makes its appearance is, without doubt, to be attributed to the smaller dispersion of the quartz spectrograph for this region of the spectrum.

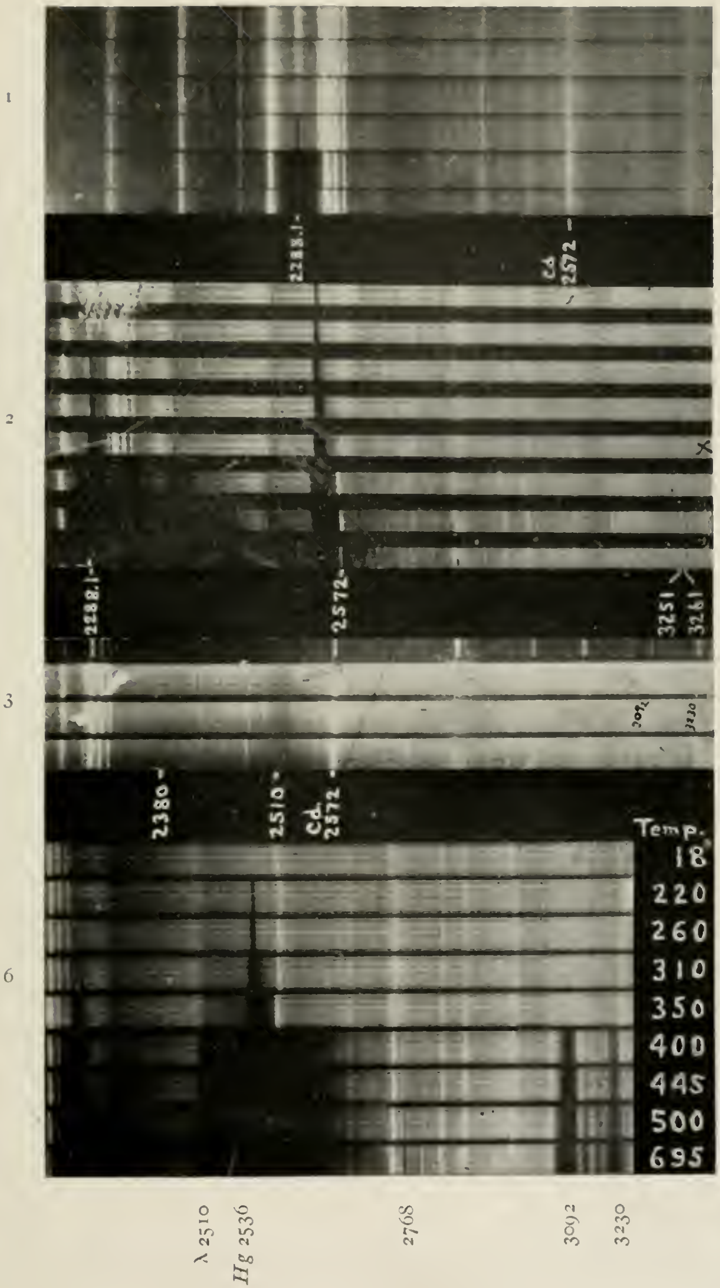
The absorption is relatively weaker than that of the line λ 2288, for the bright line λ 3261 is not obliterated until the λ 2288 line has attained the width shown in the lower two spectra of Fig. 1, Plate XX. The line λ 3261 is not shown in this figure, but it appears in Fig. 2. These two lines are the only ones in the cadmium spectrum which show any trace of absorption, though vapor of such density has been used that the line at λ 2288 broadened nearly to the cadmium line at λ 2572.

The manner of broadening of the absorption bands with increasing vapor-density was then studied. It was found that in a vacuum, no foreign substance being present with the cadmium, the band λ 2288 broadens perfectly symmetrically, the broadening amounting to considerably more than 100 Å. U. on each side at the highest temperatures employed. It was found, however, in working with zinc in which a little cadmium was present as an impurity, that when mercury was added, this absorption band appeared to broaden only on the side toward the longer wave-lengths.

In order to determine whether this was really an effect of the mercury upon the cadmium, a bulb was prepared containing cadmium and a little mercury. In this case there was a marked unsymmetrical broadening of the absorption band at λ 2288 (Fig. 2). The band broadens a little on the more refrangible side, especially at the higher temperatures, but very much more on the less refrangible side. The emission line *reverses on the less refrangible side* first, instead of in the center as before, and the absorption band has extended almost as far as the emission line λ 2307 by the time the more refrangible edge of the emission line λ 2288 has entirely disappeared (Fig. 2). Mercury vapor, even when very dense, shows no absorption band at this point, though there is a fairly strong band at λ 2338 and another very broad one at λ 2140. Measurements on one of the negatives in which the band had attained a considerable width gave the following result:

More refrangible limit.	λ 2415.1
Less refrangible limit.	λ 2270.6
Mean.	λ 2342.9

PLATE XX


1
2
3
6
2288.1
Cd 2572
2288.1
2572
3251
3261
3092
3230
2380
2510
Cd 2572
Temp.
18°
220
260
310
350
400
445
500
695
λ 2510
Hg 2536
2768
3092
3230

showing that an apparent shift of the mean wave-length of the band from the position of the emission line (λ 2288.1) amounting to about 55 Å. U. has taken place. Similar measurements on negatives representing the absorption of pure cadmium showed that the distance of each limit of absorption from the emission line was the same, to within the limit of error of observation. The shift is of course only apparent, for the densest or heaviest part of the band remains in the position of the emission line.

This effect is all the more remarkable in that it forms a contrast to the behavior of the mercury band at λ 2536, which broadens unsymmetrically in vacuo, but symmetrically when a foreign gas is present. However, this effect, as clearly as that, affords an instance of a modification in the appearance and position of an absorption band of one substance resulting from the direct influence of a foreign substance. The absorption line at λ 3261 does not appear to be affected, showing no signs of asymmetry in either case. Its width, however, at the temperature available was too slight to enable any very definite conclusion to be drawn as to the effect of the mercury vapor upon it. It can be seen in the two lower spectra of Fig. 2. The gradual disappearance of the bright line does not show well in the enlargement. The spectrum in which it has just disappeared is marked with an x.

2. *Thallium.*—The absorption of thallium vapor was next investigated. It was found that when pure thallium was sealed up in an exhausted bulb and its absorption examined, three or four faint absorption bands were to be seen, appearing at about 400° and not becoming strong even at the highest temperatures. When now a little mercury was added to the thallium, and its absorption studied under the same conditions, a number of other absorption lines were brought out which did not appear at the highest temperatures with the pure thallium, while of the absorption bands obtained with the thallium alone, all except one *disappeared.* Thus the appearance of the absorption spectrum is entirely changed by the addition of the mercury. Further, it was found that the relative intensities of the absorption bands and the temperatures at which they first appear vary in a remarkable manner with the relative proportions of mercury and thallium.

The spark spectrum of thallium was photographed and most of the absorption lines were found to correspond to emission lines, though there were a few which did not appear to coincide with any line in the spark spectrum. The absorption bands obtained with the pure thallium were (see Fig. 3):

a) A fairly sharp but faint absorption line corresponding to the emission line λ 2380. This is the only absorption line which appears under all conditions, both with and without the addition of mercury. In the present case, with pure thallium, it never became strong, even at the higher temperatures.

b) A broad and faint absorption band lying very near the emission line λ 2530.0 on the side of shorter wave-lengths. Its mean wave-length was about 2510.

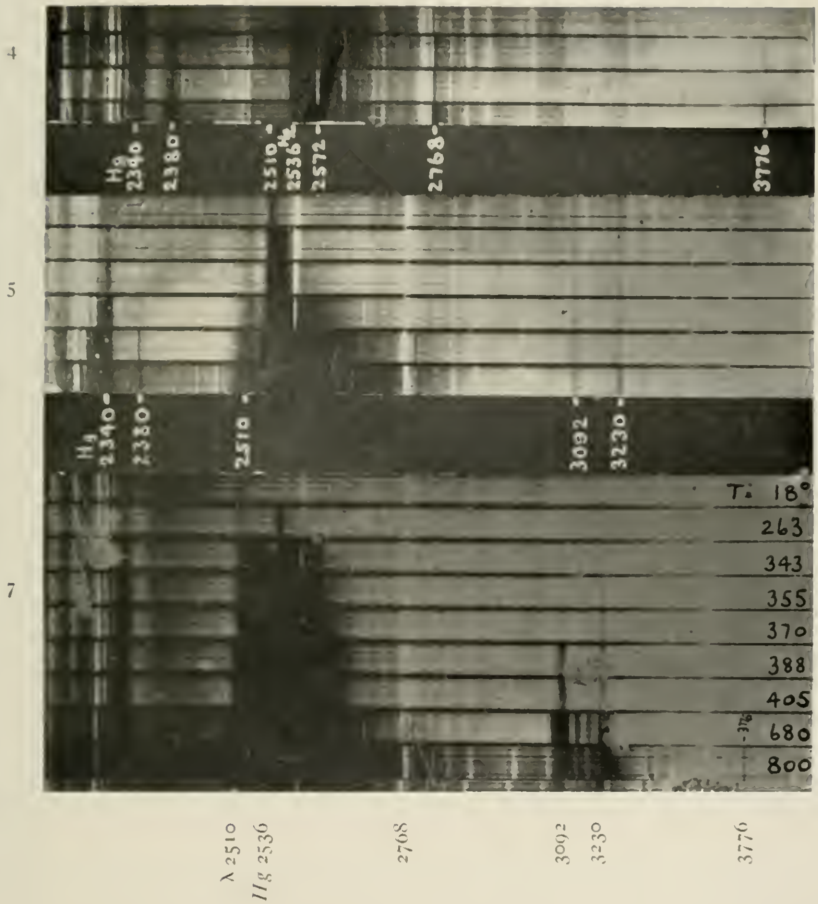
c) A band resembling the last but even fainter, and bordering on the emission line at λ 3092 but lying on the less refrangible side of this line.

d) A group of three lines, faint and not very sharp, lying very close together and uniformly spaced. The middle component coincides with the emission line at λ 3230.

Though these bands appear at a comparatively low temperature (about 400°) they change very little either in width or intensity as the temperature is further increased, a circumstance which suggests that they may be due to some easily volatilized impurity.

Some preliminary experiments with bulbs containing different proportions of thallium and mercury showed that with these mixtures absorption could be obtained in the case of several other lines, appearing at different temperatures in the different cases. In order to study the phenomenon quantitatively, a series of bulbs was examined containing definite amounts of thallium and mercury. First, 150 mg of thallium was weighed out and inserted in a bulb of about 10 cc capacity. Then, the amount of thallium being kept constant, six different amounts of mercury varying from 6 to 150 mg were tried with it. In the first case, although the amount of mercury was very small, the mercury absorption band at λ 2536 remaining quite narrow at the highest temperatures, this small quantity of mercury was sufficient to affect the absorption very markedly, bringing out very clearly absorption lines corresponding to the emission lines at λ 2580, λ 2768, and λ 3776 (Fig. 4, Plate XXI), and causing all the absorption bands found with the pure thallium except the one at λ 2380 to disappear.

PLATE XXI


4
5
7
Hg 2340
2380
2510
2536 Hg
2572
2768
3776
Hg 2340
2380
2510
3092
3230
T: 18°
263
343
355
370
388
405
680
800
3776
λ 2510
Hg 2536
2768
3092
3230
3776

Absorption in the case of the line λ 2580 can be seen only when the amount of mercury in the bulb is small, for owing to the proximity to the mercury band λ 2536 it is hidden by the broadening of this band unless the band remains quite narrow. Previous experiments made by one of us have shown that the mercury band at λ 2536 even with very dense vapor expands only a few Ångström units on the short wave-length side, unless air or some other gas is present in which case it may extend almost as far down as the thallium band at λ 2510. With thallium and mercury, it apparently extends much farther down the spectrum; this change is only apparent, however, for we are dealing with two bands which fuse together when the vapor mixture is dense. The thallium band at λ 2510 broadens symmetrically, while the mercury band at λ 2536 broadens only on the side of longer wavelengths. There are also indications, at high temperatures, of absorption corresponding to the line λ 2237.9 in one or two instances, when the amount of mercury present is small, this part of the spectrum being also obliterated in other cases by the strong absorption of mercury in the extreme ultra-violet. As the amount of mercury was increased, up to a certain point the absorption lines appeared at successively lower temperatures. Thus for the two cases where the amounts of mercury were 6 mg and 25 mg, the ratios of the amount of mercury to the amount of thallium being 1:25 and 1:2, the temperatures at which the absorption lines could be first detected were as follows:

Ratio *Hg* to *Tl*	λ 2380	λ 2580	λ 2768	λ 3776
1:25	600°	900° (est.)	600°	900° (est.)
1:2	400	700	400	500

The temperatures given are those of the air bath as indicated by the thermo-couple, except those over 700°, when the bulb was heated in the flame and the temperature estimated. For intermediate amounts of mercury the temperatures at which absorption appeared lay between these limits. As the proportion of mercury is still further increased beyond that in the second case above, the absorption lines do not appear at temperatures any lower than these.

This effect of mercury in bringing out absorption lines of thallium, which do not appear when pure thallium is examined under the same conditions, appears to be quite analogous to some observations of Liveing and Dewar (*Proc. R. S.*, 27, 132, 350, 1878), who found that the red line of lithium, λ 6103, could be obtained reversed only when both sodium and potassium were present. They found also, in studying the absorption of magnesium, that when sodium was present, a line at λ 5300 could be seen reversed and when potassium was present, two lines, at λ 6580 and λ 6475, these absorption lines being obtainable only with the mixtures. They suggest that

> a very slightly volatile vapor may be diffused in an atmosphere of a more volatile metal, so as to secure a sufficient depth of vapor to produce a sensible absorption. This would be analogous to well-known actions which take place in the attempt to separate organic bodies of very different boiling points by distillation, where a substance of high boiling point is always carried over, in considerable quantity, with the vapor of a body boiling at a much lower temperature.

It seems quite probable that something of this kind occurs in the present case.

It has been mentioned that in the series of experiments just described in which 150 mg of thallium was used in the bulb and the amount of mercury varied, all of the absorption bands found with the pure thallium except λ 2380 disappear. The amount of mercury was now kept constant (about 150 mg) and the amount of thallium varied. The surprising result was obtained that as the amount of thallium in the bulb is increased above the amount used before, these absorption bands, which had disappeared in the former case, reappear, and as the amount of thallium is still further increased, they finally become very much stronger than with the pure thallium (Fig. 5 shows this effect). The most remarkable result was obtained with a bulb containing a large amount of thallium (Figs. 6 and 7). The bands just mentioned appear in this case at between 300° and 350°, being faint at first but rapidly becoming stronger, and finally at 400° and higher becoming very broad and intense, being almost as prominent as the mercury bands. The temperatures are recorded in the figures.

The circumstance which strikes one as most remarkable is that when there is but a small amount of thallium present (Fig. 4) the absorption lines at λ 2768 and 3776 are very conspicuous, while there is no trace of the strong bands λ 3092 and 3230. With a large amount of thallium we have the latter bands very conspicuous and little or no trace of the former (Figs. 5, 6, and 7). This is especially noticeable in Fig. 7, in which the line at λ 3776 only appears in the lowest spectrum obtained at a temperature of about 800°. Whether this is a matter of the solution of thallium vapor in the vapor of mercury is a question. At the temperatures used it must be remembered that only a very minute proportion of the total amount of thallium present is vaporized in any case.

In addition to the bands which appeared in former cases, there are, in this case (Figs. 6 and 7), two strong bands lying between the bands at λ 3092 and the triplet at λ 3230, of which there were perhaps faint traces with the pure thallium, and also a number of fine lines lying near the band at λ 3230 on the less refrangible side, which are not represented on any of the former negatives. The wave-lengths of these absorption bands were measured. It was not possible to determine them very accurately, but the values given are correct to within a few tenths of an Ångström unit. For the two wider bands the limits of absorption were measured, while for the finer ones a setting was made on the middle of each, the average width being about 3 Å. U. The temperature in this case was 700°. The values obtained are as follows:

Limits of Absorption	Mean
3145·9 3166.6	3156·3
3179·1 3197·9	3188·5

Mean Wave-Length (width about 3.0)	
3267·9	3308·4
3279·7	3320·5
3289·4	3337·6
3297·4	3350·1

None of these absorption bands corresponds to any emission line. The triple line at λ 3230 appears shortly before the other bands, the most refrangible component being the strongest at first, though this is not the one that coincides with an emission line. The three lines merge into one and appear as a strong band at about 700°. The positions of the edges of these lines and of the band at λ 3092 were also measured at different temperatures, the values obtained being as follows:

	Limits of Absorption	Mean
350°	3220.2 3224.0	3222.1
	3228.4 3231.5	3229.9
	3236.8 3239.8	3238.3
400°	3220.2 3225.7	3223.0
	3228.4 3233.4	3230.9
	3236.8 3241.6	3239.2
700°*	3218.2 3262.1	3240.2

* The three merged into one.

It will be observed that when the lines first appear, the middle component of the triplet coincides with the emission line at λ 3229.9, and that with increasing temperature they broaden at first only on the less refrangible side, though after merging into a wide band a slight extension toward the side of shorter wave-lengths is perceptible. For the band near λ 3092 the following values were obtained:

	Limits of Absorption	Mean
400°	3091.9 3126.3	3109.1
700°	3081.3 3134.6	3108.0

It is to be noticed that the band when it first appears just touches the position of the emission line at λ 3091.9 on its more refrangible edge, broadening, however, so as to cover it, the broadening being probably symmetrical.

The manner of broadening of the thallium absorption bands with increasing vapor-density was also investigated. Difficulty was experienced in getting the absorption bands to broaden sufficiently to draw definite conclusions as to their behavior, but finally on heating one of the bulbs containing thallium and mercury in the hottest flame of the blast lamp, some of the bands brought out by the mercury attained a width of 15 or 20 Angström units. The broadening of the bands λ 2768 and λ 3776 was found to be quite unsymmetrical, the extension on the side of longer wave-lengths being very much greater than on the opposite side, like that of the mercury band λ 2536, though not so marked as in that case. The appearance of these bands as they broaden also resembles that of the mercury band, as the limit of absorption remains very sharply defined on the more refrangible edge, but becomes very hazy and ill defined on the other side, on which the extension mainly occurs. This can be seen in Fig. 4. The line λ 2380 appears to broaden symmetrically. As already mentioned in a former part of the work, the lines at λ 3230 broaden toward the less refrangible side, while the wide band near λ 3092 probably broadens symmetrically.

The following measurements of the wave-lengths of the limits of absorption for the bands λ 2768 and λ 3776 clearly indicate the unsymmetrical character of the broading for these wave-lengths. The observations are taken at three successive temperatures, the lowest being well above 700° and the highest in the hottest blast-lamp flame. They are estimated as 800°, 900°, and 1000°.

Temperature	Edges of Band	Mean	Emission Line at
800°	2767.2 2769.1	2768.2	2768.1
900°	2766.5 2777·3	2769.9	
1000°	2765·4 2779.2	2772·3	
800°	Just visible	3775·9	3775·9
900°	3773·6 3779·3	3776.5	
1000°	3772·4 3786.3	3779·4	

As the unsymmetrical broadening of the cadmium band λ 2288 is caused only by the addition of mercury, it would be interesting to observe the behavior of these thallium bands with the pure thallium, but at the temperatures which could be attained in the present work it was not possible to obtain these absorption bands except when mercury was added to the thallium.

It was thought possible that cadmium, which volatilizes at a much lower temperature than thallium, might have the same effect as mercury in bringing out thallium absorption bands, and so a mixture of thallium and cadmium was examined for absorption. The absorption of neither substance was modified by the presence of the other, however, the thallium absorption being the same as with pure thallium and the cadmium band λ 2288 broadening symmetrically and not showing the peculiar unsymmetrical effect caused by the addition of mercury.

Observations were also made to ascertain whether the addition of mercury to sodium has any effect upon the temperature at which the D lines appear in the absorption spectrum. For this purpose pure sodium and several mixtures of sodium and mercury in different proportions were sealed up in exhausted glass bulbs and the light from a carbon arc, after passing through the bulb contained in an air bath of which the temperature could be observed, was received on the slit of a spectroscope. No effect was observed, however, the temperature at which the D lines first appeared as absorption lines being practically the same in all cases.

3. *Other metals.*—The absorption of zinc was examined both with and without the addition of mercury. In no case did any trace of absorption appear in the region of the spectrum investigated, although zinc volatilizes at a much lower temperature than thallium. It seemed probable that results might be obtained with magnesium, which also volatilizes at a comparatively low temperature, and has several lines which are often seen reversed, but when heated the magnesium attacked the surface of the quartz, rendering it opaque, and making it impossible to work with this metal. The absorption of magnesium vapor was then investigated by passing the light of the cadmium spark through the vapor contained in a long porcelain tube, the ends of which were closed by quartz plates, which was heated by means of a Heraeus electrical furnace. Rather strong absorption was found corresponding to the emission line λ 2852.2, but nowhere else in the spectrum. In particular, visual observations revealed no trace of absorption for the lines of the *b* group, even at the highest

temperatures, which were probably well above 1000°. The influence of mercury was investigated, but its presence was found to have no effect on the absorption of the magnesium vapor.

The absorption of tin, of lead, and of bismuth was also investigated. As these metals volatilize at a higher temperature, though still lower than thallium, a little mercury was added in each case with the idea that it might help on absorption, as with thallium. No absorption was observed, however, with any of these metals.

SUMMARY OF RESULTS

The results of the foregoing investigation may be summarized as follows:

1. The absorption of cadmium vapor in the ultra-violet region of the spectrum was investigated and two absorption bands were found, one corresponding to the emission line at λ 2288, the other to λ 3261. Of these the former is much the more prominent.

2. The cadmium absorption band λ 2288 was found to broaden perfectly symmetrically when the pure cadmium was examined, but when mercury was added showed a marked asymmetry, broadening much more on the less refrangible than on the opposite side.

3. The absorption of thallium vapor was investigated, and with the pure thallium four absorption bands were found, two corresponding to the emission lines at λ 2380 and λ 3230 respectively, one lying near λ 2530, on the more refrangible side, and one lying on the less refrangible side of λ 3092.

4. With a moderate amount of thallium in the bulb all these bands (except λ 2380) disappear when mercury is added, but as the quantity of thallium present is increased they reappear, and finally become much stronger than with the pure thallium. On adding mercury other bands also appear which are not found with pure thallium. Among these are bands corresponding to emission lines at λ 2580, λ 2768, and λ 3776.

5. The behavior of these bands with increasing vapor-density was studied, and some of the bands, notably λ 2768 and λ 3776, showed a marked asymmetry, broadening chiefly on the less refrangible side.

JOHNS HOPKINS UNIVERSITY
1908

ELEVEN STARS HAVING VARIABLE RADIAL VELOCITIES

BY W. W. CAMPBELL

In the course of the regular observing programme with the Mills Spectrograph the following stars have been found to have variable radial velocities. The types of spectra are taken from the Harvard College Observatory *Annals*.

γ *Persei* ($\alpha = 2^h\ 57^m.6;\ \ \delta = +53^\circ\ 7'$)

Date	Velocity	Measured by
1897, January 19	−0.9 km	Reese
1898, September 16	+0.5	Reese
1898, November 14	−1.1	Reese
1899, July 5	−0.3	Reese
1901, December 4	−2.5	Reese
1904, October 25	+5.5	Miss Hobe
1904, October 31	+7	Moore
1904, November 23	+5	Moore
1906, September 19	+0.9	Moore
1908, October 31	−0.7	Moore
1908, November 2	+0.1	Moore

The type of spectrum is G. This star is on the Harvard list of stars showing composite spectra. The variable velocity was detected by Mr. Moore. Its period is probably long.

ξ *Tauri* ($\alpha = 3^h\ 21^m.8;\ \ \delta = +9^\circ\ 23'$)

Date	Velocity	Measured by	
1903, November 24	−38 km −39	Campbell Moore	λ 4481 only 4481
1903, December 7	+2 +1	Curtis Moore	4481 4481
1904, January 26	−59± −65±	Campbell Moore	4481 small weight 4481 small weight
1904, October 18	+34 +37	Campbell Moore	4481 4481
1908, November 11	−4 −1	Campbell Moore	4481 and λ 4549 4481 only

The type of spectrum is A, with a very broad $H\eta$, and poor lines at λ 4481 and λ 4549. The appearance of λ 4481 seems to vary.

The variable velocity was detected by Mr. Campbell.

θ^2 *Tauri* ($\alpha = 4^{\mathrm{h}}\,22^{\mathrm{m}}9$; $\delta = +15^\circ\,39'$)

Date	Velocity	Measured by
1903, December 1	+39 km	Curtis
	+37	Moore
1905, January 3	+50±	Moore
1906, October 4	+74	Moore
1908, September 8	+80	Moore
1908, October 21	+17	Moore
1908, October 25	+23	Moore

The spectrum is of the A type. The lines λ 4481 and λ 4549 are broad but measurable. The variability was discovered by Mr. Moore. Its period is probably short.

l (53) *Eridani* ($\alpha = 4^{\mathrm{h}}\,33^{\mathrm{m}}6$; $\delta = -14^\circ\,30'$)

Date	Velocity	Measured by
1900, September 25	+36 km	Wright
	+34.1	Burns
1900, October 16	+35	Wright
	+33.1	Burns
1900, October 23	+35	Campbell
	+34.4	Burns
1904, December 6	+41	Moore
	+41.7	Newkirk
1907, October 30	+43	Moore
	+43.5	Miss Allen
1908, October 13	+42	Moore

This star's spectrum is type I. The variability was discovered by Mr. Moore. Its period is probably long.

ζ *Aurigae* ($\alpha = 4^{\mathrm{h}}\,55^{\mathrm{m}}5$; $\delta = +40^\circ\,56'$)

Date	Velocity	Measured by
1898, December 4	+4 km	Campbell
	+0.1	Burns
1899, December 3	+6	Campbell
	+5.0	Burns
1903, December 29	+3	Curtiss
	+2	Moore
1906, September 30	+3	Moore
	+3.4	Newkirk
1907, October 10	+4	Moore
1908, September 27	+25	Wright
1908, October 26	+20	Wright

This is one of the stars described by Miss Maury as having a composite spectrum.

The variation was detected by Mr. Wright on measuring the sixth plate.

ρ Orionis ($\alpha = 5^h 8^m.1$; $\delta = +2^\circ 45'$)

Date	Velocity	Measured by
1902, October 14	+34 km +37.5	Curtiss Burns
1905, September 25	+39 +40.5	Moore Newkirk
1906, September 17	+34 +33.5	Moore Newkirk
1907, November 7	+50 +49.4	Moore Miss Allen
1908, October 27	+36 +35.0	Moore Miss Hobe
1908, November 2	+33	Moore

The spectrum is type I. The variability was detected by Mr. Moore.

β Canis Majoris ($\alpha = 6^h 18^m.3$; $\delta = -17^\circ 55'$)

Date	Velocity	Measured by	
1904, January 13	+39.8 km	Albrecht	Poor focus
February 1	+30.9	Curtiss	
1907, November 1	+39.9	Albrecht	
December 17	+28.3	Plummer	
1908, January 11	+28.2	Albrecht	
February 24	+33.0	Albrecht	
September 11	+31.9	Albrecht	
October 5	+40.2	Albrecht	
October 9	+40.0	Albrecht	
October 13	+37.4	Albrecht	
October 26	+35	Albrecht	
November 6	+36.5	Albrecht	
November 10	+27.0	Albrecht	
November 19	+40.9	Albrecht	
November 28	+28.7	Albrecht	
November 29	+27.0	Albrecht	
December 5	+34.1	Albrecht	

The type of spectrum is B. The variation was discovered by Mr. Albrecht. The period is short.

ν Draconis $(\alpha=17^{\rm h}\,30^{\rm m}.3;\ \delta=+55^\circ\,15')$

Date	Velocity	Measured by
1902, June 22	−13.6 km	Burns, Underexposed
1908, August 6	− 4.0	Albrecht, Underexposed
August 31	−23.2	Albrecht, Underexposed
November 2	−21.1	Albrecht
November 17	−21.5	Albrecht
	−21.4	Miss Allen

The type of spectrum is *A*. The variability was detected by Mr. Albrecht.

70 Ophiuchi $(\alpha=18^{\rm h}\,0^{\rm m}.4;\ \delta=+2^\circ\,32')$

G. M. T. of Observation	Observed Radial Velocities	Plate Measured by	G. M. T. of Observation 1895.00+	Observed Radial Velocities	Computed Radial Velocities
1897, May 11.95	−10.68 km	Burns	$2^{\rm y}.360$	−10.68 km	−10.73 km
1898, May 4.97	−10.18	Burns	3.423	−10.00	−10.33
July 4.84	− 9.82	Burns			
1901, April 21.99	− 9.26	Burns	6.330	− 9.26	− 9.10
1902, May 26.90	− 9.95	Burns	7.397	− 9.95	− 8.69
1905, June 27.88	− 7.66	Newkirk	10.486	− 7.66	− 7.78
1908, April 26.99	− 6.73	Plummer	13.447	− 7.20	− 7.21
April 26.99	− 7.62	Miss Allen			
July 30.80	− 7.21	Miss Allen			

This is a well-known double star, with components of 4.5 and 6.0 magnitudes, and revolution period of 88 years. The spectrum of the bright component is of K type. Its radial velocities have been measured as in the first and second columns of the above table. Combining the two observations of 1898 into one mean result, and the two of 1908 similarly, and expressing the times in mean solar years following the beginning of the year of periastron passage in the system, we have the fourth and fifth columns. Assuming the parallax of the star to be $0''.24$, the value of the angle of inclination in the visual orbit to be $-38^\circ.57$,[1] the relative masses of the primary and secondary to be as 3 to 2, and the radial velocity of the center of mass of the system to be −7.4 km, the computed values[2] of the radial velocities of the primary are as in the sixth column. The fair agreement of the observed and computed velocities may be held to prove that the angle of inclination of the orbit plane is negative; but a considerably longer arc of the ellipse must be described before the speed of the center of the mass can be determined accurately; and the speed of the secondary is needed for a computation of the value of the parallax.

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[1] Lohse's Elements, III, has been used (*Publ. des Astroph. Obs. zu Potsdam*, **20**, 143, 1908).

[2] The method is described in *L. O. Bulletin*, **3**, 81, 1905. The velocity equation used is $v=+3^{\rm k}.077-6^{\rm k}.340\cos(v+w)$, v being the velocity of the secondary with reference to the primary.

111 Herculis $(\alpha = 18^h\,42^m.6;\ \delta = +18^\circ\,4')$

Date	Velocity	Measured by
1902, June 15	−63 km	Burns
	−64	Wright
1905, June 21	−36	Wright
1908, August 4	−54	Wright
1908, October 28	−45	Wright
1908, November 4	−43	Wright
1908, November 15	−26	Wright

This star has a spectrum approaching the *Orion* type, but contains some additional lines. The above observations, which are to be considered as approximate, depend upon measurements of but one line, λ 4481.

The variable speed was detected by Mr. Wright.

φ Cygni $(\alpha = 19^h\,35^m.5;\ \delta = +29^\circ\,56')$

Date	Velocity	Measured by
1907, August 29	−24 km	Moore
	−24 and +30 km	Moore
1908, July 13	+ 3	Plummer
July 24	+ 2	Moore
November 16	0	Moore

The spectrum is classified as type I in the *Draper Catalogue*. The velocity was suspected by Mr. Plummer to be variable from his measure of the second plate. On examining the first plate, recently, Mr. Campbell noticed that two sets of spectrum lines are visible in nearly equal prominence. This star is thus of the class of binaries in which the spectra of both components are recorded.

LICK OBSERVATORY
December 1908

FIVE STARS HAVING VARIABLE RADIAL VELOCITIES

BY HEBER D. CURTIS

The following stars have recently been shown to be spectroscopically double, in the course of the work of the D. O. Mills Expedition to the Southern Hemisphere.

ζ *Canis Majoris* ($\alpha = 6^h\ 16^m5$; $\delta = -30° 2'$)

Date	Velocity	Measured by
1906 Nov. 7	+35 km	Paddock
	+31	Curtis
Dec. 17	+38	Paddock
	+35	Curtis
1907 Oct. 4	+11	Paddock
	+12	Curtis
Oct. 19	+19	Paddock
	+21	Curtis
Oct. 30	+22	Paddock
Nov. 7	+20	Paddock
Nov. 8	+21	Paddock
Nov. 12	+17	Paddock
Nov. 23	+26	Paddock
	+25	Curtis
Nov. 26	+29	Paddock
	+29	Curtis
Dec. 2	+32	Paddock
Dec. 4	+27	Paddock

This star is of type B3A and five lines of fair quality are available for measurement. The binary character of this star was discovered by Mr. Paddock from measures of the third plate. The period is probably long.

τ *Puppis* ($\alpha = 6^h\ 47^m4$; $\delta = -50° 30'$)

Date	Velocity	Measured by
1903 Nov. 15	+39 km	Palmer
1904 Oct. 12	+34.4	Palmer
Dec. 9	+33.0	Palmer
1905 Jan. 24	+33.1	Palmer
Nov. 10	+34	Wright
1907 Jan. 15	+39.7	Curtis
Jan. 19	+40.3	Curtis
Oct. 4	+36.6	Curtis
Oct. 13	+37.2	Curtis
Nov. 23	+34.0	Curtis

This star is of type K with very good lines. The variable radial velocity was discovered by Mr. Curtis from definitive measures of the sixth plate and independently discovered by Mr. Palmer from measures made at Mount Hamilton of the first four plates taken. Though the variation is small, the character of the lines is such as to leave no doubt of its reality. The period is long.

o Velorum ($\alpha = 8^{\mathrm{h}}\ 37^{\mathrm{m}}.4$; $\delta = -52^{\circ}\ 34'$)

Date	Velocity	Measured by
1904 Feb. 4.657	+14 km	Palmer
	+14.3	Wright
Nov. 15.833	+11.1	Albrecht
1905 Jan. 31.628	+10.0	Palmer
Apr. 13.574	+11.0	Palmer
1906 Dec. 29.826	+16.0	Paddock
1907 Mar. 1.644	+19.8	Paddock
Apr. 26.510	+19.4	Paddock
Oct. 14.885	+18.6	Paddock
Nov. 1.859	+17.4	Paddock
	+16.7	Curtis
1908 Jan. 13.770	+27.0	Curtis
Feb. 10.636	+24.7	Curtis (II Prism)

This star is of type B3A; it has available six lines of *C*, *H*, *He*, and *Mg* of quite fair quality. The variation in the velocity was discovered by Mr. Curtis. The period is probably long.

d Carinae ($\alpha = 8^{\mathrm{h}}\ 38^{\mathrm{m}}.5$; $\delta = -59^{\circ}\ 24'$; Mag. = 4.4)

Date	Velocity	Measured by
1907 Nov. 7	+72 km	Paddock
Nov. 14	+62	Paddock
Nov. 23	+53	Curtis
Nov. 25	+46	Curtis
Nov. 26	+48	Curtis
Dec. 2	+38	Curtis

This star is of type B2A and has available six lines due to *C*, *H*, *He*, and *Mg*, besides a few other lines too faint to use. The variation was discovered by Mr. Paddock from the measure of the second plate. The period is probably of some length.

q Velorum ($\alpha = 10^h\ 10^m.6$; $\delta = -41° 37'$; Mag. 4.0)

Date	Velocity	Measured by
1904 Apr. 10	− 1 km	Curtis
1907 Jan. 26	+25	Paddock
	+19	Curtis
Apr. 30	− 2	Paddock
	− 1	Curtis
Nov. 29	+21	Curtis

This star is of type A2F with very broad $H\gamma$ and two other broad lines; the measures are accordingly subject to considerable uncertainty. The variation was discovered by Mr. Paddock.

THE D. O. MILLS EXPEDITION
SANTIAGO, CHILE
February 1908

TWO STARS HAVING VARIABLE RADIAL VELOCITIES

BY W. H. WRIGHT

Spectrograms of the following two stars, secured by the D. O. Mills Expedition to the Southern Hemisphere, show their radial velocities to be variable:

ν Puppis ($\alpha = 6^h\ 34^m.7;\ \delta = -43^\circ\ 6'$)

The only line in the region covered by the plates is λ 4340.6 *Hγ*. This presents the appearance of being made up of a fairly sharp line in the center of a broad absorption, suggesting that both components are bright. The central nucleus is easily measurable and very fair velocity determinations can be made from the single line. Variability was suspected from the fifth plate. The last three plates, kindly secured by Dr. Curtis, confirm this variation.

Date			Velocity	Measured by
1904	January	1	+33.0 km	Wright
	October	17	+33.7	Wright
	December	31	+34.9	Wright
1905	January	18	+33.4	Wright
	December	13	+28.2	Wright
1907	January	22	+29	Curtis
			+29	Paddock
1908	January	6	+20	Curtis
	January	14	+20	Curtis

ν Octantis ($\alpha = 21^h\ 30^m.4;\ \delta = -77^\circ\ 50'$)

This star has a spectrum of the solar type.

Date			Velocity	Measured by
1904	June	8	+29.0 km	Palmer
	October	13	+29	Palmer
			+28.5	Wright
1905	August	7	+39	Wright
			+36.5	Palmer
	September	6	+36.2	Wright
1906	October	31	+33	Paddock
1907	May	6	+30	Curtis
	June	22	+27	Paddock
	September	19	+32	Curtis

The variation was detected on measuring the third plate. The last four plates, which are fully confirmatory, were taken under the direction of Dr. Curtis.

MT. HAMILTON
October 6, 1908

SPECTROGRAPHIC NOTES

By EDWIN B. FROST

I. REANNEALED PRISMS

Those who have had to use large prisms in their spectroscopes are aware of the difficulty often experienced in obtaining glass of adequate homogeneity for refined work. This obstacle was encountered in the construction of the Bruce spectrograph, as recorded in this *Journal* (**15**, 14-15, 1902). The prisms were 57 mm high, of 66° angle, with faces of 117, 127, and 135 mm; for λ 4405, $n=1.6374$. Having heard of the occasional success of reannealing, I took up the question with Mr. Brashear, who learned that the maker of the glass, M. Parra-Mantois, would be glad to undertake to reanneal the prisms which had proved so disappointing.

The process was completed and the prisms were returned from Paris to Pittsburg last spring. An examination of one prism which was at once figured by the Brashear Company indicated that the improvement was enough to justify putting new faces on all of the prisms, which was done. The prisms were now cut down somewhat, so that all three are of nearly the same dimensions: 54 mm high and 115 mm long in face.

A number of trial spectrograms of spark spectra and of sunlight were obtained with these prisms in the Bruce spectrograph, chiefly by Dr. D. V. Guthrie, who was spending the summer quarter here. The outcome of the trials was that although greatly improved, the three prisms together did not equal in definition the performance of the train of prisms of Jena flint O 102 regularly used with the instrument. The prism which was most cut down before it was figured, however, gave as fine definition as any one of the Jena prisms, and it was accordingly used for one-prism work with the spectrograph during a part of the summer, over 100 stellar spectrograms being obtained with it. The inconvenience of having the scale of the plates slightly different from that of the 1600 other spectrograms taken with one prism led to the subsequent restoration of the Jena prism to its position. The inference may be drawn by spectroscopists, however, that prisms lacking in homogeneity may in some cases be successfully reannealed, with great improvement in their optical performance. I am glad to put on record our appreciation of the interest and assistance of the Brashear Company in this experiment and of the generosity of M. Parra-Mantois in doing the work of reannealing without expense to the observatory.

II. PECULIAR SPECTROSCOPIC BINARIES

A peculiar interest attaches to those stars of the *Orion* type in which the characteristic lines are broad and diffuse but the H and K lines of calcium are narrow and sharp. Examples of such stars for which data have been published are δ *Orionis* and 9 *Camelopardalis*. In his investigation of the spectroscopic orbit of the former, Hartmann reached the conclusion "that the calcium line at λ 3934 does not share in the periodic displacements of the lines caused by the orbital motion of the star."[1] In measuring the Bruce spectrograms of 9 *Camelopardalis*, it was found[2] by Mr. Adams and myself that the H and K lines were very sharp and well suited for measurement and that they yielded a variable radial velocity differing from that given by the broad lines usually measured and from which the binary character of the star was first inferred. The detection of some peculiarities in other stars having such spectra led me about two years ago to examine all our spectrograms which included H and K. I was not prepared to find that such spectra were so common, for I was able to list about twenty-five stars having this spectrum. I was struck by the fact that about half of them had already been detected here as binaries from the variable displacements of the broad lines. For nearly all of the others, the data at hand were far too meager for definite inferences on this point.

This would accordingly seem to indicate that we are dealing in such cases with composite spectra. The general principle of such a discrimination was laid down long ago by Professor E. C. Pickering,[3] and it has been applied in the *Harvard Annals*, **28** and **56**, by Miss Maury and by Miss Cannon. This particular variety of composite spectrum has, however, not been described by the Harvard observers.

As the discovery of spectroscopic binaries here is incidental to our regular programmes, we have not yet been able to follow up the stars of this sort on our lists and find what precentage of those having sharp H and K, with the other lines broad, are *not* spectroscopic binaries.

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[1] *Astrophysical Journal*, **19**, 273, 1904.

[2] *Ibid.*, **19**, 350, 1904.

[3] "The Discovery of Double Stars by Means of Their Spectra," *Astronomische Nachrichten*, **127**, 155, 1891.

When I noted recently the presence of sharp K in ϵ *Orionis*, I arranged to have a few spectrograms taken, in the confident expectation that variable velocity would be indicated, although the earlier observations by myself and Mr. Adams (with three prisms, spectrum measurable between λ 4340 and λ 4713) did not show variation. Mr. Lee has measured all of our one-prism plates with the following results:

RADIAL VELOCITY OF ϵ *Orionis*

Plate	Date	G. M. T.	Taken by*	Broad Lines		H and K		Quality of Plate†
				No.	Velocity	No.	Velocity	
					km		km	
IB 227	1903 Dec. 26	17^h 20^m	F.	6	+30	..	....	..
484	1905 Jan. 21	15 49	F.	6	22	..	....	g.
1327	1908 Jan. 14	15 55	B.	5	27	2	+31	g.
1337	Jan. 17	18 10	L.	5	30	2	41	g.
1353	Jan. 20	15 1	B.	6	28	2	23	w.
1463	Feb. 17	13 39	P.	6	17	2	20	g. w.
1496	Feb. 21	13 19	F.	8	30	1	39	g.
1770	Oct. 2	23 1	L.	7	35	2	23	v. g.
1788	Oct. 9	23 3	L.	6	30	..	....	g too weak for *Ca* lines
1805	Oct. 16	23 15	L.	8	27	..	....	v.g
1813	Oct. 30	23 53	L.	8	30	2	28	p.
1905	Dec. 14	21 0	B.	6	37	2	28	g.
1926	Dec. 26	15 30	F.	8	30	2	30	v.g.
1946	1909 Jan. 1	18 0	B.	8	16	2	20	v.g.

* B.=Barrett; F.=Frost; L.=Lee; P.=Parkhurst. Mr. Sullivan assisted as usual in guiding.
† g=good; p=poor; w=weak.

The earlier measures[1] of three-prism plates are added for completeness:

Plate	Date	G. M. T.	No.	Adams	No.	Frost
A 208	1901 Sept. 4	22^h 10^m	3	+28 km	..	......
B 228	Nov. 13	19 50	6	29	5	+24 km
B 298	1902 Mar. 13	15 11	5	26	4	26
B 316	April 9	14 21	3	26	3	28

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[1] "Radial Velocities of Twenty Stars Having Spectra of the *Orion* Type," *Publications of Yerkes Observatory*, **2**, 210, 1903.

The inference is that in this case the H and K lines share in the oscillations of the broad lines, so that this would not be an instance of a composite spectrum.

I reserve for a later discussion the remarkable feature of the binary character of some of these stars, already alluded to, that the component responsible for the sharp H and K lines appears to have a different velocity (γ) of its orbital center of gravity from that of the component having broad lines. The first inference in such a case would be that we are dealing with a quaternary system—two pairs of spectroscopic binaries; if so, however, a change in the relative velocity of the center of gravity would be expected within a few months or years, and in at least one instance our observations cover a longer period of time.

I have in mind particularly the star ξ *Persei*, the first observations of which were made by Mr. Adams[1] in 1903. The five plates then measured yielded a velocity of +85 km, which would be entirely exceptional for a star of the *Orion* type not a spectroscopic binary. We therefore expressed the opinion that later observations might be expected to show the star to be a binary. A range of 30 km was given by a plate taken later in that year, establishing the variation in velocity, and we have been observing the star occasionally each year since, so that we now have over 30 spectrograms, of which about one-half are measured. We do not appear to have caught the star with a negative velocity on any plate, the total range shown on our measured plates being about 60 km. Mr. Plaskett reports his measures of 7 plates taken in the autumn of 1908 in the *Journal of the R. A. S. of Canada* (**2**, 325, 1908), giving a range of velocity from −2 to +143 km.

The lines are exceptionally broad and diffuse, even for spectra of that type, but H and K are quite sharp, although H cannot always be separated from $H\epsilon$. In 1906 at my request Mr. Ichinohe measured the ultra-violet portion of a number of the earlier plates, and obtained for the calcium lines only moderately positive velocities, having a range of about 25 km, but differing greatly from the values derived from the broad lines (generally by not less than 50 km). I have also measured some later plates with similar results. The star evidently is of the sort of 9 *Camelopardalis* and δ *Orionis*, but its thorough investigation will be a difficult task on account of the evil character of the broad lines.

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[1] *Astrophysical Journal*, **18**, 388, 1903.

I am well aware that Professor Julius finds an explanation of such stars in anomalous dispersion, regarding the broad lines as dispersion bands.[1] Perhaps some physical effects will have to be invoked to explain the peculiarities of such stars; but I am not yet convinced that anomalous dispersion is here a sufficient cause.

III. SPECTROSCOPIC BINARIES AMONG STARS IN PROFESSOR BOSS'S STREAM IN *TAURUS*

In a brief paper read at the Baltimore meeting of the American Association I reported on the beginning made here in observing the radial velocities of the stars in the "moving cluster in *Taurus*" described by Professor Lewis Boss in No. 604 (**26**, 31, 1908) of the *Astronomical Journal*. As my remarks were printed in *Science* (**29**, 156, 1899), practically in full, I will not repeat myself further than to state that observations of those 40 stars have now been in progress, as circumstances permitted, for about six months with the Bruce spectrograph, a dispersion of one prism, which is as high as many of the spectra will stand, being employed.

About seventy spectrograms have been so far obtained, chiefly by Mr. Barrett and Mr. Lee, of 21 of the stars. The six stars of that list which we had found to be spectroscopic binaries at the time that paper was prepared, were: *90 Tauri* and *B. D.* 15° 637 found by Mr. Barrett; *64* and *97 Tauri*, found by Mr. Lee; and θ^2 and v *69 Tauri*, found by myself. Two that I then regarded as suspicious, *71 Tauri* and *92 Tauri*, have since proved by measurement to be also variable in velocity. The particulars of the measures thus far made of six of these stars will be given here and in the following note by Mr. Lee. Mr. Barrett will report later on his measures of *90 Tauri* and *B. D.* 15° 637. It must be acknowledged that the proportion of binaries is surprisingly large, 8 out of the 14 stars for which we have two or more spectrograms.

The observations with the Bruce spectrograph indicate a proportion of spectroscopic binaries of not less than 1:3 among stars of the *Orion* type, but the proportion here would seem to be still greater. The 14 stars, with one exception, have spectra of the first type or of one intermediate between the first type and the early solar type: in the current Harvard designation they are of types A, A2, A3, or A5, with the exception of θ' *Tauri*, of type K.

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[1] "Dispersion Bands in the Spectra of δ *Orionis* and *Nova Persei*," *Astrophysical Journal*, **21**, 286, 1905.

The spectrum of θ^2 *Tauri* is of type A5. With a dispersion of two prisms (scale at λ 4500 : 1 mm = 15 t.-m.) the lines are very diffuse for measurement; with one prism the lines are better, but some seem to be complicated by the presence of companions due to a second component. The radial velocity is therefore liable to considerable uncertainty. On being informed by Professor Boss regarding his cluster, I asked Mr. Lee to measure the two plates of this star then on hand, both taken with two prisms. The other measures are by myself. The variable radial velocity was at once apparent after we began to get spectrograms with one prism.

θ^2 *Tauri* ($\alpha = 4^h\ 23^m$; $\delta = +15^\circ\ 39'$; Mag. = 3.7)

Plate	Date	G. M. T.	Taken by	No. Lines	Velocity	Remarks
IIB 86	1906 Aug. 31	21^h 11^m	F.	4	+42 km	
180	1907 Nov. 4	18 32	L.	4	(+29)	Very weak
IB 1707	1908 Aug. 25	22 32	B.	5	+64	
1734	Sept. 8	22 45	B.	6 2	+88 +40	Second Component
1741	Sept. 18	20 32	L.	7	+10	
1794	Oct. 12	20 25	B.	4 3	+39 + 8	Second Component
1836	Nov. 8	18 46	L.	5	+36	
1895	Dec. 7	15 1	L.	5	+38	
1900	Dec. 11	15 2	F.	4	+31	

The spectrum of *69 Tauri* is a very difficult one to deal with. To classify it as of type A5 fails to express the diffuse character of the lines. My re-examination of our first plate, taken three years ago, led me to believe that the spectrum is composite and this was confirmed by the measures of the subsequent plates. It is difficult and often incorrect to combine into means the velocities from lines which in some cases belong to different components; where the components are not separated, the setting on the center of the apparently single line has no definite meaning. My measures are as follows:

69 (Upsilon) *Tauri* ($\alpha = 4^h\ 20^m$; $\delta = +22^\circ\ 35'$; Mag. = 4.5)

Plate	Date	G. M. T.	Taken by	No. Lines	Velocity
IIB 24	1905 Nov. 24	$17^h\ 26^m$	B.	2 2	− 81 km + 16
IB 1747	1908 Sept. 21	19 40	B.	2	+ 102
1896	Dec. 7	16 21	L.	1 2	+ 82 + 130
1901	Dec. 11	16 1	F.	6	+ 38
1954	1909 Jan. 3	18 28	B.	2	+ 34

Yerkes Observatory
February 17, 1909

VARIABLE RADIAL VELOCITIES OF FOUR STARS IN THE *TAURUS* STREAM

By OLIVER J. LEE

The following measures have been made on Bruce spectrograms taken with one prism. They were reduced with the aid of tables constructed in the manner employed by Professor Schlesinger.[1]

64 Tauri ($\alpha = 4^{h}\ 18^{m}$; $\delta = +17^{\circ}\ 13'$; Mag. $= 5.1$)

Plate	Date	G. M. T.	Taken by	No. Lines	Velocity km	Quality
IB 1786	1908 Oct. 9	$20^{h}\ 47^{m}$	L.	15	+32	v.g.
1848	Nov. 10	19 29	L.	13	+44	v.g.
1856	Nov. 13	19 18	L.	12	+38	v.g.
1864	Nov. 16	16 24	L.	12	+33	v.g.
1886	Dec. 4	19 51	L.	13	+44	v.g.

The spectrum of the star *64 Tauri* is of type A2 in the notation of the *Revised Harvard Photometry*, or Vogel's Ia2. The lines are numerous and well defined, so that there can be no doubt of the binary character of the star, although the range thus far observed, 12 km, is rather small. A period of 12 days is suggested by the observations thus far, but later plates may indicate a sub-multiple of this period. A higher dispersion could be used to advantage with this star, and spectrograms will be obtained with two prisms.

The three other stars have complex spectra, and it was the double appearance of some of the lines on the first spectrograms obtained that rendered the stars suspicious. It was necessary to make settings upon what seemed to be the separate components, a difficult procedure in many cases. Where the lines appeared single the settings were made upon the centers, but this may introduce error where the lines were actually double but too confused for separate recognition. It will thus be understood that the values given for centers are not averages of the settings on the component. The measures on the centers show a velocity variable by as much as 30 km for each of the stars, probably chiefly due to unequal light effect of the two components, and partly to lines present in only one component.

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[1] *Publications Allegheny Observatory*, 1, 9, 1908.

97 i Tauri ($\alpha = 4^h\ 45^m$, $\delta = 18° 40'$; Mag. = 5.3)

Plate	Date	G. M. T.	Taken by	Center		Violet Comp.		Red Comp.		Quality
				No.	Vel	No.	Vel	No.	Vel	
					km		km		km	
IB 1787	1908 Oct. 9	22h 8m	L.	4	+41	3	−14	1	+ 88	v.g.
1832	Nov. 6	22 9	L.	4	+53	.		. .		g.
1907	Dec. 19	17 3	L.	7	+19	2	−52	3	+ 90	g.
1912	Dec. 21	17 3	L.	3	+28	5	−11	2	+103	g.

This spectrum is classified as A5, but the lines are so diffuse and complicated that accuracy of measurement is not possible. In many cases normally simple lines are split up into two or three parts. On one plate only, No. 1832, do the lines seem relatively simple.

71 Tauri ($\alpha = 4^h\ 21^m$; $\delta = +15° 24$; Mag. = 4.8)

Plate	Date	G. M. T.	Taken by	Center		Violet Comp.		Red Comp.		Quality
				No.	Vel.	No.	Vel.	No.	Vel.	
					km		km		km	
IB 1748	1908 Sept. 21	20h 47m	B.	6	+12	4	−61	6	+44	v.g.
1902	Dec. 11	17 17	F.	8	+ 9	2	−18	2	+57	v.g.
1910	Dec. 21	14 37	F. L.	10	−21		.		. .	v.g.
1962	1909 Jan. 25	13 48	F. L.	6	− 9	4	−97		.	p.
1972	Feb. 1	12 2	L. B.	4	+18	7	−90	2	+82	v.g.

The spectra of this star and the next are classed as simply A in the *Revised Harvard Photometry*, but there is some difference in the accuracy of measurement possible. In *71* the lines are diffuse, with confusing indications of a second component on three of the plates. On plate No. 1910 there was no indication of the presence of a second component. The centers were measured as in the case of *97 Tauri*. The results are liable to considerable uncertainty.

92 σ^2 Tauri ($\alpha = 4^h\ 34^m$; $\delta = +15°\ 43°$; Mag. = 4.5)

PLATE	DATE	G. M. T.	TAKEN BY	CENTER		VIOLET COMP.		RED COMP.		QUALITY
				No.	Vel.	No.	Vel.	No.	Vel.	
					km		km		km	
IB 1804	1908 Oct. 16	22h 10m	L.	7	+42	6	0	3	+ 97	g.
1838	Nov. 8	21 20	L.	6	+27	5	−26	6	+ 71	v.g.
1944	1909 Jan. 1	15 18	B.	6	+ 2	3	−67	7	+ 46	g.
1952	Jan. 3	13 44	B.	7	+17	7	−42	8	+ 79	v.g.
1963	Jan. 25	14 37	L.	8	+18	2	−88	2	+102	p.
1973	Feb. 1	12 56	B. L.	11	+31	..	....		....	g.

In this star the double character of the lines is quite evident, and measurements of the separate components are certainly more accurate than in the case of *71*. Only one plate, No. 1973, gives simple lines.

YERKES OBSERVATORY
February 17, 1909

SERIES IN THE BARIUM SPECTRUM

By W. RITZ

The previously unknown triplet series of barium has recently[1] been discovered and published by F. A. Saunders. Very few terms of the series were observed, and since these were not represented by formulae known to be applicable to Mg, Ca, and Sr, the choice of lines may seem at first to be somewhat doubtful. The following remarks will, I think, serve partly to verify his results.

The lines of the first subordinate series may be accurately represented by either of the two following formulae:

$$\nu = A - \frac{109675.0}{\left[m + a + \frac{b}{m^2}\right]^2} \tag{1}$$

$$\nu = A - \frac{109675.0}{[m + \alpha + \beta 10^{-5}(A - \nu)]^2} \tag{2}$$

which, as I have shown,[2] serve to represent with great accuracy the series of other elements. In these formulae A, a, b, α, and β are constants; m a whole number, and ν the wave-number (per cm) in vacuum.

For the strongest (and most completely observed) lines we obtain, for instance:

$m=$	3	4	5	6	7
λ obs..................	5777.84	4489.50	4084.94	3895.2	3787
ν obs..................	17302.8	22268.1	24473.4	25665.4	26399
Error (in Å.)...........	0.00	0.00	+0.02	+0.6	−0.5

The differences are throughout smaller than the errors of observation. In this case $A = 28472$; $a = +0.29581$; $b = -1.4600$. In the corresponding cases for Mg, Ca, Sr, the constant $a + 1$ has the values 0.842, 1.090, and 1.222; the value for Ba, 1.296, fits in well with these. The alkalies furnish a progression which is very similar.

The wave-number difference $\nu = 878.0$, divided by the square of the atomic weight (μ) gives $47.0 \cdot 10^{-3}$, while the values for Mg, Ca, and Sr are $68.8 \cdot 10^{-3}$, $66.1 \cdot 10^{-3}$, and $51.5 \cdot 10^{-3}$. As I have pointed out,[1] the quotient $\frac{p_1-p_2}{\mu^2}$, calculated from the limits of the series, varies still less from element to element,

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[1] *Astrophysical Journal*, **28**, 223, 1908.

[2] *Annalen der Physik*, **12**, 264, 1903.

$$\frac{(p_1-p_2)10^5}{\mu^2}=0.146(Mg)\ ;\ 0.177(Ca)\ ;\ 0.157(Sr)\ ;\ 0.148\ (Ba)\ .$$

Finally, the separations of the satellites of the strongest lines of the first members ($n=3$) vary approximately as the square of the atomic weights. This requirement is also fulfilled in this case. From these facts it is evident that the first subordinate series of *Ba* is correctly given by Saunders.

The same conclusions hold for the first two triplets of the second subordinate series. Since the quotient $\frac{\beta}{a+0.5}$ varies but little among the members of a group of chemically related elements, I have already[2] given a probable value of 0.825 for this number in the case of *Ba*. Granting that the limits of the two series are the same, we have left in the equation of the second subordinate series only a single constant, a, to be calculated. In the red and ultra-red, where the first term of the series must lie, only one triplet was found with the proper separations. If we use this to calculate a, the formula so derived leads to the second triplet given by Saunders to within 5 Å., and this is a very small error when we consider that both the number 0.825 and the limit of the first subordinate series[3] are very uncertain. Thus both these triplets doubtless belong to the second subordinate series. If we now determine A and a from these two triplets, we find $\lambda=4248.4$ for the strongest line of the third triplet, while Saunders gives 4239.91. The use of the formula therefore leads us to doubt the propriety of classifying this triplet (and those following) in the series. It is possible, as Saunders remarks, that the formula does not fit this series, notwithstanding the fact that it successfully represents the second subordinate series of all the elements; but further investigation of the missing lines of the triplets of this series must be made before the question can be definitely settled.

GÖTTINGEN
January 1909

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[1] *Astrophysical Journal*, **28**, 237, 1908; *Physikalische Zeitschrift*, **9**, 521, 1908.

[2] *Annalen der Physik*, **12**, 309, 1908.

[3] The formulae (1) and (2) represent, in general, the first subordinate series (especially of *Ca*) less exactly than the second; if only a few terms are known, and these only inaccurately, considerable errors in A may result.

Reviews

The Study of Stellar Evolution: An Account of Some Recent Methods of Astrophysical Research. By GEORGE ELLERY HALE. Chicago: The University of Chicago Press, 1908. Pp. xi + 252; with 104 plates. $4.00 net.

Hale's book on the *Study of Stellar Evolution* will be welcomed with the keenest pleasure and interest by a host of readers and students of astrophysics. The book was first planned to serve as a handbook for the use of the numerous visitors to the Yerkes Observatory; but when Professor Hale went to organize the new Solar Observatory of the Carnegie Institution, now splendidly established on Mount Wilson, in California, he decided to enlarge the scope of the book and make it a handbook for the use of visitors to both of the named observatories. By describing a connected series of investigations in stellar evolution, Professor Hale makes the opportunity of indicating from personal experience the observational methods adopted at both places. The result is an intensely interesting narrative from the pen of a pioneer who is in the thick of his adventurous work. It discloses to minds that are not conversant with the aims of the investigations of practical astronomers something of the hopes, difficulties, and achievements that go to make up the zest of life for one who is successfully engaged in developing a new attack on a problem, that has in large measure defied the efforts of the last forty years. To such minds it will perhaps convey the idea that the whole matter is to be settled by this onslaught; for Hale writes with that deep conviction that serves as motive power to an intrepid investigator, and he has set forth the subject with such consistent simplicity that the lay mind is sure to feel that, after this work is completed, there will be little left for astrophysicists to do.

Hale's narrative brings interest to very varied minds, and consequently lends itself to review from many points of view. The most difficult part of the task of the reviewer is to decide which point of view he is to take. I propose to resist the temptation to be purely descriptive, preferring rather to let myself be caught by the spirit of the pioneer and to join Hale, however inadequately, in looking out over the country to be explored. My point is that students of astrophysics as well as eager visitors to the large observatories owe a debt to Professor Hale for having devoted much of his time to writing this handbook. And how can a reviewer, who is interested in the subject that has been helped by Hale's book, show his appreciation better than by trying to put down thoughts that have been stimulated by the perusal of the book?

Any attempt, such as Hale makes, to lay before a wide audience a consistent representation of a subject like astrophysics must needs start from some more or less generally accepted standpoint. Hale carefully sets forth what may be called the case for temperature variation, and he uses it freely in summarizing the varied phenomena of stellar and solar spectra. A reviewer would be doing poor service if he were to seem to pin an author to statements in a handbook as if they were parts of his final creed. He must rather feel that he has, as it were, joined a field excursion and has been hearing a pioneer sketch out to a crowd of varied minds the lie of the country. According to the amount of study he has devoted to the subject the student will recognize the importance which the pioneer attaches to some of those dim ranges in the landscape, to which he has only passingly referred. His thoughts will go wandering on; the definite words to which he has been listening, used for stating an accepted view, may only tend to fix in his mind a diametrically opposite view. And if I use the opportunity afforded by this review to give utterance to thoughts that have long occupied my mind, I am sure I shall not be understood as offering cheap criticism of an admirably fair-minded and stimulating exposition of the present situation in an important part of astrophysics. I wish rather to plead for the consideration of a new aspect of the whole subject. Tersely put, the new aspect to be considered is: can it be that the main characteristic spectroscopic phenomena of the sun and the stars are dictated mainly by matter brought from without, and not mainly by matter brought from within the body of the sun and the star?

First let me quote a few lines from a notice written by myself in the *Observatory* (October 1908, p. 378):

> Realize how the search is baffled whether on the purely physical side of the interpretation of general spectroscopic phenomena or the astrophysical applications. The statement of the problems today—as given, for instance, in chaps. ix, x, xi, xvii, xviii, xx—is nearly the same as the statement forty years ago. It seems as if the methods of analysis and discussion of isolated phenomena on physico-mathematical lines had proved too refined to cope with the broad aspects of the involved conditions of the questions to be elucidated. There were no signs of an immediate development of a more embracing method of solution on these lines.
>
> But now comes Hale's method of the spectroheliograph, which automatically co-ordinates in a single record the many discordant isolated observations of the earlier days, and even those of today. Strange to say, it is as if the sight of the faggot is likely to help us to break the single sticks, an inversion of *divide et impera.*

Such an achievement leads us to hope that the work is to land us, if it has not already landed us, on a new platform, where the horizon and extent of the subject is so much enlarged that the mind finds itself again. The way is opened afresh for the physico-mathematical attack on broader lines.

As an instance of the baffled search, let us consider the case of widened lines in the spectra of vapors. Such widening is generally attributed to pressure, and Rayleigh and Michelson have indicated the paths along which explanation of the effect may be sought by calculating (1) the magnitude of the Doppler effect arising from the velocities of the light-giving atoms or molecules; and (2) the amount of the ill definition of lines arising from the shortness of sequences of undisturbed undulations (limitation of free path). These results show good agreement between theory and observation for the narrower lines in spectra. Michelson's expression for widening produced by shortness of sequences (limitation of free path) is of the form

$$\delta_2 = \frac{\lambda^2}{\rho}\,\frac{v}{V},$$

where δ_2 is the widening, ρ the free path, v the temperature-velocity of the molecules, and V the velocity of light. Thus when ρ is of the order of magnitude of the wave-length λ (and this is the case for pressures of the order of 100 mm of mercury for hydrogen), the widening by limitation of free path is of the same order of magnitude as the widening from the Doppler effect.

But there appears to be a source of widening which evades this analysis unless we are willing to admit a limitation of free path far more stringent than that imposed by the ordinary molecular kinetic theory. To take a single case, the experiment of passing electric sparks through unignited coal-gas in the absence of air shows that exceedingly wide lines may be produced at atmospheric pressure. The experiment is performed by putting sparkling terminals inside a glass tube, the ends of which are then connected by rubber tubing with a gas-main and with any convenient burner to get rid of the gas that passes through the tube. The spectrum of the sparks exhibits the hydrogen lines; and by manipulation of the spark-strength the hydrogen lines may be made either quite narrow or so broad as eventually to meet and so give a continuous spectrum. The widening is in the main symmetrical, a fact which would seem to rule out an explanation based on a change of molecular periods whether by loading or by increase of the forces under which molecular vibrations take place. Apart from a special and consequently unlikely combination of the two causes alluded to, an explanation lies in a new order of limitation of free paths, involving collisions or disturbances about a thousand or ten thousand times more frequent than those provided for in the ordinary kinetic theory.

Such an increase in the number of disturbances might fairly be ascribed to multitudes of corpuscles (J. J. Thomson) set free in the passage of the electric spark. But putting explanation aside, the experiment shows us that we are able to fix, at any rate momentarily, a mode of excitation of luminosity which gives exceedingly wide lines at low pressures; and it points to the existence of a peculiar condition, of which we have not yet complete control nor complete knowledge.

As another instance of baffled search, we find (1) that in the spectrum of almost every element under apparently any given conditions of pressure and temperature and electrical excitation some lines are broad, others narrow; and (2) that if any one of the conditions is varied, the relative width and brightness of the lines is in general changed. Thus in observations of the electrical arc under pressure (e. g., Duffield, *Phil. Trans.*, A, **208**, 157, 1908) it would appear that alteration of pressure under fixed electrical conditions may be accompanied by the same sort of changes in certain lines as are produced by a change of electrical excitation under fixed conditions of pressure. We are not yet in possession of a knowledge of the relation that must subsist between the variables defining the state of a gas, in order that a prescribed condition of luminosity may be evoked. All that we seem entitled to say is that we can in many cases evoke a prescribed state of a given line in a spectrum by proper adjustment of the conditions of excitation, pressure, and temperature.

I wish then to emphasize the statement that there is strong evidence that widening of lines may arise from a special mode of (electrical?) excitation even at low pressures.

Now widened lines, like those of hydrogen, helium, calcium, magnesium, etc., are a marked criterion in our accepted classification of stellar spectra. I would ask, do we not allow ourselves to adopt them without duly understanding their significance?

Let us look at the usual classification of the stars according to their spectra. We will make the authoritative statements in a type appropriate to the textbook, while the queries of an impetuous lay-reader shall be hushed in smaller type:

The usual classification begins with the blue stars; their spectra are marked by sparse and fine metallic lines together with broad hydrogen lines and sometimes broad helium lines; these are early stars with diffuse atmospheres.

Then, if breadth of line is a sign of pressure, does this mean that the hydrogen is under higher pressure than the metallic vapors in these early stars?

Perhaps, but let us proceed with the classification. The next class are the more condensed yellow stars of solar type.

Then, if the stars are more condensed, the lines are broader generally?

No, the lines are sharper; sharper hydrogen lines, and sharper metallic lines, except the calcium lines like H and K, which are strong and broad, forming a characteristic feature in stars of solar type.

Then the calcium is probably in the lower strata of the atmospheres of such stars?

Well, solar prominences point to its being outside in a rarified condition, and the spectroheliograph indicates that some rarified calcium is glowing brightly and irregularly over the surface of the sun, whilst the darker patches are possibly beneath and at much greater pressure.

Then, near the edge of the sun's disk the effect of the pressure will be more marked? I mean because the paths are relatively longer in the lower strata; are they not?

Yes, it is true that through three superposed strata, each of thickness 1000 kilometers, the paths of light near the sun's limb in the direction of the earth would be 120,000, 50,000, and 40,000 kilometers, if we neglect refraction.

Then, I suppose the widened edges of the K line are much darker at the edges of the sun's disk?

No. Hale's observations show that they are weaker, but then you have to take account of the filamentary structure of the photosphere. But we must return to the classification. The third class are still further condensed than solar stars. The characteristic features in their spectra are the marked bands which are believed to be due to titanium oxide and perhaps other compounds which only begin to be formed at temperatures lower than those at which *elements* (as opposed to compounds) are vaporized.

Then the compounds form first on the cooler outskirts of the star's atmosphere, I suppose? The metallic lines are of course much broader in this third class?

No, the metallic lines are still sharp.

Oh. And wait a moment, why don't the titanic oxide strata form outside the solar stars too? Such stars must get cooler in the upper strata of their atmospheres.

Yes, but it is a question of depth of strata as well as relative temperatures.

Then the bands might be visible at the limb of the sun?

Yes, they might be: but they are not visible there, at any rate not nearly so strongly as in sun-spots.

Oh, they are visible *locally* on the sun. Does not that point to special supplies of the proper material? Curious that the metallic lines are not widened! Are they widened in the fourth class?

The fourth class are marked by bands turned the opposite way to the titanium oxide bands; they are due to carbon compounds. The metallic lines are still narrow.

I advise you to look after those fine metallic lines. They don't look like high pressure.

We find the classification very convenient.

Doubtless a better presentment could easily be given of the position of both textbook and reader, but this is enough to serve my point: namely, to insist that in our attempt to justify a convenient empirical classification we seem unconsciously to reserve for ourselves the right to pick out of a multitude of criteria now one, now another, without ever consistently searching for binding relations between the criteria or rigorously following them out to their inevitable conclusions. We blind ourselves to the fact that laboratory phenomena clearly show that almost any element taken at random under chance conditions of luminosity will by its different lines afford examples of all the different affections of spectrum lines seen in a given star spectrum. We refuse to admit that it is the star which exercises eclectic choice, and commits itself in general to a fixed and constant type of spectrum—full of apparent inconsistencies.

Is a fixed and constant spectrum what we are entitled to expect of a star? I would say no. We realize from the comparison of a few of Hale's spectroheliograms how unwise it is to expect that suns, in which one of the essential factors in the mechanism of radiation is the *radial* convection currents, should show any spectroscopic symptoms of permanent tangential stratification. Consider this aspect of the case a little in detail.

I wish to bring forward three points, A, B, and C.

A. First, we will for a moment confine attention to the calcium (K_2) spectroheliograms. In such pictures we find that neighboring regions on the sun exhibit very different intensities of the incandescence of K_2 calcium. The dark regions are those where the incandescence of K_2 is feeble or where the motion in the line of sight is so great that the K_2 line is displaced off the second slit of the spectroheliograph. If we set aside this latter cause of dark flocculi on the ground that it is probably capable of producing only a small part of the observed darkening, we may take it that these spectroheliograms prove that calcium glows with very different intensities in different parts of the sun's surface.

If it should happen that the integrated area of the bright regions over the whole disk of the sun were greater than the integrated area of the dark regions, then the spectrum of integrated sunlight would exhibit a bright calcium line. Its easy detection as a bright line depends, however, on the existence of the dark wings of K_1; for if these wings did not exist, the bright K_2 line would appear as bright as the neighboring continuous spectrum, since it seems to be the observed fact that the bright K_2 line over a bright flocculus is about equal in intensity to those parts of the continuous spectrum which are not darkened by the wings of K_1. Thus but for the darks wings of K_1 the calcium line would appear as a narrow dark line over a dark flocculus; it would not appear at all over a bright flocculus. In the case of the sun, however, the dark wings of K_1 are always present. Hence (*a*) if the actual sun were observed as a remote star, at a time when the integrated bright flocculi were in excess, its stellar spectrum would exhibit a broad and dark K_1 line with a bright K_2 center; whereas if the integrated bright flocculi were in defect, the bright center would not be visible. And (*b*) if we had to do with a sun, whose spectrum did not exhibit dark wings of K_1, then the stellar spectrum would show a dark K_2 in times of defect of bright flocculi, whereas in times of excess the line would be simply obliterated.

These considerations (relating, it is true, to a single line) are enough to show in what marked degree a stellar spectrum may be a *residual* phenomenon in the conflict of two opposed conditions, represented by dark flecculi and bright flocculi. (The bright calcium flocculi are most noticeable in the obviously disturbed regions of sun-spots. They call for special consideration in a later paragraph.) Apart from the sun-spot zones, the calcium flocculi are distributed in a mottled structure over the whole disk of the sun in a way that suggests the active influence of radial convection currents, which seem to reach almost the upper surface of the reversing layer. But the persistence of the darkness of the K line in the *integrated* solar spectrum shows that in this general mottled network of flocculi the dark regions are greatly in excess of the bright flocculi; there is in fact in ordinary solar conditions no risk of that degree of variability in the K line, that would make it bright at one time and dark at another. In other words, the stability of the condition that gives a dark K line is considerable and is not likely to be upset by the convection currents beneath the reversing layer. It is otherwise evident from a comparison of the numbers of stars of different special types that the solar spectrum is in a sense an indication of a condition of fairly stable equilibrium of some sort.

With respect to the radial convection currents let me call to mind how

Kelvin's estimate of the power of sunlight per square meter of sun's surface has been put by Schuster in the following form:

> Taking the pressure of the vapor near the surface to be one atmosphere, we may say that all the heat contained in a layer having a thickness of 370 meters is lost by radiation in each second of time, and this number does not depend on the nature of the vapor or on its temperature. A layer of that thickness would have to be replaced by convection in every second if the temperature of the surface is to be maintained.—(*Astrophysical Journal*, **17**, 173, 1900.)

I would therefore put it that though these powerful radial convection currents form a mechanism which is well fitted for maintaining a photosphere, they are hardly suited for producing a permanent spectrum ascribed to vapors floating over it. Hale's spectroheliograms in fact seem to afford proof that the reversing layer is one of considerable thickness. The brightest flocculi, however, in the disturbed regions near sun-spots appear at first sight to demand mechanism of a different kind. But I incline to believe that one general scheme subject to local differences can be imagined which will serve to co-ordinate most of the phenomena of stellar and solar spectra.

I find myself provisionally regarding all stellar spectra and also local spectroscopic phenomena on the sun as conditioned, not mainly by matter which comes from within the body of the sun or star, but by matter which comes from without and is more or less slowly working in upon the photosphere.

Schwarzschild's idea of radiative equilibrium (*Nachrichten d. K. Ges. d. Wiss.*, Göttingen, 1906) is very helpful in this view, but, as I understand it, he is inclined to regard it as applicable to the case of what one may describe as a finished sun radiating in empty space.

My view would rather be that we must never lose sight of the constant activity of the gravitational attraction of the sun. Matter is always coming in, sometimes in the shape of meteoric stones and rocks, sometimes in finer dust or vapor. The rapid approach of cold dust to a raging sun and the rapid recession of hot dust from it will inevitably result in splintering and splitting. Radiation pressure will exert a sifting action, and thus the most finely divided matter—much in molecular state and the rest nearly so—will be that which will "settle" in superbly equably mixed state upon the sun after circulating round it for perhaps months. Leaving on one side for the moment the consideration of the question of organized rotation in the reversing layer, the mechanism which I suggest seems to be more in conformity with our knowledge of what goes on in the solar system; and so far as I have been able to test it, it seems to be a more resourceful aspect of the phenomena of stellar spectroscopy.

B. The second point is as follows. Hale's success in taking hydrogen spectroheliograms has led him to the discovery of the remarkable fact that in general where calcium flocculi are bright there are dark hydrogen flocculi. The discovery is, not only that hydrogen flocculi are distributed over the sun's surface irregularly, bright here and dark there, but also that the distribution of the hydrogen flocculi is complementary in a sense to that of the calcium flocculi. Now this would seem to indicate that the seat of the flocculi is the same, and that there is not a difference in level between the strata in which the calcium and hydrogen flocculi appear, but that the excess of incandescence of hydrogen is connected in some way with the defect of incandescence of the calcium. Were the observed fact to be referred to changed conditions *in different levels*, it would appear that the only way of accounting for the complementary nature of the phenomena would be to admit one of three views: (1) that the incandescence of calcium shields the hydrogen above it in some way; or (2) that the elevated "thunderhead" of calcium displaces the generally glowing hydrogen into an upper region where it is incapable of continuing to glow; or (3) that the rising of the calcium "thunderhead" displaces the hydrogen sideways, making it fall into the calcium valleys, where it glows.

The third view seems to introduce a mechanism which works in opposite ways for calcium and hydrogen: for it is the uprising calcium that glows in K_2, but it is the displaced hydrogen that glows in descending to a lower level.

The second view involves the idea of upheavals persisting for several days. The first view demands a special mechanism; it may be forthcoming, but it is unnecessary to develop it.

I incline to accept none of these three views, but rather to think that both the persistence of bright flocculi (in particular, calcium K_2 flocculi, involving the radiation of large amounts of energy for many days, or even months) and also the complementary nature of hydrogen and calcium flocculi may be more easily referred to the continued descent of matter from without upon the sun's surface. Hale's view of ascensional currents, unless I misunderstand it, leaves us in the position of having to supply energy to provide for radiation under conditions of dynamical loss during expansion. The descensional currents seem to afford a more hopeful aspect.

C. In the third place, let us consider the spectrum of a bright calcium flocculus in a disturbed region of the sun's surface—i. e., near a sun-spot. Probably in the great number of investigations which Hale is carrying out or has on his programme, one of the most interesting and important will be that which deals with the brightness of different parts of the spectrum in the wings of the K line—the photometric researches. It would appear that the bright K_2 line is nearly if not quite as bright as the continuous spectrum outside the dark wings of K_1. It would also appear that the narrow metallic absorption lines involved in the shaded edges of the broad K_1 line are not reversed bright. Now if the dark edges of the K_1 line are due to lower strata of calcium, then these strata must darken the photosphere and impair its power to make metallic lines due to vapor above them appear as absorption lines. The fact that the metallic lines remain absorption lines in the edges of K_1 affords means of arriving at an estimate of limiting values of the difference in effective temperatures of the strata in which the absorption of the broad edges of K_1. and the narrow metallic lines involved in them originates. It is evident that the same argument which would stand in the way of our assigning a lower level to the calcium vapor will stand also in the way of our inverting the layers. And thus until we have photometric observations to help us, we are, I think, driven to regard the seat of absorption for both the broad calcium line and the narrow metallic lines as residing in one and the same reversing layer. And we are left with the need to find explanation for the formation of both broad and narrow absorption lines in that one layer.

Hale's imagery in dealing with solar flocculi is of solar "thunderheads," analogous with the cumulus clouds connected with terrestrial thunderstorms. If in attempting to account for the persistent emission of radiant energy from such ascensional clouds of calcium we try to develop Schwarzschild's idea of radiative equilibrium, we are met by the difficulty that if a bright K_2 flocculus is incandescent calcium vapor floating over cooler calcium which gives a broad absorption line (K_1), then a great part of the photospheric radiation available for keeping the flocculus glowing is cut off by the cooler strata of calcium below; and unless we are willing to admit that the glowing of the flocculus is of the nature of fluorescence, we are driven back upon electrical or chemical modes of maintaining the radiation of the flocculus. A simple mechanism seems available, if we admit a combination of the recognized emission of corpuscles by an incandescent photosphere with gaseous or vaporous matter settling in upon the sun from without.

Hale has carefully set forth in his book the case for temperature variation as capable of being made the basis of explanation of sun-spot and stellar phenomena. My own conviction is that that case is only strong enough for a general exposition of the subject and for showing how the new methods give hope of our being able to substitute a better aspect. This is, of course, Hale's point of view. But no one can read his recent scientific papers without realizing that he is committing himself more and more deeply to that -doxy. (I naturally refrain from prefixing to that hyphen either ortho- or hetero-.)

In his experiments with the electric furnace, he is dealing with vapor permanently inclosed in an incandescent tube, which according to all the work of H. A. Wilson, Richardson, Horton, and others must be full of free corpuscles. The substitution of the furnace in place of the arc must give us most important advances of knowledge of the radiation of vapor in a considerable space under conditions specifiable in terms of temperature. But it must never be forgotten that the furnace is electrical both in name and nature.

Nothing can be less suitable for our purpose of studying temperature variation than methods based upon arc and spark. Each epitomizes a vast range of conditions both electrical and thermal. The one gives instantaneous explosions of the most complicated conditions; the other exhibits phenomena which depend on continued flux of vapor through exceedingly varied conditions, electrical and otherwise. Out of this agnostic *impasse* safety lies along the path of the method of experimenting adopted by Hale, but the goal is not yet in sight.

I may summarize the aspect which I venture to suggest, as follows:

We have suns with incandescent photospheres maintained by convection currents which carry the energy from within the sun to the radiating photosphere. Such suns by their gravitationally attracting power are constantly drawing matter in upon their surface. But their radiation exerts by the agency of light-pressure a selective action on matter so attracted. We may divide the attracted matter into three categories: (1) molar matter, in masses large enough to be drawn in spite of light-pressure, having diameters longer than about 2½ times the wave-length of light, (2) molecular matter, in masses small enough to escape in virtue of diffractional effects the repulsion of light, having diameters less than about one-tenth of the wave-length of light, and (3) light-driven matter, in masses intermediate between molar matter and molecular matter, and subject to the repulsion of light.

The reversing layer is provided by molecular matter constantly streaming across what we may call a critical envelope concentric with the sun and probably is in a slightly modified state of radiative equilibrium. The constant flux of matter here involved is provided by those constituents of the planetary whirl of molecular matter circulating round the sun, which are directed inward upon the sun in virtue of properly directed collisions. This constant flux appears to me to be of the essence in the suggested aspect.

The molar matter may be active in producing local disturbances which result in sun-spots and prominences.

The light-driven matter is probably intimately connected with the apparent repulsion of comets' tails.

The corona is formed and maintained by the joint action of molecular matter and light-driven matter, the former probably being involved in the arches and the latter being responsible for most of the long-drawn streamers.

Under the influence of local disturbances, regions of the sun may emit corpuscles which succeed in penetrating the lower strata of the reversing layer and cause higher strata to glow as if their effective temperature were higher than lower strata, and thus bright flocculi arise.

The difference between stars of the first and second type might be referred partly to the difference in the relation between the critical envelope and the photosphere, as conditioned by the mass and density of the nucleus within the photosphere, and partly to the inability of solar stars to retain hydrogen and helium (Schuster, *Astrophysical Journal*, **17**, 197, 1903).

The peculiar features of stellar spectra of third and fourth types might be viewed as the result of the accidental nature of the molecular matter circulating round them. The third-type stars would require titanium meteorites, while those of fourth type would be fed by comets.

The most difficult part of the whole matter seems to lie in accounting for the organized rotation of the vapor which we observe in the spectroscopic determination of the solar rotation. The difficulty lies in the reconciliation of the high planetary velocities that must exist in the whirl of gas, with the low value of the observed velocity on the sun's equator. It would seem certain that the survival of narrow Fraunhofer lines must be a residual phenomenon.

If the sun's mass were to increase by 4×10^9 grams per second for a million years, the period of the earth in its orbit (the length of the year) would be diminished by $\frac{1}{1000}$ of a second. Such an increase of mass would be brought about by a continuous stream of matter over the whole surface of the sun at a rate of about 6.6×10^{-14} grams per second per square centimeter; that is, by 30 particles (each of diameter $\lambda/10$ and sp. gr. 5.5) per second per square centimeter, or by gas of molecular weight of air streaming in with velocity 1 cm/sec. at pressure $75\times10^{-12}\times$atmospheric pressure, or by 3000×10^6 molecules of air per second per square centimeter.

No one can be more conscious than myself of the immaturity of this suggested aspect. I have tried in setting it forth to steer a course between the unintelligible by reason of brevity and the unendurable by reason of length, and the statement of the case needs must suffer. But it would seem well to submit it for what it is worth to the tender mercies of workers who may be ready to co-operate in developing the best sides of it and eliminating the bad.

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ON CERTAIN IMPLICATIONS OF POSSIBLE CHANGES IN THE FORM AND DIMENSIONS OF THE SUN, AND SOME SUGGESTIONS TOWARD EXPLAINING CERTAIN PHENOMENA OF VARIABLE STARS[1]

By F. R. MOULTON

The shape of the sun and the possibility of changes in its dimensions have been matters of keen interest to astronomers for about a century. Among those who have contributed to the discussion of these questions we may mention Von Lindenau (1809), Bianchi (1831), Secchi (1871), Auwers (1873 and 1895), Newcomb and Holden (1874), Ambronn (1905), and Poor (1908). In the last of Poor's papers[2] there are full references and complete abstracts of all the most important discussions of the subject. It is sufficient to state here that up to Auwers' work in 1885 the observations were obtained with meridian instruments, that the later observations of Auwers and those of Ambronn were made with the heliometer, and that Poor's work, aside from a rediscussion of the heliometer measures of Auwers and Ambronn, was based upon twenty-two Rutherford plates taken in the years 1870–72, nine Northfield plates taken in 1893–94, and six Yerkes Observatory plates taken in 1907. In using these photographic plates, Poor limited his discussion to the question of the differences in the equatorial and polar diameters.

[1] This paper was prepared by the author as a research Associate of the Carnegie Institution. Since it was written Professor E. B. Frost called the writer's attention to the fact that Ritter had treated certain phases of the problems considered here, in his celebrated memoirs in *Wiedemanns Annalen*, Vols. 5-20 (1878–83). The methods employed by Ritter are altogether different from those used here. It will be noted below where the results are common.

[2] "An Investigation of the Figure of the Sun and of Possible Variations in Its Size and Shape," *Annals of the N. Y. Acad. Sci.*, **18**, No. 9, Part III. 385–424, 1908.

The results obtained in the various investigations have been most discordant. Von Lindenau reached the conclusion that there are periodic variations in the sun's diameter; from the same observation Auwers came to the opposite conclusion; Secchi found the diameter varying inversely with the number of sun-spots; Auwers, from the same data, found no variations; Newcomb and Holden reached the result that there is no long-term variation; in his first discussion of the heliometer observations Auwers found that the diameter varies directly as the number of sun-spots, and in a second discussion that the apparent variation was due to varying personal equations; and Ambronn was convinced that during the interval of thirteen years covered by his observations there was no periodic or secular change in the diameter of the sun exceeding 0."1.

The conclusions respecting the shape of the sun were equally varied, the polar diameter being found a little greater than the equatorial about as often as the opposite. Poor infers from all the evidence that "the exact shape of the sun is not known with certainty;" and that the difference in the polar and equatorial radii is probably not more than 0."25. He states that the heliometer observations show a fluctuation in the size of the sun of probably not over 0."10. Ambronn thinks there is no observational evidence of changes in its dimensions.[1]

In scientific work there is rightly no higher evidence than the observational. The facts derived from observations lead to theories and sometimes overthrow them. Yet theories and theoretical work may often guide observations, as in the case of the discovery of *Neptune*, or reveal inconsistencies and errors in them. At the present time theories are most apt to be of service in subjects of a dynamical character, for the foundations of mechanics are laid deep in an enormous mass of experience, and the fundamental principles of it have been shown to be consistent with the phenomena of the physical universe with a thoroughness and degree of accuracy approached in no other part of science. Now the questions of the form of the sun and the possible variations in its dimensions are of a dynamical nature, and may well be considered from the dynamical standpoint.

In considering the shape of the sun we may inquire:

I. What the character of the figure of equilibrium of such a mass is.

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[1] *Astrophysical Journal*, **23**, 344, 1906.

II. The extent of its deviation from sphericity.

III. The character and periods of the oscillations it may undergo if its equilibrium is disturbed.

In considering the possibility of changes in the dimensions and shape of the sun we may inquire:

IV. The effect upon its rate of rotation;

V. The effect upon its energy of rotation;

VI. The effect upon its potential energy;

VII. The effects upon its temperature and rate of radiation.

VIII. Whether the results may not be used to explain some of the phenomena of variable stars of certain types.

We shall consider these questions briefly in this order.

I. THE FIGURE OF THE SUN

If the sun were a homogeneous liquid its figure of equilibrium for steady rotation (omitting certain theoretical figures having, for its density and rate of rotation, quite different shapes) would be strictly an oblate spheroid whose degree of oblateness would depend upon its density and rate of rotation. It is practically certain that the density of the sun increases below its surface, at least as far as it remains approximately gaseous in character, and the increase in density may continue even to its center. If this central condensation were slight the form of equilibrium would remain nearly an oblate spheroid. It is known from a consideration of the limiting case of an atmosphere of vanishing mass surrounding a rigid spherical center, and from the investigations of Clairaut,[1] that a heterogeneous body of this character will be less oblate than a homogeneous mass of the same mean density and having the same rate of rotation. This is verified by actual observations in the case of the earth, *Jupiter*, and *Saturn*, the difference being very marked in the case of *Saturn*.

On the other hand, a prolate spheroid is not a figure of equilibrium for the homogeneous fluid. Neither is it a figure of equilibrium for a fluid mass whose density increases continuously toward its center of gravity. Therefore, if the sun is a figure of equilibrium we are justified in concluding that it is oblate and approximately spheroidal in form, and that its oblateness is less than that of the corresponding homogeneous mass.

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[1] Tisserand, *Mécanique céleste*, 2, chap. xiii.

II. THE OBLATENESS OF THE SUN

The relation between the rate of rotation, density, and oblateness of the homogeneous spheroid is the well-known equation[1]

$$\frac{\omega^2}{2\pi k^2\sigma}=\frac{1}{\lambda^2}\left\{(3+\lambda^2)\frac{\tan^{-1}\lambda}{\lambda}-3\right\}, \tag{1}$$

where ω is is the angular rate of rotation, k^2 the gravitational constant, and σ the density. Let a represent the equatorial radius, b the polar radius, and then λ is defined by $a^2=b^2(1+\lambda^2)$. Let $\frac{\omega^2}{2\pi k^2\sigma}=\eta$ and expand the right member. Then we get

$$\eta=\tfrac{4}{15}\lambda^2-\tfrac{8}{35}\lambda^4+\tfrac{4}{21}\lambda^6\ldots \tag{1'}$$

Inverting this series we have

$$\lambda^2=\tfrac{15}{4}\eta+\tfrac{6}{7}\left(\tfrac{15}{4}\right)^2\eta^2+\tfrac{37}{49}\left(\tfrac{15}{4}\right)^3\eta^3\ldots, \tag{2}$$

a convenient formula for computing λ^2 when η is small.

Taking the mean solar day as the unit of time, the mass of the sun as the unit of mass, the mean distance from the earth to the sun as the unit of distance, the period of the sun's rotation as 25.3 days, and its density as 1.41 times that of water, we find in the case of the sun

$$\eta=2\times10^{-5}.$$

Hence equation (2) gives

$$\lambda^2=75\times10^{-6}.$$

If we let ϵ represent the ellipticity we have

$$\epsilon=\frac{a-b}{a}=1-\frac{1}{\sqrt{1+\lambda^2}}=\frac{\lambda^2}{2}-\tfrac{3}{8}\lambda^4\ldots=37\times10^{-6}.$$

The actual ellipticity of the sun must be less than this quantity. The difference in the equatorial and polar diameters of the sun must be less than

$$2a-2b=2b(\sqrt{1+\lambda^2}-1)=b(\lambda^2-\tfrac{1}{4}\lambda^4\ldots). \tag{3}$$

Taking $2b$, as seen from the earth, equal to 1920″ we find that the difference in equatorial and polar diameters of the sun must be less than 0″.07 as seen from the earth. Even if only the order of this result is correct, the quantity in question is beyond observation with present means.

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[1] Tisserand, *op. cit.*, chap. vi.

III. OSCILLATIONS OF THE SUN

Consider an incompressible homogeneous liquid in a stable figure of equilibrium. For the moment we shall neglect the rotation and suppose the figure is a sphere. If it is deformed slightly and left to its own gravitation it will oscillate around the spherical shape, the amplitude of the oscillation depending upon the extent of the original deformation, and its period upon the character of the deformation. It is a well-known theorem that any slight deformation of a sphere may be represented by a converging series of spherical harmonics. The period of oscillation of each harmonic term will depend upon the order of the harmonic. The problem of determining the characteristics of this sort of gravitational oscillations of a sphere was first solved by Lord Kelvin,[1] and he showed that the period of oscillation of the harmonic of order n is

$$T_n = 2\pi\sqrt{\frac{2n+1}{2n(n-1)}}\sqrt{\frac{a}{g}}\,, \tag{4}$$

where a is the radius and g the surface-gravity. Since g is directly proportional to the density and first power of the radius it follows that T_n depends only upon the order of the harmonic and the inverse one-half power of the density.

The first value of n having physical application to the problem under consideration is $n=2$, and the period of the corresponding term is

$$T_2 = 2\pi\sqrt{\tfrac{5}{4}}\sqrt{\frac{a}{g}} = 3^{\mathrm{h}}\ 8^{\mathrm{m}} \tag{5}$$

for a homogeneous mass having the dimensions and mean density of the sun. The higher the order of the harmonic the shorter the period of oscillation.

In considering the oscillations of a viscous spheroid Professor Lamb has shown that the time it takes the amplitude of a harmonic oscillation to decrease to $\frac{1}{e} = 0.37\ \ldots$ of its original value is[2]

$$\tau = \frac{1}{(n-1)(2n+1)}\cdot\frac{a^2}{\nu}\,, \tag{6}$$

where $\nu = \frac{\mu}{\sigma}$, σ being the density and μ the coefficient of viscosity.

[1] *Phil. Trans.*, **153**, 612, 1863.

[2] *Proceedings of the London Math. Soc.*, **13**, 61, 1881–82.

For water at moderate temperatures $\nu = 0.014$; then, taking $n = 2$, as in applying (4), equation (6) gives for a body having the size and mass of the sun $\tau = 22 \times 10^{14}$ years. The rate at which friction destroys the oscillations increases as the order of the harmonic increases. In a body having the size and mass of the sun a gravitational oscillation represented by a harmonic of low order, once started, will persist for an extremely long time.

But these results are not applicable to the sun without modification. In the first place the sun is undoubtedly much condensed toward its center, and this will have an effect on the period of the gravitational oscillations. Because of the increased mean potential the period is decreased by this factor. In the second place the material is compressible and probably highly elastic as regards compressibility. This factor probably will also somewhat decrease the period of an oscillation.[1] These considerations lead to the conclusion that if the sun is undergoing oscillations, their periods are only a few hours, though separate oscillations of different periods may unite at long intervals like "beats" in musical notes. It is not improbable that the sun is yet undergoing slight oscillations started by some ancient disturbance.

IV. EFFECT OF OSCILLATIONS UPON THE RATE OF ROTATION

The hypothesis is explicitly made here that the sun is not now subject to sensible external disturbances. Hence all changes in its dimensions and rotation must come from internal forces, and it follows that its moment of momentum will be constant. The formula for the moment of momentum, M, is

$$M = CmA^2\omega ,$$

where m is the mass, A one of the linear dimensions, ω the angular rate of rotation, and C a constant depending upon the shape of m, the definition of A, and the distribution of density. For simplicity we shall suppose that the sun is always homogeneous, though it is almost certainly densest far below its surface. We shall suppose also that its axis of rotation is always an axis of symmetry. The general character of the results cannot be changed by this assumption. Then, whatever may be its shape at any time, the distance, r, from its center of gravity to its surface can be expressed uniquely in the form

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[1] In Emden's *Gaskugeln*, which was received after this paper was in type, the computation is made (p. 452) under a number of simplifying assumptions, and the period of oscillation for the sun is found to be two hours.

$$r=a[1+p_0+p_1P_1(\phi)+p_2P_2(\phi)+\ldots], \tag{7}$$

where ϕ is the colatitude, $P_n(\phi)$ Legendre's coefficient of the nth order, and p_n numbers depending upon its shape. If the mass changes in shape, the $P_n(\phi)$ remain fixed while the p_n vary. In particular, if the mass undergoes periodic oscillations the p_n vary periodically, and, so far as first-order disturbances are concerned, independently and with different periods. A change in p_0 implies a simple spherical dilatation,[1] while a change in the other p_n involves a change in shape as well as in volume. For example, the sphere modified by the term depending on p_2 is prolate or oblate according as p_2 is positive or negative, while the term depending on p_3 deforms the sphere into an egg-shaped figure. The periods of the oscillations depend upon many uncertain factors such as the elasticity of the sun, the distribution of its density and temperature, and the effects of the enormous internal stresses to which its matter is subject. On this account it is difficult, if not impossible, to determine the periods of the p_n in the actual case with any degree of certainty. But the exact periods are not important in the present connection. The work of Kelvin gives their general order of magnitude.[2]

Since the origin has been taken at the center of gravity we have

$$\int\rho\cos\phi dm=\sigma\int_0^{2\pi}\int_0^{\pi}\int_0^{r}\rho^3\cos\phi\sin\phi d\rho d\phi d\theta=0, \tag{8}$$

where ρ is the distance from the center of gravity to the element of mass dm. Integrating with respect to ρ, substituting for r from equation (7), and making use of the properties of the integrals of spherical harmonics, we have up to terms of the order $p_2 p_3$,

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[1] Ritter treated infinitesimal dilatational oscillations under the hypotheses that the mass is always spherical and homogeneous, and that it obeys the laws of gases in its expansion and contraction, *op. cit.*, **8**, § 25, 174 (1879); and finite oscillations under the same hypotheses, *op. cit.*, **13**, § 41 (1881). His analysis did not cover the more important oscillations specified by p_1P_1, p_2P_2, ...

[2] Under his hypotheses, Ritter found for the sun $T=5$h. 50m. *op. cit.*, **8**, 178, (1879). The dilatational oscillations were not included in Kelvin's analysis, for he treated only incompressible fluids.

$$p_1 = -\tfrac{27}{35} p_2 p_3 \cdot \cdot \cdot \cdot \qquad (9)$$

Therefore p_1 is small relatively to p_2 and p_3 and it may be neglected. As was seen from equation (6) the terms depending upon harmonics of higher orders are most quickly reduced by friction. Therefore it will be unnecessary to go to terms of high order.

Since the continuity condition of incompressible fluids has not been imposed, the volume and mean density will in general change with the shape as well as with the dilatation expressed through p_0. Letting v represent the volume, we have

$$v = \frac{1}{\sigma}\int dm = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho^2 \sin\phi d\rho d\phi d\theta \ ,$$

which becomes after integrating, as above,

$$v = \tfrac{4}{3}\pi a^3 (1+p_0)^3 \left[1 + \frac{3p_2^2}{5(1+p_0)^2} + \frac{3p_3^2}{7(1+p_0)^2} + \frac{2p_2^3}{35(1+p_0)^3} + \ldots \right] . \qquad (10)$$

We shall now compute the moment of inertia of this mass with respect to its axis of symmetry, which we shall assume is also its axis of rotation. It is defined by the equation

$$I = \frac{M}{\omega} = \sigma \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho^4 \sin^3\phi d\rho d\phi d\theta \ .$$

Making use of the fact that $\sin^2\phi = \frac{2}{3}(1-P_2)$ and carrying out the integration as in the preceding cases, we find

$$\frac{M}{\omega} = \tfrac{8}{15}\pi\sigma a^5 (1+p_0)^5 \left[1 - \frac{p_2}{1+p_0} + \frac{2p_2^2}{(1+p_0)^2} + \tfrac{10}{7}\frac{p_3^2}{(1+p_0)^2} + \ldots \right] . \qquad (11)$$

Making use of (10) this equation becomes

$$M = \tfrac{2}{5} m a^2 (1+p_0)^2 \left[1 - \frac{p_2}{1+p_0} + \tfrac{7}{5}\frac{p_2^2}{(1+p_0)^2} + \frac{p_3^2}{(1+p_0)^2} + \ldots \right] \omega ,$$

or, with sufficient accuracy when p_0, p_2, . . . are small,

$$M = \tfrac{2}{5} m a^2 [1 + 2p_0 + p_0^2 - p_2 - p_0 p_2 + \tfrac{7}{5} p_2^2 + p_3^2 + \ldots] \omega \ . \qquad (12)$$

In astronomical units for the sun, $m = 1$, $a = 466 \times 10^{-5}$, $\omega = \frac{2\pi}{25 \cdot 3}$, and p_0, p_2, . . . are very small. Hence equation (12) gives $M = 216 \times 10^{-8}$ for the sun.

Oscillations in the sun either in dimensions or in form will change its rate of rotation. We may inquire whether the possible direct changes in dimensions and form, or the consequences of these changes upon the rate of rotation, may the more easily be observed. We note immediately that the gravitational and elastic oscillations have short periods, and that there are no permanent markings on the sun from which its period can be accurately determined. Consequently we see that this indirect process is probably doomed to failure. But we may still inquire whether the possible secular shrinking may not more quickly be detected by observations of the acceleration of its rotation than by direct measurements of its dimensions. If we represent its period by P, we have from (12), when $p_0 = p_2 = \ldots = 0$

$$P = \frac{2\pi}{\omega} = \frac{4\pi m a^2}{5M} \, .$$

Suppose the radius shrinks the fraction of its diameter p_0. Then the period of rotation becomes

$$P' = \frac{4\pi m a^2 (1 - p_0)^2}{5M} \, .$$

Suppose the shrinkage in diameter is 1″.0. Then

$$p_0 = \tfrac{1}{1920}$$

and we have

$$P' - P = -\frac{8\pi m a^2}{5M} p_0 \left(1 - \frac{p_0}{2}\right) = -0.0265\text{d.} = -38 \text{ minutes.} \tag{13}$$

That is, the diminution of period is so small that it could not be determined with certainty, and we conclude that the change in diameter will be observed before the change in the rate of rotation.

V. EFFECTS OF OSCILLATIONS OF THE SUN UPON ITS ENERGY OF ROTATION[1]

The rotational energy of the sun is

$$E = \tfrac{1}{2} I \omega^2 = \tfrac{1}{2} M \omega \, . \tag{14}$$

As a consequence of (12) this reduces to

$$E = \frac{5M^2}{2ma^2} [1 - 2p_0 + 3p_0^2 + p_2 - 3p_0 p_2 - \tfrac{7}{5} p_2^2 - p_3^2 \ldots] \, . \tag{15}$$

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[1] Ritter recognized this factor, **13**, 370, but made no discussion of its importance, even in the case of simple expansion and contraction which he considered.

The sun is rotating so slowly that the changes in its energy of rotation produced by any admissible oscillations will be found in the applications which follow to be relatively unimportant.

VI. EFFECTS OF OSCILLATIONS OF THE SUN UPON ITS SELF-POTENTIAL

We shall now compute the potential of the whole mass upon itself. Representing this potential by W we have

$$W=\tfrac{1}{2}k^2\int dm\int\frac{dm'}{\Delta}\,, \tag{16}$$

where dm and dm' are any two distinct elements of mass, Δ the distance between them, and where the integrals must separately be extended over the entire body. We shall first compute

$$V=\int\frac{dm'}{\Delta}\,. \tag{17}$$

Let the co-ordinates of dm' be ρ', ϕ', θ' and those of dm be ρ, ϕ, θ. Then

$$\left.\begin{aligned} dm'&=\sigma\rho'^2\sin\phi' d\rho' d\phi' d\theta',\\ \Delta^2&=\rho^2+\rho'^2-2\rho\rho'\cos\gamma,\ \text{where}\\ \cos\gamma&=\cos\phi\cos\phi'+\sin\phi\sin\phi'\cos(\theta-\theta')\,. \end{aligned}\right\} \tag{18}$$

Then we have

$$\left.\begin{aligned} V&=V_1+V_2,\ \text{where}\\ V_1&=\sigma\int_0^{2\pi}\int_0^{\pi}\int_0^{\rho}\frac{\rho'^2\sin\phi'}{\Delta}d\rho' d\phi' d\theta',\\ V_2&=\sigma\int_0^{2\pi}\int_0^{\pi}\int_{\rho}^{r}\frac{\rho'^2\sin\phi'}{\Delta}d\rho' d\phi' d\theta'\,. \end{aligned}\right\} \tag{19}$$

In the region of integration covered in V_1 we have

$$\Delta^{-1}=\frac{1}{\rho}\left[1+\frac{\rho'}{\rho}Y_1+\left(\frac{\rho'}{\rho}\right)^2 Y_2+\ldots\right], \tag{20}$$

and in the region of integration covered in V_2,

$$\Delta^{-1}=\frac{1}{\rho'}\left[1+\frac{\rho}{\rho'}Y_1+\left(\frac{\rho}{\rho'}\right)^2 Y_2+\ldots\right], \tag{21}$$

where $Y_n\ (\phi,\phi',\theta,\theta')$ is Laplace's coefficient of the nth order. Consequently

$$V_1=\sigma\int_0^{2\pi}\int_0^{\pi}\rho^2\left[\tfrac{1}{3}+\frac{Y_1}{4}+\ldots+\frac{Y_n}{n+3}+\ldots\right]\sin\phi' d\phi' d\theta',$$

which, by the properties of the integrals of spherical harmonics, becomes

$$V_1 = \tfrac{1}{3}\pi\sigma\rho^2. \tag{22}$$

Similarly V_2 becomes

$$V_2 = \sigma\int_0^{2\pi}\int_0^{\pi}\left[\frac{\rho'^2}{2} + \rho\rho' Y_1 + \rho^2 \log \rho' Y_2 + \ldots \right.$$
$$\left. + \frac{\rho^n Y_n}{(-n+2)\rho'^{n-2}} + \ldots\right]_\rho^r \sin \phi' d\phi' d\theta'.$$

Substituting the limits, this equation may be written

$$\left.\begin{aligned}
&V_2 = V_2^{(1)} + V_2^{(2)}, \text{ where}\\
&V_2^{(1)} = -\sigma\int_0^{2\pi}\int_0^{\pi}\left[\frac{\rho^2}{2} + \rho^2 Y_1 + \rho^2 \log \rho Y_2 + \ldots\right.\\
&\qquad\qquad\left. + \frac{\rho^2}{-n+2} Y_n + \ldots\right] \sin \phi' d\phi' d\theta',\\
&V_2^{(2)} = \sigma\int_0^{2\pi}\int_0^{\pi}\left[\frac{r^2}{2} + \rho r Y_1 + \rho^2 \log r Y_2 + \ldots\right.\\
&\qquad\qquad\left. + \frac{\rho^n r^{-n+2}}{-n+2} Y_n + \ldots\right] \sin \phi' d\phi' d\theta'.
\end{aligned}\right\} \tag{23}$$

We find at once

$$V_2^{(1)} = -2\pi\sigma\rho^2. \tag{24}$$

The value of r from (15) must be substituted in the expression for $V_2^{(2)}$. Making use of the general equations

$$\int_0^{2\pi}\int_0^{\pi} P_{n_1}(\phi') Y_{n_2}(\phi, \phi', \theta, \theta') \sin \phi' d\phi' d\theta' = 0, \; n_1 \mp n_2,$$

$$\int_0^{2\pi}\int_0^{\pi} P_n(\phi') Y_n(\phi, \phi', \theta, \theta') \sin \phi' d\phi' d\theta' = \frac{4\pi}{2n+1} P_n(\phi),$$

$$\log r = \log a(1+p_0) + \sum_{i=1}^{\infty}(-1)^{i+1}\left[\frac{p_2}{1+p_0}P_2 + \frac{p_3}{1+p_0}P_3 + \ldots\right]^i,$$

we get

$$V_2^{(2)} = A + B + C + D,$$

where

$$A = \frac{\sigma}{2}a^2(1+p_0)^2\int_0^{2\pi}\int_0^{\pi}\left[1 + \frac{p_2}{1+p_0}P_2 + \frac{p_3}{1+p_0}P_3 + \ldots\right]^2 \sin \phi' d\phi' d\theta'$$

$$= 2\pi\sigma a^2(1+p_0)^2\left[1 + \frac{p_2^2}{5(1+p_0)^2} + \frac{p_3^2}{7(1+p_0)^2} + \ldots\right],$$

$$B = \sigma\rho a(1+p_0)\int_0^{2\pi}\int_0^{\pi}\left[1 + \frac{p_2}{1+p_0}P_2 + \ldots\right] Y_1 \sin \phi' d\phi' d\theta'$$

$= 0$ except for terms involving p_1,

$$C=\sigma\rho^2\int_0^{2\pi}\int_0^{\pi}\left\{\log a(1+p_0)+\sum(-1)^{i+1}\left[\frac{p_2}{1+p_0}P_2+\frac{p_3}{1+p_0}P_3+\ldots\right]^i\right\}Y_2\sin\phi' d\phi' d\theta'$$

$$=\frac{4\pi}{5}\sigma\rho^2\frac{p_2}{1+p_0}P_2+\ldots,$$

$$D=\frac{-\sigma\rho^n}{(n-2)a^{n-2}(1+p_0)^{n-2}}\int_0^{2\pi}\int_0^{\pi}\left[1+\frac{p_2}{1+p_0}P_2+\frac{p_3}{1+p_0}P_3+\ldots+\frac{pn}{1+p_0}P_n+\ldots\right]^{-n+2}Y_n\sin\phi' d\phi' d\theta',\ n\geqq 3,$$

$$=\frac{4\pi\sigma\rho^n}{(2n+1)a^{n-1}(1+p_0)^{n-1}}p_nP_n(\phi)+\ldots,\ n\geqq 3.$$

Collecting results, we have finally

$$\left.\begin{aligned}V=&-\tfrac{2}{3}\pi\sigma\rho^2+2\pi\sigma a^2(1+p_0)^2\left[1+\frac{p_2^2}{5(1+p_0)^2}+\frac{p_3^2}{7(1+p_0)^2}+\ldots\right]\\&+\frac{4\pi}{5}\sigma\rho^2\frac{p_2}{1+p_0}P_2(\phi)+\frac{4\pi}{7}\frac{\sigma\rho^3}{a(1+p_0)}\frac{p_3}{1+p_0}P_3(\phi)+\ldots.\end{aligned}\right\}\quad(25)$$

Therefore (18) becomes

$$W=\tfrac{1}{2}k^2\int V dm=\tfrac{1}{2}k^2\sigma\int_0^{2\pi}\int_0^{\pi}\int_0^{r}V\rho^2\sin\phi d\rho d\phi d\theta.$$

Substituting the value of V from (25), performing the integration with respect to ρ, making use of (7), and completing the integration, we find

$$W=\tfrac{16}{15}\pi^2k^2\sigma^2a^5(1+p_0)^5\left[1+\tfrac{4}{5}\frac{p_2^2}{(1+p_0)^2}+\tfrac{25}{49}\frac{p_3^2}{(1+p_0)^2}+\ldots\right].\quad(26)$$

Since we are supposing that the density is uniform we have, making use of (10),

$$W=\tfrac{3}{5}k^2\frac{m^2}{a(1+p_0)}\left[1-\tfrac{2}{5}\frac{p_2^2}{(1+p_0)^2}-\tfrac{17}{49}\frac{p_3^2}{(1+p_0)^2}\ldots\right],$$

or, with sufficient approximation,

$$W=\tfrac{3}{5}k^2\frac{m^2}{a}[1-p_0-\tfrac{2}{5}p_2^2-\tfrac{17}{49}p_3^2\ldots].\quad(27)$$

The first term of this expression is, of course, the potential of a sphere upon itself with respect to infinite dispersion as the standard of reference.[1]

VII. EFFECTS OF OSCILLATIONS OF THE SUN UPON ITS TEMPERATURE AND RATE OF RADIATION

Let T represent the absolute temperature of the sun, s its specific heat, and H its rate of radiation. For simplicity we shall suppose T and s are uniform throughout the whole mass, and that s is constant with respect to the time. Let K represent the kinetic energy of the oscillations. Consider any two epochs t_1 and t_2 and let the various quantities at these epochs be distinguished by the corresponding subscripts. Then we have by the law of the conservation of energy

$$E_1-W_1+msT_1+K_1=E_2-W_2+msT_2+K_2+\int_{t_1}^{t_2}Hdt\,. \tag{28}$$

Dividing by t_2-t_1 and passing to the limit as $t_2-t_1 \doteq 0$, and then making use of equations (15) and (27), we get

$$\begin{aligned}\frac{dT}{dt}=\tfrac{3}{5}\frac{k^2m}{a}\left\{-\frac{dp_0}{dt}-\tfrac{2}{5}\frac{dp_2^2}{dt}-\tfrac{17}{49}\frac{dp_3^2}{dt}\ldots\right\}&\\ +\frac{5M^2}{2sm^2a^2}\left\{2\frac{dp_0}{dt}-3\frac{dp_0^2}{dt}-\frac{dp_2}{dt}\ldots\right\}-\frac{1}{ms}\frac{dK}{dt}-\frac{1}{ms}H\,.&\end{aligned} \tag{29}$$

In order to treat this equation we must express H in terms of T. Stefan's law of radiation for a black body is that H is proportional to the product of the radiating surface and the fourth power of the absolute temperature. While it is not to be supposed that this law applies strictly to a star, still it seems probable that it will give the order of the true results. Hence we may write

$$H=cST^4\,, \tag{30}$$

where c is a constant depending upon the units, and S is the whole surface of the radiating body.

If we represent an element of the surface by ds, we have

$$S=\int ds=\int_0^{2\pi}\int_0^{\pi}r\sqrt{r^2+\left(\frac{dr}{d\phi}\right)^2}\sin\phi d\phi d\theta\,. \tag{31}$$

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[1] Moulton, *Introduction to Celestial Mechanics*, pp. 37 ff. (49).

By means of equation (7), and the general formula

$$\frac{dP_n}{d\phi}=-[(2n-1)P_{n-1}+(2n-5)P_{n-3}+(2n-9)P_{n-5}+\dots]\sin\phi\,,$$

equation (31) becomes

$$\begin{aligned}S=\frac{a^2}{(1+p_0)^2}\int_0^{2\pi}\int_0^{\pi}\Big\{&1+\frac{2p_2}{1+p_0}P_2+\frac{2p_3}{1+p_0}P_3+\frac{p_2^2}{2(1+p_0)^2}[2P_2^2+9P_1^2\sin^2\phi]\\&+\frac{p_2p_3}{(1+p_0)^2}[2P_2P_3+3P_1\,(1+5P_2)\sin^2\phi]+\\&\tfrac{1}{2}\frac{p_3^2}{(1+p_0)^2}[2P_3^2+(1+5P_2)^2\sin^2\phi]+\dots\Big\}\sin\phi d\phi d\theta\,.\end{aligned}\tag{32}$$

Making use of the fact that $\sin^2\phi=\frac{2}{3}(1-P_2)$and of the properties of the integrals of spherical harmonics, this equation reduces to

$$S=4\pi a^2\{1-2p_0+\tfrac{4}{5}p_2^2+p_3^2+\dots\}\,.\tag{33}$$

Substituting (30) and (33) in (29), we have

$$\begin{aligned}\frac{dT}{dt}=\tfrac{3}{5}\frac{k^2m}{sa}&\Big\{-\frac{dp_0}{dt}-\tfrac{2}{5}\frac{dp_2^2}{dt}-\tfrac{17}{49}\frac{dp_3^2}{dt}\dots\Big\}\\&+\frac{5M^2}{2sm^2a^2}\Big\{2\frac{dp_0}{dt}-3\frac{dp_0^2}{dt}-\frac{dp_2}{dt}\dots\Big\}\\&-\frac{1}{ms}\frac{dK}{dt}-\frac{4\pi a^2c}{ms}\Big\{1-2p_0+\tfrac{4}{5}p_2^2+p_3^2\dots\Big\}T^4\,.\end{aligned}\tag{34}$$

In a body like the sun c is very small. Hence for intervals of not more than a few weeks T may be integrated as a converging series of the form

$$T=u_0+u_1c+u_2c^2+\dots.\tag{35}$$

Substituting (35) in (34), and integrating, we find

$$\begin{aligned}u_0=T_0+\tfrac{3}{5}\frac{k^2m}{sa}&\Big\{-[p_0(t)-p_0(t_0)]-\tfrac{2}{5}[p_2^2(t)-p_2^2(t_0)]-\tfrac{17}{49}[p_3^2(t)-p_3^2(t_0)]+\dots\Big\}\\&+\frac{5M^2}{2sm^2a^2}\Big\{2[p_0(t)-p_0(t_0)]-[p_2(t)-p_2(t_0)]-\frac{1}{ms}[K(t)-K(t_0)]\Big\}\,,\end{aligned}\tag{36}$$

$$u_1=-\frac{4\pi a^2}{sm}\int_{t_0}^{t}\Big\{1-2p_0(t)+\tfrac{4}{5}p_2^2(t)+p_3^2(t)+\dots\Big\}u_0^4dt\,,$$

$$u_2=-\frac{16\pi a^2}{sm}\int_{t_0}^{t}\Big\{1-2p_0(t)+\tfrac{4}{5}p_2^2(t)+p_3^2(t)+\dots\Big\}u_0^3u_1dt$$

where T_0 is the temperature at $t=t_0$. Equation (30) becomes

$$H=cS[u_0+u_1c+\dots]^4\,.\tag{37}$$

We shall apply these equations to the sun for those short-period oscillations which alone can exist. There may be, of course, a secular contraction or expansion which might be accounted for through p_0. Since the present radiation of the sun could lower its temperature at the most only a very few degrees in a year, we may for intervals of not more than a few days regard T as equal to u_0. As was seen following equation (12) $\frac{M^2}{m^2}$ for the sun is very small relatively to the second term of u_0, and the term involving this factor may be neglected. Then the expression for T becomes with sufficient approximation for applications to the sun

$$T=T_0+\tfrac{3}{5}\frac{k^2m}{sa}\Big\{-[p_0(t)-p_0(t_0)]-\tfrac{3}{5}[p_2^2(t)-p_2^2(t_0)]$$
$$-\tfrac{17}{49}[p_3^2(t)-p_3^2(t_0)]+\ldots\Big\}-\frac{1}{ms}[K(t)-K(t_0)]\,, \qquad (38)$$

and the rate of radiation is[1]

$$H=4\pi a^2c\Big\{T_0+\left[-\tfrac{3}{5}\frac{k^2m}{sa}+\frac{T_0}{2}\right][p_0(t)-p_0(t_0)]+$$
$$\tfrac{1}{5}\left[-\tfrac{6}{5}\frac{k^2m}{sa}+T_0\right][p_2^2(t)-p_2^2(t_0)]+\left[\frac{-51}{245}\,\frac{k^2m}{sa}+\frac{T_0}{4}\right][p_3^2(t)-p_3^2(t_0)]+$$
$$\ldots-\frac{1}{ms}[K(t)-K(t_0)]\Big\}^4. \qquad (39)$$

We shall now reduce these equations to numbers. For this purpose we shall select as units of mass, distance, and time, the kilogram, meter, and second, respectively. We shall suppose that s has the high value unity. (A smaller value would be more favorable to our most important conclusion.) Letting r_E and m_E represent the radius and mass of the earth, and g the value of gravity at its surface, we have

$$g=9.80\,, \qquad \frac{m}{m_E}=324{,}439\,,$$
$$r_E=6{,}371{,}000\,, \qquad a=692{,}428{,}000\,,$$
$$k^2=g\frac{r_E^2}{m_E}\,, \qquad \text{Calory}=425g\,.$$

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[1] Ritter gave the formula for the dilatational term, *op. cit.*, 13, 367.

Consequently equation (38) gives, since for $s=1$ the number of calories of heat per unit mass equals the temperature,

$$T=T_0+27\times10^6\left\{-[p_0(t)-p_0(t_0)]-\tfrac{2}{5}[p_2^2(t)-p_2^2(t_0)]-\right.$$
$$\left.\tfrac{17}{49}[p_3^2(t)-p_3^2(t_0)]+\ldots\right\}-\frac{K(t)-K(t_0)}{ms}. \qquad (40)$$

If the sun were denser at its center and if the law of density were preserved during the oscillations, the numerical coefficient would be larger than this obtained under the hypothesis of homogeneity.

The term p_0 is the dilatation and is expressed (see equation 7) in terms of the sun's mean radius as unity. At both its maximum and its minimum $K(t)=0$. An oscillation of this type of 0″1 in the sun's diameter means that p^0 varies over the range $\frac{1}{19,200}$, or from $-\frac{1}{38,400}$ to $+\frac{1}{38,400}$. This makes a change in T of 1400° C. That is, *a variation in the sun's apparent dimensions of 0″1, which is at the limit of direct observational determination, if not actually beyond it, would produce a change of its temperature of 1400° C.* This result is not so surprising when we remember how small an annual shrinkage in the sun will make up for its loss of radiant energy.

The term p_2 is the coefficient of the second order zonal harmonic. When it is positive (neglecting now all other terms) the mass is prolate, and when it is negative the mass is oblate. If it varies periodically, the mass varies from the prolate form to the oblate periodically, and equation (40) gives the temperature changes during these oscillations. Since $P_2(0)=1$ and $P_2(\frac{1}{2}\pi)\doteq-\frac{1}{2}$ the oscillations in the polar diameter are twice those of the equatorial. This deviation from sphericity enters in the second degree, and therefore a small value of it produces a much smaller change in T than that produced by the dilatational term. For example, if p_2 produces an extreme oscillation in the polar diameter of 2″0 as seen from the earth the change in T for the sun is only 3° C. The temperature is least when it is prolate or oblate, being equal in the two cases. The number just computed is really the upper limit of change under the hypothesis made, for at the time when $p_2=0$ the kinetic energy of oscillation is a maximum. It is impossible to compute the amount of energy which is kinetic without determining completely the circumstances of the oscillations. But in an oscillation involving considerable density changes, where the greater part of the mass moves but slightly, it will be relatively small.

Neglecting the kinetic energy of oscillation, we find that a body having the size and mass of the sun, oscillating in such a way that the maximum value of p_2 is $\frac{1}{76}$, will undergo temperature variations of 2200° C. Its greatest oblateness is only $\frac{1}{47}$, or one-fourth that of the planet *Saturn* at the present time. This temperature change would be reduced by the fact that some of the energy would be kinetic at the time the body was spherical.

Let us now obtain some idea of the upper limit of the kinetic energy at the time the mass is spherical and passing from the oblate form to the prolate. Since kinetic energy is proportional to the product of the first power of the mass and second power of the velocity, we shall obtain the largest possible kinetic energy by supposing the change of form and dimensions has been accomplished by the motion of the smallest possible amount of matter. We shall satisfy this condition if we suppose the equatorial bulge is transferred to the poles. We shall have a distance of motion greater than the greatest possible, if we suppose each particle slides along the surface from the equator entirely to the pole. In the case of the sun the distance is 108×10^7 meters. Suppose that the whole period of oscillation is 3 hours, and consequently that the motion in one direction would be made in 90 minutes. Suppose that it is uniform. Then the velocity is 2×10^5 meters per second. Since a mass falling 425 meters under the acceleration of the earth's gravity has a velocity of 91 meters per second, this velocity in a mass of specific heat unity is equivalent to a temperature of one degree. A velocity of 2×10^5 meters per second is equivalent to a temperature of $\left(\frac{2 \times 10^5}{91}\right)^2 = 48 \times 10^5$ degrees. But equation (10) shows that the mass moved is only 125×10^{-6} of the whole sun. Consequently the motion is equivalent to a temperature of 600 degrees for the whole sun. This is to be subtracted from the 2,200 degrees of variation computed above, leaving a temperature change of 1,600 degrees. While these results can lay no claim to numerical precision, they are probably correct as to order of magnitude, and if so are sufficient for present purposes.

We shall now return to the effects of the oscillations upon the rate of radiation. We shall consider first the dilatation expressed by the term p_0. From (39) we see that we must compute the ratio of

$$\left\{ T_0+\left[\tfrac{3}{5}\frac{k^2m}{sa}-\frac{T_0}{2}\right]p_0(t) \right\}^4 \text{ to } \left\{ T_0+\left[\tfrac{3}{5}\frac{k^2m}{sa}-\frac{T_0}{2}\right]p_0(t_0) \right\}^4$$

in order to obtain the percentage of change of rate of radiation. This makes it necessary to know, or to assume, the value of T_0. We shall assume that for the sun $T_0=6000°$. The term $\frac{1}{2}\,p_0T_0$, which comes in from the change in area of the radiating surface, is, in the present case, negligible in comparison with the other term. Taking $p_0(t_1)=\frac{-1}{38{,}400}$ and $p_0(t_2)=\frac{+1}{38{,}400}$ as above, and the origin of time such that $p_0(t_0)=0$, we find from (39) that

$$H_1=1.56H_0 \text{ and } H_2=0.61H_0\,.$$

This means that if the sun were undergoing a dilatational oscillation of extreme range of 0″.1 in diameter, the rate of radiation at minimum radius would be 2.56 times that at maximum radius. That is, the variation would be more than one star magnitude.

The sun is certainly undergoing no changes in rate of radiation comparable to the results just found. Consequently we conclude that it cannot be undergoing dilatational oscillations capable of being detected by direct measurements of its diameter. However, Langley and Abbot thought they had observational evidence of short-period variations of about 10 per cent.[1] If we assume that the observed fluctuations are real and caused by oscillations in the sun of the type under consideration, we find from the formulae above that the apparent variations in the sun's diameter as seen from the earth are only 0″.01.

Now suppose $p_0=0$ and that the body is undergoing an oscillation specified by p_2. As above, we shall suppose that its maximum value is $\frac{1}{10}$. The temperature change is about 1600 degrees, and the ratio of the rate of radiation at maximum to that at minimum is found to be 2.6, or slightly more than a star magnitude. To account for a variation in radiation of 10 per cent. we should have to give p_2 the value 0.0039, which in the sun would amount to a periodic oscillation in the equatorial diameter, as seen from the earth, of 7″.4.

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[1] *Astrophysical Journal*, **19**, 305, 1904.

VIII. APPLICATIONS TO VARIABLE STARS[1]

The telescope and the spectroscope prove that binary stars are very numerous. The planes of the orbits of a certain fraction of them lie so that, as seen from the earth, the two members periodically mutually eclipse each other, at least partially. Those stars in which the duration of obscuration is only a small fraction of the period of revolution are known as the *Algol* variables.

Observations of the *Algol* variables have revealed a number of important facts. These stars are all of the first spectral type. The relations between their periods and their durations of obscuration show that they are very tenuous. A remarkable fact connected with them is that in many cases their periods slowly oscillate.

There is another class of variable stars, of which δ *Cephei* is the type, having the following peculiarities: their spectra are of the solar type; their brightness changes continuously; the intervals from their maxima to their minima are much longer than those from their minima to their maxima; their spectral lines shift in their light-periods, but with such phase that they cannot be eclipsing variables; besides the chief maxima and minima they have secondary maxima and minima which are often pronounced; their periods are short (generally 3 to 10 days); and their maxima and minima are not always the same.

The variables of the β *Lyrae* type constitute another important class. Their brightness changes continuously, the maxima being divided nearly symmetrically by two unequal minima. The lines in the spectrum of β *Lyrae* shift regularly in the same period as the light-variations, but the significance of the changes is at present not certainly understood.

We shall now raise the question whether oscillations in stars of the type considered above may not be factors in their variability. That oscillations sometimes exist is not improbable. If a star suffers a violent disturbance, such as a collision with another body of planetary dimensions or larger, oscillations will be set up and will persist for a very long time. If in the birth of a binary star there are violent changes in dimensions or shape, oscillations will be produced. In fact, two stars of a binary system revolving near each other in an eccentric orbit will disturb each other in such a way as to produce oscillations. The disturbing potential is a sum of zonal harmonics, P_2, P_3, ... , like those considered in equation (7). Consequently we may reasonably conclude that close binaries may be undergoing oscillations, and it is not unreasonable to suppose that single stars sometimes do also.

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[1] Ritter made applications of his results to variable stars, *op. cit.*, **8**, § 26, 179 (1879), and **13**, § 42, 366 (1881). As has been stated, he limited his discussion to the dilatational oscillations, which are much less apt to be produced by disturbances than those specified by harmonics of the second and third orders, etc. Moreover, in their quantitative effects, the dilatational oscillations stand in a class by themselves, and the results regarding them alone might easily be misleading.

Consider an eclipse variable undergoing slight oscillations. The periods of the vibrations will depend primarily upon the density of the mass and the orders of the harmonics describing them. They will produce alternate maxima and minima with continuously changing light. These light-variations will compound with the apparent changes produced by the eclipses. If the former are relatively small they will introduce slight modifications of the eclipse light-curve. Since the periods of the oscillations will in general be quite different from that of revolution, the phases will progressively change relatively to each other. This will produce periodically changing maxima and minima, and will change the period from minimum to minimum as they move along the slopes of the light-curve. In a word, apparently some of the peculiarities of the *Algol* variables can be explained by supposing the effects of oscillations of the type considered here supplement the eclipses.

Consider a single star undergoing large oscillations such as treated above. The radiation will vary continuously. The maxima of light, occurring when it is spherical, will all be the same. (This would not be strictly true if the coefficients of the harmonics P_3, P_4, . . . were relatively sensible.) Considering the term depending on P_2 alone, the alternate minima will be more or less unequal; one will occur when the body is prolate with the expansion at the ends of an axis, and the other when it is oblate with the expansion along a great circle. They will be different both because of the difference in the shape of the body at the two times, and also because of its unsymmetrical orientation relatively to the earth. There will undoubtedly also be oscillations specified by harmonics of higher order, but they will have been more rapidly destroyed by viscosity of the mass and hence, in general, will be of secondary importance. Consequently if the magnitude of a star were made to vary by this cause alone, its chief maxima would all be about the same, it might have two chief minima, and there would be secondary maxima and minima depending upon the higher-order oscillations. The spectral lines might be periodically modified and shifted by the pressure changes.

We see from equation (38) that for an oscillation of a given fraction of a star's radius, the temperature changes are directly proportional to its mass and inversely proportional to its mean radius. Or, the temperature change is proportional to the product of the density and the square of the radius. It follows from (39) that the higher the mean temperature the less the change in radiation for a given change in temperature. Equation (4) enables us to get an idea of the order of density for an oscillation of given period. Variable stars of the δ *Cephei* type have periods of from less than one day (*R Muscae*, 21^h) to nearly 30 days (*T Monocerotis*, 27^d). Most of them are considerably under 10 days. Let us compute the density from (4) for an oscillation of type P_2 having a period of 5 days. We easily find $\sigma = 10^{-3}$ on the water standard. This is small, but the density would come out less than three times as great if we attempted to explain the variation by supposing that two stars revolving essentially in contact eclipse each other. The same ratio of densities by the two hypotheses is maintained for all periods, both depending upon the inverse square of the period. For example, the density of β *Lyrae*, period 12.9 days, comes out about 14×10^{-5} that of water, while Myers found under the eclipse theory a mean density for it less than that of our atmosphere at sea-level.[1]

If the densities obtained are suspiciously small, the eclipse hypothesis is slightly preferable to the oscillation hypothesis; but in most other respects, such as the irregularities in many of the light-curves, the secondary maxima, varying maxima and minima, varying periods, and perhaps in spectroscopic behavior, the latter seems less incredible. It has been shown above that light-variations of the observed range might be produced by very moderate oscillations. If we are entitled to any conclusion at all in the matter, it is that oscillations are probably sensible factors in the light-variations in close binaries, and that they may possibly be the sole cause of variation in some stars. We shall almost certainly be wrong if we suppose the light-variations in all variable stars are produced by any one cause.

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[1] *Astrophysical Journal*, 7, 1, 1897.

IX. SUMMARY

The problems considered in this paper are the shape of the sun, assuming that it is in a state of equilibrium of rotation, and some of the consequences of its possible oscillations.

Under the hypothesis that the sun is rotating without oscillations, we find that it must be nearly an oblate spheroid, and that the difference between its equatorial and polar diameters must be less than 0.″07 as seen from the earth.

It is well known that such a fluid mass as the sun may undergo oscillations whose amplitude and character depend upon the nature of the disturbances producing them, and whose periods depend upon the character of the oscillations, and upon its mass, density, and elasticity. Whatever the nature of the oscillations, the equation of the surface of the body can be represented as the sum of a converging series of spherical harmonics whose coefficients are periodic functions of the time. For simplicity, it is assumed that the mass always has an axis of symmetry so that its surface can be defined by zonal harmonics. It is also assumed that at every instant the mass is homogeneous throughout. With these restrictions, the surface, volume, moment of inertia, and self-potential of this mass of general variable form are computed.

The first practical question is whether a dilatational oscillation will not change the rate of rotation so that the change of period of rotation can be observed even though the change in diameter is beyond direct observation. The answer is in the negative, for a shrinkage of diameter of 1.″0, as seen from the earth, would decrease the period of rotation by only 38 minutes.

The second question is regarding the change in temperature produced by a dilatational oscillation. An oscillation of this type such that the whole change in the sun's diameter as seen from earth is only 0.″1 is, assuming the specific heat of the sun as unity, 1400° C.

It is found then, assuming Stefan's law of radiation, and that the absolute temperature of the sun at mean radius is 6000°, that a variation of temperature 700° each side of this mean would result in a variation from 61 per cent. to 156 per cent. of that at mean radius. That is, the radiation at maximum would be 2.56 times that at minimum, or a variation of little more than a star magnitude. Consequently, if the sun were oscillating in this way the changes in its rate of radiation would be observed long before those in its diameter. An extreme oscillation of 0."01 in the sun's diameter as seen from the earth would account for the variations of 10 per cent. in the solar radiation observed by Langley and Abbot.

The question of oscillations defined by zonal harmonics of the second order is considered. The potential energy is a minimum when the coefficient of this harmonic passes through zero, that is, at the instant when the body is spherical, and has two equal maxima when the body is most prolate and most oblate. But when the potential energy is a minimum the kinetic energy of the oscillation is a maximum. Consequently the range of variation of temperature cannot be exactly computed without knowing the motion of each element of mass at the critical times. Since this is an unsolved problem, a rough computation was made to determine an upper limit to the kinetic energy at the time the mass is spherical, and therefore a lower limit to the range of variation of temperature. It was found that a variation of this type such that the equatorial diameter changes by $\frac{1}{70}$ its value would cause a variation of temperature of 1600° C. At its greatest oblateness it would be relatively flattened one-fourth as much as *Saturn* is now. The radiation at maximum would be 2.6 times that at minimum, or more than a star magnitude greater. An oscillation of this type producing a change of 10 per cent. in the solar radiation would cause a variation in the sun's equatorial diameter, as seen from the earth, of 7."4.

The period of these oscillations is of interest. Kelvin's formula shows that if the sun were a perfect incompressible fluid the period of the oscillations depending upon the harmonics of the second order would be 3 hours and 8 minutes. The period is independent of the mass and varies inversely as the square root of the density. The work of Lamb shows that if the mass had the viscosity of water the amplitude of the oscillation would be reduced to 37 per cent. of its original value in 22×10^{14} years. The former of these two results is almost certainly of the right order of magnitude for the actual sun; the latter is probably much too great.

The results obtained above suggest the idea that certain classes of variable stars may owe their variability, at least partially, to oscillations. Reference is made particularly to stars whose light changes continually, such as those of which δ *Cephei* is the type, and those of the β *Lyrae* type. The spectroscope proves that in the δ *Cephei* stars the light-variations are not eclipse phenomena, and in case of β *Lyrae* the phenomena are so complicated as to suggest that eclipses are not the sole cause of the variability. If oscillations of sufficient magnitude are admitted they would explain rather satisfactorily most of the phenomena of these classes of stars. For a star of the mass and density of the sun an oscillation defined by the second-order harmonic amounting to $\frac{1}{50}$ of its radius would account for all the light-changes in these classes of stars. However, the periods of variability demand very low densities, yet greater than one-third of those required if we attempt to explain the light-variations on the eclipse theory. For a period of 5 days the density is 10^{-3} on the water standard.

The question at once arises as to the cause of the possible oscillations. In the case of a binary system having an eccentric orbit, the periodically changing disturbance of one star by the other is an efficient cause. A single star would be set vibrating by a collision with another body even of planetary mass, and by passing near another star. The fact that the sun now has a remarkable equatorial acceleration, as well as an unexplained 11-year cycle, tends to make us open-minded regarding the much simpler oscillations treated in this paper.

THE UNIVERSITY OF CHICAGO
January 20, 1909

IMPROVEMENTS AND NEW RESULTS IN SOLAR CONSTANT DETERMINATIONS[1]

By C. G. ABBOT and F. E. FOWLE, JR.

Our measurements of the "solar constant" of radiation, published as Part I of Vol. II of the *Annals of the Astrophysical Observatory of the Smithsonian Institution*, were based on bolometric observations between wave-lengths 0.387 μ and 2.428 μ of the intensity of the solar spectrum of a 60° flint-glass prism. In order to connect the bolometric intensities with a fixed scale of energy, we summed up the total area included within the spectral energy-curve, first applying certain corrections for instrumental absorption, and for the small amounts of solar radiation which reach the earth's surface, having respectively less and greater wave-lengths than those observed. These corrected areas we made proportional to pyrheliometer readings obtained simultaneously with the bolometric observations. A full description of the process employed will be found in chap. iii of Part I of the *Annals*.

INFLUENCE OF EXTRA-ATMOSPHERIC ENERGY-CURVE ON THE "SOLAR CONSTANT"

One by-product of the research was a determination of the form of the solar spectrum energy-curve on the normal or wave-length scale, as it would be outside the atmosphere. We made use of the corresponding curve on the prismatic scale in our reductions, but the discussion of sources of error made it clear that no very high degree of accuracy in the form of it was required for our purposes. In further illustration of this point we now give the results of two independent computations of the "solar constant" from observations of September 15, 1908, reduced with the two following widely different assumptions as to the form of the prismatic extra-atmospheric energy-curve. The figures give the intensities for the mean form of 1905–1906 and for the form determined from the observations of September 15, 1908.

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[1] Published by permission of the secretary of the Smithsonian Institution.

Wave-Lengths	$0^{\mu}.390$	0.4494	0.5094	0.5525	0.6098	0.6925	0.818	1.034	1.508	2.060
1905–1906 form	201	477	672	831	1005	1202	1396	1393	833	230
1908 form	102	392	564	726	861	1105	1379	1409	906	242
"Solar constant"	First result: 2.020					Second result: 2.034				

The two results differ by little more than one-half of 1 per cent., and in the opposite direction to that which should follow from the two different assumptions involved in the reduction, so that their difference is attributable to accidental changes made in the graphical and numerical reductions.

COMPUTED EFFECTIVE TEMPERATURE OF THE SUN

Not requiring, and hindered by great difficulties from reaching, high accuracy in the form of the extra-atmospheric solar energy-curve, we contented ourselves in 1905–1906 with the determinations given on p. 105 of the *Annals*. Recently Goldhammer[1] has employed our early observations at Washington and those of Langley at Mt. Whitney from which to compute the effective absolute temperature of the sun by means of the Planck formula.

$$e_{\lambda}\lambda^{5}\left(e^{\frac{c_2}{\lambda T}}-1\right)=c_1\,.$$

At our suggestion Mr. L. B. Aldrich has employed the values given in the *Annals*[2] according to Goldhammer's process, with the following results:

λ	T	λ	T	λ	T	λ	T	λ	T
$0^{\mu}.3853$	5677°	$0^{\mu}.4343$	6279°	$0^{\mu}.5226$	6230°	$0^{\mu}.7280$	6053°	$1^{\mu}.508$	6242°
.3897	6125	.4417	6336	.5370	6242	.7690	6006	1.648	6558
.3942	6353	.4494	6404	.5525	6215	.818	5968	1.786	6816
.3991	6463	.4578	6366	.5697	6200	.877	5901	1.924	5823
.4042	6404	.4666	6355	.5889	6237	.946	5824	2.060	5311
.4091	6418	.4762	6354	.6098	6163	1.034	5839	2.196	4783
.4147	6300	.4861	6312	.6333	6137	1.127	5846	2.316	4550
.4210	6285	.4974	6265	.6610	6089	1.239	5953	2.428	4082
.4275	6289	.5094	6241	.6925	6073	1.367	6036		

[1] *Annalen der Physik* (4), **25**, 905–920, 1908.

[2] Change top numbers of last three columns of Table 16 of the *Annals* as follows:

6.940	798	0.3853
6.603	1326	.3897
6.270	1670	.3942
5.956	1850	.3991
5.641	1782	.4042
5.390	1817	.4091
5.048	1636	.4147

These results indicate an effective solar temperature not far from 6200° Abs.

SOLAR SPECTRUM AND SPECTRUM OF "BLACK BODY"

Mr. Aldrich computed by Planck's energy formula the "black body" curve for absolute temperature 6200° and also for 7000° and compared them with the mean observed curve as published by us in Plate XVI of the *Annals*. The intensities having been matched at wave-length 0.5697, there are found the following percentage differences at other wave-lengths:

λ	O.–C. Per Cent.		λ	O.–C. Per Cent.	
	T=6200°	T=7000°		T=6200°	T=7000°
0.40.....	+22.0	+3.0	0.80....	− 9.2	+ 1.8
.41.....	+17.0	0.0	0.90....	−18.3	+ 0.5
.43.....	+ 7.2	−8.4	1.00....	−20.7	+ 1.7
.44.....	+10.0	−2.8	1.10....	−14.7	+ 5.4
.45.....	+12.3	+0.7	1.20....	−12.9	+10.1
.47.....	+11.2	+2.8	1.30....	− 9.7	+12.4
.50.....	+ 4.3	−1.9	1.50....	+ 1.8	+28.9
.60.....	− 2.2	−0.3	1.70....	+ 7.3	+29.2
.70.....	− 7.5	+1.1	2.00....	− 2.8	+22.0

It will be seen that neither computed curve fits the observed curve closely, but except at wave-lengths greater than 1.2 μ the 7000° curve agrees better.

CORRECTIONS FOR SOLAR SPECTRUM NOT OBSERVED

In connection with these comparisons we were led to inquire whether the fractional increases of observed areas of curves, which we had allowed for solar energy of less wave-lengths than 0.387 μ and greater than 2.43 μ, were justified. We had allowed approximately as follows:

Wave-Lengths	0.000 to 0.387 μ	2.428 μ to ∞
Mt. Wilson........	0.0040*	0.0050*
Extra-atmosphere...	0.0130	0.0055

* These two are average values, the real corrections depending on the intensity at near-by regions of the spectrum.

We are now of the opinion that the allowances are too small, especially outside the atmosphere. If the extra-atmospheric solar energy-curve is as near in form to that of the "black body" in these unobserved regions as in the parts of the spectrum observed, we have greatly underestimated the corrections. In the last three months of the stay of the Smithsonian expedition on Mt. Wilson, in 1908, the bolometric observations were pushed to the very end of the ultraviolet spectrum transmissible by the spectroscope, namely to wave-length 0.360 μ. In all the work we have observed as far down in the infra-red as the glass prism would permit. We intend to employ in future work a quartz prism and magnalium mirrors, and hope to extend the observations from wave-length 0.29 μ to wave-length 4.0 μ.

CHANGES OF SENSITIVENESS

One of the most unsatisfactory things in our published work was the necessity stated on p. 77 of the *Annals*, of applying a correcting factor for an apparent change of sensitiveness of the bolometric apparatus. This correction was determined by comparing the corrected area of the energy-spectrum curves with the readings of the pyrheliometer. It was invariably found that the areas were smaller as compared with the pyrheliometer readings when the sun was high than when it was low. To see if this depended on the pyrheliometry, a comparison was made between two pyrheliometers, one of which pointed directly at the sun, and the other of which, pointing at a mirror, could be read either horizontal or vertical without altering the angle of incidence of the sun's rays upon the mirror. No change of sensitiveness depending on the inclination of the pyrheliometer was found, hence this comparison showed clearly that no appreciable part of the above-mentioned correction was traceable to the pyrheliometer readings. We at first thought the bolometric apparatus altered in its sensitiveness on account of change of temperature. But great successive improvements in temperature-control gave no lessening of the range of the correcting factors between 1905, 1906, and 1908. Indeed some recent comparisons perhaps indicate that the range of sensitiveness has increased in these successive years. We thought it possible that the cause of the change might be the diurnal variation of the earth's magnetism. In September 1908 a great change in the system of magnetic control of the galvanometer was made, including a very careful astaticizing of the needle system, so that the earth's magnetic field thereafter played a much subordinated part. We have not yet reduced enough of the work of October 1908 to be sure whether this diminished the range of sensitiveness. We now think it possible that the deficient allowance heretofore made for ultraviolet radiation observed by the pyrheliometer but not by the bolometer may account for a part of the troublesome apparent change of sensitiveness.

CORRECTION OF GALVANOMETER SCALE

In studying the possible causes of the effect just discussed it was noted that the correcting factors in 1908 showed a marked stairlike succession. That is, the sensitiveness of the bolometric apparatus seemed to increase fast from the first observation to the second, then nearly halt from second to third, increase fast from third to fourth, and so on. This indicated a non-uniformity of the galvanometer scale. For the second bolographic record was always started at about 1½ cm to the south of the first; after it the plate was reversed, and the third and fourth records were placed like the first and second. By trial it was found in August 1908 that the galvanometer was in fact somewhat more sensitive for deflections on the north end of the scale. In September this defect was twice entirely removed; first, accidently, by raising the time of swing, and to that end rearranging the two control magnets which were above and below the galvanometer; second, designedly, after many experiments, by reastaticizing the needle system, removing the lower control magnet altogether, and rotating the plane of the needle system and coils from the true north-and-south plane, which they had always before occupied, to the magnetic north-and-south plane, which they will hereafter always occupy, and have occupied since September 20, 1908. Since that time there has been no scale-error great enough to be discovered by the most careful search over the whole 20 cm long scale of the galvanometer, or indeed for 20 cm on the negative side of its zero.

NEW AND ALTERED EXTRA-ATMOSPHERIC SOLAR ENERGY-CURVE

In September and October 1908 with these favorable scale conditions six days were devoted to complete independent determinations of all the elements of a "solar constant" measurement. When the results of this work were reduced we were shocked to find that the form of the extra-atmospheric solar enegy-curve differed very greatly from the mean result of 1905 and 1906, which we had used as the basis of a large number of computations of the "solar constant" of radiation. It has been our general practice heretofore to assume that the work of the year 1905 sufficed to determine accurately enough for our purposes the form of the extra-atmospheric solar energy-curve, and that there is probably no great change in it from year to year. Hence we have seldom determined the reflecting power of the coelostat and the transmission of the spectroscope by direct observation, but have generally computed the transmission of the whole apparatus from the known form of the energy-curve outside the atmosphere, combined with the bolographic work of the given day. Fortunately, as already demonstrated above, even the wide difference between the extra-atmospheric curve of September 15, 1908, and the mean curve of 1905–1906 produces no change in the computed value of the "solar constant." This may seem at first surprising, but the reader must remember that a value within a few per cent. of the true "solar constant" can be obtained from pyrheliometry alone, as was done by Pouillet, with no knowledge whatever of the distribution of solar energy in the spectrum.

At our request Miss Graves has compared all the elements of the determination of September 17, 1908 (which is near the mean of recent results), with corresponding elements of the determination of August 7, 1906. In order to make a fair comparison we have selected original bolographic records taken with about the same zenith distance of the sun. The air-masses range from 1.292 to 1.262 for the earlier day, and from 1.268 to 1.252 for the latter. The columns of the above table give (1) wave-lengths, (2)–(6) ratios, $\frac{\text{September 17, 1908}}{\text{August 7, 1906}}$: (2) observed prismatic intensities; (3) multipliers for correcting for

Column 1	2	3	4	5	6
0$^\mu$.4091	0.751	1.030	1.017	1.024	0.830
.4417	.825	1.048	1.063	1.038	0.866
.4861	.886	1.074	1.016	1.060	0.977
.5525	.891	1.056	0.986	1.049	1.004
.6610	.977	1.011	1.016	0.994	1.037
.877	1.031	1.006	1.077	0.982	1.147
1.367	1.041	1.037	1.107	1.013	1.230
2.060	1.026	1.053	1.176	1.042	1.297

coelostat absorption; (4) multipliers for correcting for spectroscope absorption; (5) atmospheric transmission coefficients; and (6) extra-atmospheric prismatic intensities.

APPARENT EXTRAORDINARY WEAKNESS OF VIOLET IN 1908

The table shows plainly that, excepting the last somewhat doubtful point where glass absorption and water-vapor absorption are both large, the transmission of the atmosphere and of the apparatus was almost alike on the two days, *but the original energy-curve* and hence the extra-atmospheric curve were far higher in the violet in August 1906 than in September 1908. This result, if true, indicates an enormous decrease in the sun's violet-emission. But the "solar constants" for the two days indicate no change in the sun's total emission. They are: August 7, 1906: 2.033; September 17, 1908: 2.046.

We now regret that we have not complete independent determinations of the transmission of the apparatus for all parts of the years 1906 and 1908. However, taking days when the mirrors had been freshly silvered, or freshly polished, it is to be supposed that the transmission would then be as good as on the days just compared. Two such days are June 21 and September 2, 1908, and on the latter the transmission of the apparatus was probably even better than on September 17, 1908. We now give the ratio of intensities of original bolographic curves taken at about the same air-masses for these three days of 1908 as compared with the curve of August 7, 1906. The atmospheric transmission was almost exactly the same on all these days.

Wave-Length in μ		0.4091	0.4417	0.4861	0.5525	0.6610	0.877	1.367	2.060
Intensity Ratio	June 21.....	0.888	0.928	0.941	0.984	1.011	1.040	1.022	1.136
	Sept. 2.....	0.867	0.935	0.918	0.935	0.977	1.013	0.998	1.045
	Sept. 17.....	0.830	0.866	0.977	1.004	1.037	1.147	1.230	1.297

All three days indicate the same result, namely: in 1908 the intensity of violet rays in sunlight was less as compared with red rays than in August 1906. The reader may see by inspection of Table 16 of Volume II of the *Annals* that in August 1906 the violet was apparently weaker as compared with the red than on the average of days in 1905.

Either these results are affected by some unknown experimental error or the distribution of energy in the solar spectrum is subject to great secular changes without great attending changes of the solar constant. We shall test the matter in future work; and with the new quartz-and-magnalium outfit in addition to the present glass-and-silver one, instrumental errors can hardly remain a possible explanation if such changes are in future observed.

A NEW STANDARD PYRHELIOMETER

As stated in the *Annals*, all our "solar constant" results are given in terms of a scale of energy whose unit is probably a few per cent. smaller than the gram-calory or therm. We have made many efforts to determine the ratio of our unit to the therm, and have designed and tried several new supposedly standard pyrheliometers, in which the hollow chamber or "black body" feature was introduced, and the solar heating was determined by measuring the rise of temperature of a current of water. Known electrical heating could also be measured in the same way as a check. Not satisfied with the results obtained with the instrument of this kind employed in 1905–1906, a new one of improved construction and altered size was used in 1908. This behaved in an exemplary manner, and many comparisons were made between it and the secondary pyrheliometers IV and VII used in daily "solar constant" measurements. Also many check measurements were made by means of two electrical heating coils. The results are so concordant throughout, and the electrical heat introduced was so exactly found in the water-flow in every instance, that we now believe the standard scale is established. Nevertheless our 1908 results indicate that the readings of the supposed standard of 1906 published at p. 45 of the *Annals* are 5.9 per cent. too high, and all the pyrheliometer and "solar constant" results published in the *Annals*, if reduced to our new provisional scale of real calories, would be 7.6 per cent. lower. Kimball[1] has found our arbitrary scale to be 11.0 per cent. above that of new Ångström pyrheliometers. Our new provisional standard scale is then about 2.6 per cent. above Ångström's.

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[1] *Bulletin Mt. Weather Observatory*, Vol. I, Part 2.

"SOLAR CONSTANT" CORRECTIONS

We now suppose that our "solar constant" values of 1902–1906 must be lowered 7.6 per cent. on account of pyrheliometry, and raised perhaps 10 per cent. on account of deficient correction for ultra-violet and infra-red unobserved energy. With our new quartz-magnalium spectrograph and a projected new pyrheliometer we hope to assure ourselves of these points in this present year, 1909.

Astrophysical Observatory
Smithsonian Institution
April 1909

CAMERA OBJECTIVES FOR SPECTROGRAPHS

By J. S. PLASKETT

It is well known that the camera objectives in general use in stellar spectrographic work have a very limited field of good definition, not exceeding in general 2°, which covers, in the usual dispersion of three prisms, about 200 tenth-meters. While this is a sufficient range for spectra of the second type, which are rich in lines, it is not sufficient for early-type spectra which may contain only one or two lines in this region and in which, consequently, the errors of measurement will be high. As practically the whole photographic region of the spectrum, $H\beta$ to K, may be obtained in one exposure with either refractor or reflector, it is evident that a considerable gain in the measurable material in such spectra would be obtained, without increase in exposure time, if a camera lens giving a considerably wider field were available.

Various attempts at the solution of this problem have been made, of which the most successful known to me is that described by Hartmann.[1] This objective made by Zeiss, known as the "Chromat," is constructed of the same material as the prisms, and is composed of two simple meniscus elements, one positive, one negative, separated by a small air space. As there is no chromatic correction the spectra are brought into focus by inclining the plates toward the violet, about 16° from the normal to the axis with a dispersion of three prisms. According to Hartmann this objective gives a flat field of 14°. A Zeiss "Chromat" has been in use in Ottawa for considerably over a year, entirely fulfilling expectations and giving, after slightly increasing the separation of the elements, the whole field used, from $H\beta$ to $H\delta$ (about 8°), almost absolutely flat with excellent definition. There can be no doubt that the field would extend farther if necessary.

Unfortunately, as was learned upon inquiry from Zeiss, this type of objective cannot be successfully made of a larger angular aperture than about *f*. 12. This was confirmed by the performance of a shorter focus lens of the same type (aperture ratio *f*. 8) constructed by Brashear which gave inferior definition. More recently, however,

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[1] *Zeitschrift für Instrumentenkunde*, 24, 257, 1904.

Ross Limited, London, have designed and constructed especially to conform to our requirements a lens similar in form to their "Homocentric," consisting of four separated elements, but following the principle of the "Chromat" in being entirely made of the prism material and consequently requiring inclination of the plate to bring the spectrum into focus. This lens, which will be more fully discussed later, gives, at an aperture ratio of *f.* 5.6, excellent definition and a flat field. The above remarks apply to a dispersion of three prisms, for which this type is especially adapted. If they were used with one prism, in addition to the limitations as to aperture, the plate inclination required (about 50°) would be inconvenient and practically inadmissible in radial velocity work.

There are consequently required short-focus objectives giving a flat field with three prisms, and objectives giving a flatter field than the regular triplet with a single prism. This need, together with what had already been accomplished by Hartmann and Zeiss, was laid before the J. A. Brashear Co. who, with their usual willingness, put their best efforts at our disposal and, in collaboration with Professor Hastings, produced two eminently successful objectives. In both of these the employment of one kind of glass only is followed, although not, as in the "Chromat," of the same material as the prisms, and the consequent chromatic differences in focus are overcome by inclining the plate. The objective first produced, to which they have given the name "Single Material," is composed of two widely separated positive elements of crown glass of the lowest dispersion and is especially adapted for use with one prism, giving exquisite definition and a field flat within 0.1 mm over the whole visible and considerably into the ultra-violet spectrum. The other is similar to the "Chromat" in form but made of light crown glass, giving also a flat field and good definition with three prisms with a plate inclination of only slightly over half that of the "Chromat."

The limiting aperture ratio of the former of these objectives is about *f.* 8, of the latter *f.* 12, so that evidently they cannot supply the need of short-focus lenses of *f.* 6 or thereabouts for either single or three-prism work.

The only prospect of success in this respect seemed to lie in some of the modern anastigmat photographic lenses, and a number of different makes were accordingly obtained for trial. The definition of several of these, though good enough for ordinary photographic work, would not stand the critical test of spectrum photography, owing probably to some residual spherical aberration. Two, however, the Ross "Homocentric" and the Zeiss "Tessar," gave good definition and the forms of their fields were accordingly determined.

It may not be out of place to give a brief description of the method employed in determining the fields of the ten lenses tested. The dispersion for eight of them was produced by the Ottawa spectrograph having the following optical constants: Hastings "Isokumatic" collimator objective of 35 mm aperture and 525 mm focus; one or three prisms of Jena glass O.102, angles 63° 50′ each; ray at minimum deviation λ 4415. Two of the objectives were tested with a new single-prism spectrograph having "Isokumatic" collimator of 51 mm aperture and 763 mm focus, O.102 prism of angle 63° 30′, ray at minimum λ 4325.

The positions of focus in different parts of the field were determined by a modification of Hartmann's method[1] of extra-focal exposures. By means of a revolvable semicircular diaphragm behind the collimator lens and an occulting diaphragm in front of the slit, a narrow strip of spectrum, photographed through the half of the prisms near the refracting edge, was placed between and touching two narrow strips made through the base half of the prisms. Evidently, when the plate is in the focus of the camera lens for any particular line in the spectrum, the adjacent portions of this line will be continuous, while, if not in focus, the central section will be displaced to red or violet of the outside sections, the direction and magnitude of this displacement giving a measure of the position of the focal point for the line in question. Two such plates, one inside and one outside the focus, will suffice to determine the form of the field. In order to avoid the labor of measurement and computation and on account of the diffuseness of the lines and consequent inaccuracy of measurement, when the plate is more than a millimeter from the focus, I have generally preferred to make a number of spectra, by the method outlined above, at camera settings about 0.25 mm apart within and without the focus. Five of these have in general sufficed to determine the focal curve and, as the camera back can be moved laterally, they can all be made on one plate, thus allowing ready comparisons. Simple inspection of these spectra under a microscope or even by a hand magnifier enables the focus of any line to be determined to about 0.05 mm by observing at which of two successive spectra the central section has opposite displacements with respect to the outside sections. Interpolation to the above accuracy can then generally be made. This takes only one-tenth the time and is probably equally as accurate as the method of measuring the displacements and computing the distance from focus. I may say that the camera setting in our regular work is always determined in this way, enabling the plate to be certainly placed considerably within 0.1 mm of the true focus.

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[1] *Zeitschrift für Instrumentenkunde*, 24, 1, 1904.

This method is probably open to the objection that it will not give the true focal point when the system has aberration, but it must be remembered that, to prevent systematic displacements in radial velocity work due to non-uniform illumination of the collimator objective, this method, which determines the focus by the absence of such displacement, is certainly the one that should be used. Moreover in this case tests at full aperture, so far as the focus can be determined by definition, confirmed the results of the former method and there is no reason to doubt the accuracy of the focal curves determined.

The following ten lenses, given in the order of procuring and testing, were tested for their curvature of field.

OBJECTIVES TESTED FOR FIELD

Number	Objective	Aperture	Focal Length	Tested with Dispersion of
1........	Brashear Single Material	45	525	1 Prism, 3 Prisms
2........	Zeiss Chromat	45	525	3 Prisms
3........	Ross Homocentric	40	254	3 Prisms
4........	Zeiss Tessar	48	305	3 Prisms
5........	Brashear Light Crown	45	525	3 Prisms
6........	Brashear Telescope Flint	45	525	3 Prisms
7........	Brashear O.102 ("Chromat")	45	375	3 Prisms
8........	Brashear Triplet	57	480	New 1 Prism 3 Prisms
9........	Brashear Single Material	57	457	New 1 Prism
10........	Ross Special Homocentric	40	254	3 Prisms

The form of field of each of the lenses is given in the accompanying figures where the horizontal lines represent differences of focus of

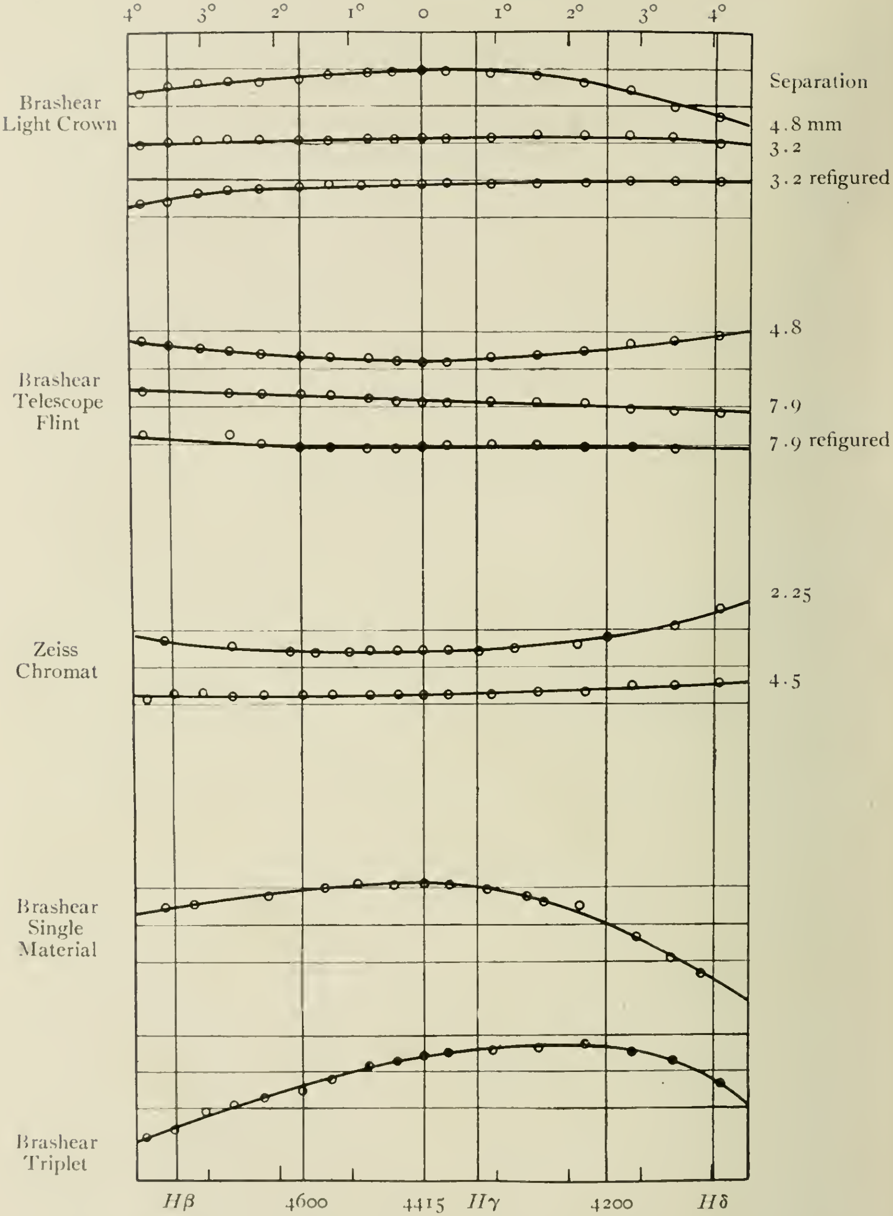

FIG. 1.—Long-focus objectives with three prisms.

one millimeter, where the wave-length and angular distance from the optical axis are shown by the vertical lines, and the diameters of the circles representing the observed points are 0.2 mm. Wherever the curves are not horizontal indicates that the inclination of the plate-holder required changing slightly, but this of course has no effect on the form of the field. In order to group the curves according to the type and purpose of the objectives, the order given in the above table has been changed and those of longer focus used with a dispersion of three prisms will be first considered (Fig. 1).

BRASHEAR SINGLE MATERIAL (NO. 1)

This consists of two simple converging lenses, the front double convex, the rear convex meniscus, of crown glass of lowest index, separated by nearly one-third the focal length. As will be seen later, this objective gives a beautiful field with one prism but is strongly concave toward the lens with three prisms, with about the same curvature of field as the regular Hastings Triplet. The inclination of the plate toward the violet is slightly over 5°. Allowing deviation from focus of 0.1 mm, slightly over 2° of field is usable.

ZEISS CHROMAT (NO. 2)

This consists of two strongly curved meniscus elements of Jena Glass O.102, the front diverging, the rear converging and of about half the focus of the combination. When received it gave a field convex toward the lens, as shown in the upper curve. When the separation between the elements was increased from 2.25 to 4.5 mm, the field became almost absolutely flat over the whole 8°, giving at the same time excellent definition. Inclination of the plate toward the violet about 16°.

BRASHEAR LIGHT CROWN (NO. 5)

This is a lens of the same form as the "Chromat" except that it is made of light crown glass. With the original separation the field was concave but became flat on decreasing the separation from 4.8 to 3.2 mm. This change in separation resulted in loss of defining power. The objective was refigured at the new separation and gave good definition and field flat over practically the whole range. Inclination of plate to the violet about 9°.

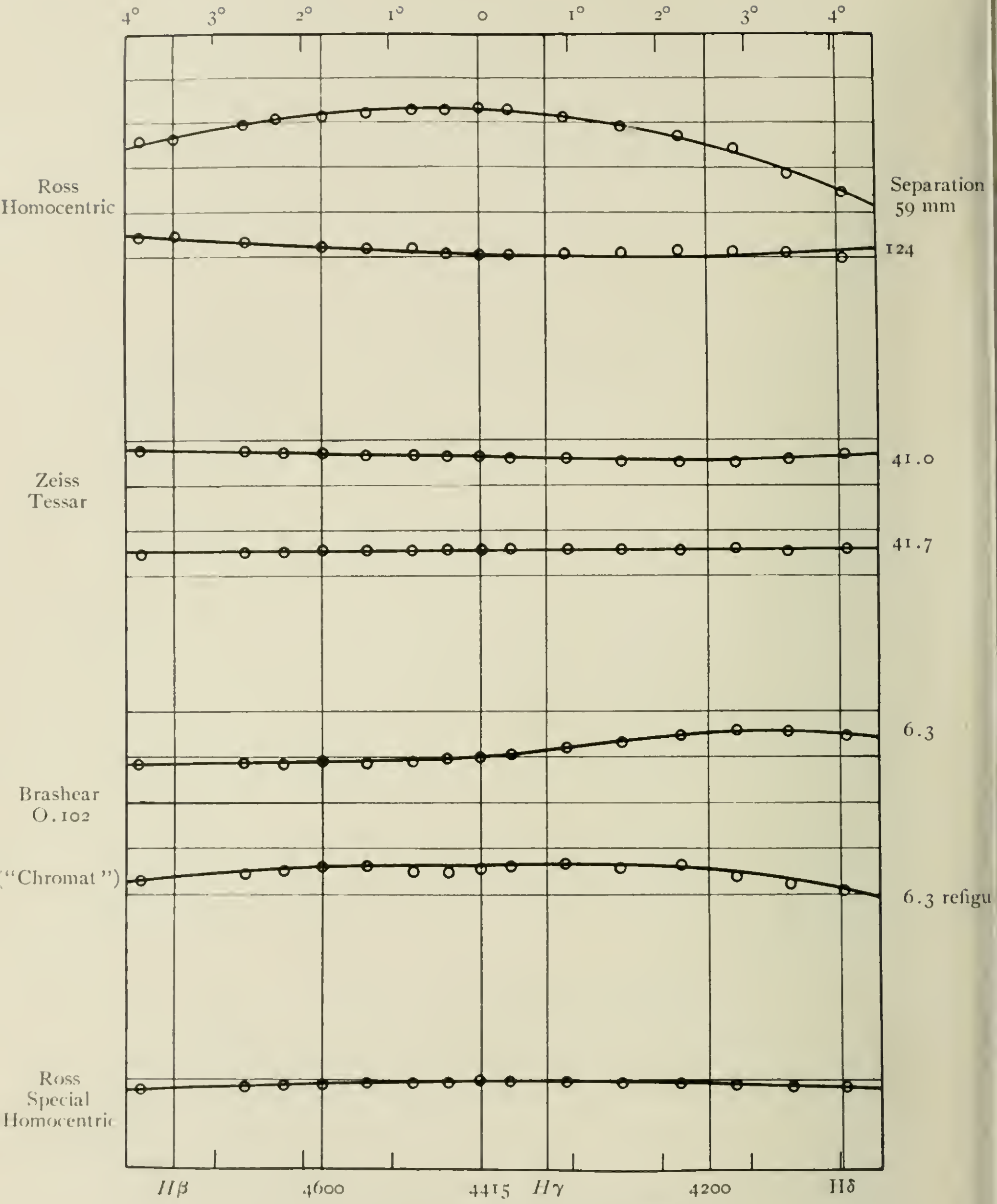

Fig. 2.—Short-focus objectives with three prisms.

BRASHEAR TELESCOPE FLINT (NO. 6)

An objective similar to the previous one only made of telescope flint glass. The field was originally convex but became flat on increasing the separation from 4.8 to 7.9 mm. Refiguring did not give so much improvement as in the light crown objective. Field is now practically flat. Inclination of plate to the violet about 13°5.

BRASHEAR TRIPLET (HASTINGS) (NO. 8)

This is a lens of the same type as used in the Mills, Bruce, and Lowell spectrographs. The field is, as shown, strongly concave toward the objective with a usable portion, allowing deviation of 0.1 mm, of about 2°5. The definition at the center of the field is about the same as in the "Chromat," but toward the margins even when in focus is much inferior.

All the above objectives are of relatively long focus, small angular aperture, about *f.* 12, tested with dispersion of three prisms. Let us now examine the fields given by shorter focus objectives, using the same dispersion (Fig. 2).

ROSS HOMOCENTRIC (NO. 3)

This standard photographic objective gives good definition but a strongly concave field. An increase in separation from 59 to 124 mm appears to flatten the field but at the expense of defining power and the lens is not usable at the increased separation. Useful field is not more than 2°.

ZEISS TESSAR (NO. 4)

This objective was one of the standard form taken from the stock of Bausch & Lomb. It gives good definition and a field very slightly convex. This convexity is removed by an increase in separation from 41.0 to 41.7 mm, but with a slight loss in defining power, so that it is probably preferable to use it at the normal separation. Another lens of the same series, aperture, and focus was tested, giving practically the same field but considerably poorer definition. This is of interest as showing the differences between the performance of two objectives presumably identical and indicates the desirability of specially selecting and testing the lens to be used.

BRASHEAR O.102 ("CHROMAT") (NO. 7)

This objective, of the same type and material as the Zeiss Chromat but of larger angular aperture, gives a field nearly flat with a separation of 6.3 mm but with poor definition even after refiguring. This shows that this type cannot be successfully constructed of larger aperture ratio than *f*. 11, say. Inclination of the plate to the violet about 16°.

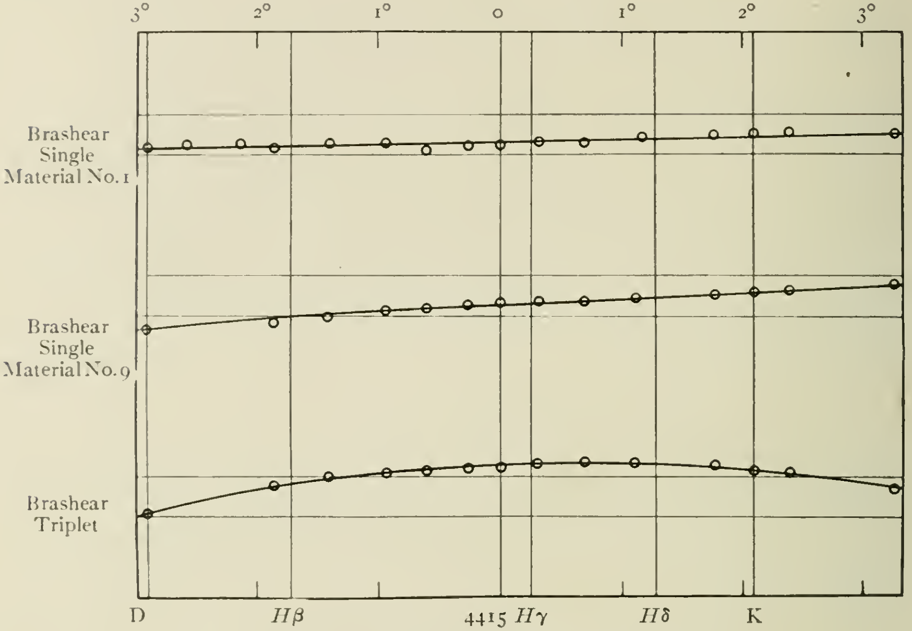

FIG. 3.—Objectives tested with one prism.

ROSS SPECIAL HOMOCENTRIC (NO. 10)

This objective was, by the kindness of the makers, Messrs. Ross Limited, especially computed and constructed for us. It has an aperture ratio of *f*. 5.6, is of practically the same form as their Homocentric but with all four elements of O.102 glass. It gives beautiful definition and a field nearly flat, usable over 8°. Change of separation is without appreciable effect on the form of field. Inclination of plate to the violet about 16°.

Two types of objectives of medium and long focus have been tested with a dispersion of one prism (Fig. 3).

BRASHEAR SINGLE MATERIAL (NOS. 1 AND 9)

Both of these objectives, whose form was described above, are of the same type, No. 1 of an aperture ratio *f.* 11.5; No. 9 of *f.* 8. When used with three prisms they give the strongly concave field shown in Fig. 1, but with a single prism the field is almost absolutely flat over the whole range of visible and as far into the ultra-violet spectrum as the prism will transmit. The definition given is excellent and the objectives leave nothing to be desired for single-prism work. The inclination of plate to the violet is about 16°. I am glad to express here my appreciation of the efforts as well as my admiration for the skill of Mr. McDowell in figuring these objectives. As both components are converging, the only means of removing the positive spherical aberration is by departure from spherical surfaces. While, as Mr. McDowell says, this was comparatively easy for No. 1, of the smaller angular aperture, it taxed even his skill to remove it entirely in the other, and it was only after a second trial and the use of a special device that the objective was finally made perfect.

BRASHEAR TRIPLET (NO. 8)

This, as with three prisms, gives a field concave toward the objective but with considerably less curvature. The usable field is somewhat over 2°. Definition good.

The final results of the investigation may be summarized as follows:

For a dispersion of three prisms with a camera of fairly long focus two objectives are much superior to the others, the Zeiss "Chromat" and the Brashear Light Crown. The former gives a flatter field and slightly better definition than the latter, but on the other hand the smaller plate inclination of 8° instead of 16° and the smaller absorption of the Brashear are an advantage. The definition of either of these is fully equal to the regular triplet in the center of the field and much superior at the margins.

For short-focus lenses with three prisms both the Zeiss Tessar and the Ross Special Homocentric give good definition and flat fields. The Ross can be used of shorter focus than the Tessar, and gives exquisite definition, but the field of the Tessar is flatter and the plate is normal to the axis.

In single-prism work the Brashear Single Material is much superior to the type of Triplet usually employed, both in definition and extent of field, and cannot be surpassed or even equaled for its purpose.

In conclusion, I wish to express my indebtedness to Dr. King, director of the Observatory, not only for the interest he has shown in this work, but also for his willingness to meet the expense of the various objectives obtained.

Dominion Observatory, Ottawa
March 1909

NOTE ON THE POLARIZING EFFECT OF COELOSTAT MIRRORS[1]

By CHARLES E. ST. JOHN

The discovery of the Zeeman effect in sun-spots by Hale[2] made it important to determine the action of the silver-on-glass mirrors of the tower telescope of the Mount Wilson Solar Observatory upon circularly polarized light. It was at once recognized that circularly polarized light would be changed to elliptically polarized by reflection from the silver surfaces and that its effect would vary with the angles of incidence and hence with the position of the sun. A description of the tower telescope has been given by Hale[3] and it only needs be said here that the second mirror sends the light vertically downward through the 12-inch objective and that about 4 feet below the objective—focal length 60 feet—the beam was received upon the analyzer which served to fix the position of the axes of the elliptically polarized light and to determine their ratio.

Light circularly polarized in a known direction was obtained by passing sunlight through a Nicol prism and then through a Fresnel rhomb whose plane of incidence formed an angle of 45° with the short diagonal of the Nicol. The direction in which it was necessary to rotate the rhomb to bring its plane of incidence into coincidence with the plane of vibration of the Nicol was taken as the direction of revolution of the circularly polarized light[4] and it was distinguished as clockwise and counter-clockwise as viewed by the observer when looking in the direction of the source. This polarizer was mounted in a screen and interposed between the sun and the first mirror so that no light reached the mirror except that passing through the polarizer. The analyzer consisted of a Fresnel rhomb mounted upon a divided circle, and a Nicol likewise with a graduated mounting that could follow the eccentric movements of the rhomb as the latter was rotated. Monochromatic light was obtained by interposing ruby glass in the train and its intensity was further reduced by means of a smoked glass. The observer lay on his back while making the observations which were carried out in the following way. The incident light was polarized in a clockwise direction and after reflection from the two mirrors fell upon the rhomb; a position of the rhomb was then found for which the Nicol could produce extinction of the light; the rhomb was rotated 90° and the position of the Nicol for extinction again determined; the operation was repeated with reversed polarization of the incident light. The readings of the scales give the azimuth of the plane of incidence of the rhomb and the corresponding azimuth of the long diagonal of the Nicol for extinction. The difference between the two azimuths of the Nicol corresponding to the first and second settings of the rhomb gives twice the angle between the major axis of the elliptically polarized light and the plane polarized light produced by the rhomb when its plane of incidence coincides with the major axis of the ellipse. This angle is given in the column of the table marked I. The ratio of the axes is of course given by the tangent of this angle. The difference between positions of the Nicol for clockwise and counter-clockwise light for the same setting of the rhomb is given in the column marked II, and approximates 90°.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 37.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 30; *Astrophysical Journal*, **28**, 315, 1908.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 23; *Astrophysical Journal*, **27**, 204 1908.

[4] Wood, *Physical Optics*, p. 277.

The great change in the position of the axis of the ellipse between morning and afternoon, as shown by the position of the rhomb, followed the transfer of the coelostat mirror from the west to the east side of the second mirror at noon. The condition of the silver surfaces affects the results appreciably, but under normal working conditions of the surfaces the changes due to this cause were not great enough to be troublesome.

In the practical application of these results in studying the Zeeman effect along the lines of force in the magnetic field of sun-spots, the rhomb was set with its plane lying northeast and southwest in azimuth 20° for the forenoon, in azimuth 70° from noon to about 3 P. M., and then in azimuth 55°. The Nicol was set with its long diagonal in an azimuth equal to that of the rhomb plus 25° in the forenoon and plus 30° in the afternoon to extinguish clockwise light, and rotated

	Forenoon					Afternoon				
		Rhomb	Nicol	I	II		Rhomb	Nicol	I	II
Clockwise	Sept. 27	23° E of N	50° E of N							
Clockwise	7:00	67 W of N	10 W of N	30°						
Clockwise	Sept. 26	25 E of N	58 E of N			Sept. 18	73° E of N	104° E of N		
Clockwise		65 W of N	0				17 W of N	52 E of N		
Counter-clockwise	8:20	25 E of N	30 W of N			1:30	73 E of N	15 E of N		
Counter-clockwise		65 W of N	87 W of N	29	87°.5		17 W of N	39 W of N	26°.5	90°
Clockwise	Sept. 26	15 E of N	48 E of N			Sept. 16	70 E of N	105 E of N		
Clockwise		75 W of N	5 W of N				20 W of N	50 E of N		
Counter-clockwise	9:35	15 E of N	42 W of N			2:00	70 E of N	17 E of N		
Counter-clockwise		75 W of N	90 W of N	25	91.5		20 W of N	40 W of N	28	89
Clockwise	Sept. 18	23 E of N	55 E of N			Sept. 25	70 E of N	105 E of N		
Clockwise		67 W of N	0				20 W of N	45 E of N		
Counter-clockwise	10:00	23 E of N	35 W of N			2:40	70 E of N	20 E of N		
Counter-clockwise		67 W of N	88 W of N	26	89		20 W of N	45 W of N	31	87.5
Clockwise	Sept. 26	12 E of N	42 E of N			Sept. 18	54 E of N	90 E of N		
Clockwise		78 W of N	8 W of N				36 W of N	25 W of N		
Counter-clockwise	10:30	15 E of N	47 W of N			3:50	54 E of N	0		
Counter-clockwise		75 W of N	93 W of N	24	87		36 W of N	60 W of N	31	87.5
Clockwise	Sept. 26	10 E of N	40 E of N			Sept. 26	63 E of N	100 E of N		
Clockwise		80 W of N	8 W of N				27 W of N	38 E of N		
Counter-clockwise	11:00	10 E of N	50 W of N			3:10	63 E of N	7 E of N		
Counter-clockwise		80 W of N	100 W of N	24.5	91		27 W of N	53 W of N	30.5	92
Clockwise	Sept. 18	23 E of N	55 E of N			Sept. 26	55 E of N	95 E of N		
Clockwise		67 W of N	2 E of N			4:05	35 W of N	30 E of N	32.5	
Counter-clockwise	11:20	23 E of N	40 W of N							
Counter-clockwise		67 W of N	88 W of N	25	92.5					
Clockwise	Sept. 16	20 E of N	47 E of N			Sept. 26	50 E of N	85 E of N		
Counter-clockwise	12:00	20 E of N	45 W of N	26		5:00	40 W of N	20 E of N	32.5	
Mean		22° E of N		26°			62° E of N		30°	

90° to extinguish counter-clockwise light. Calculation of the effect produced by the plane of the rhomb not coinciding with the major axis of the elliptically polarized light shows that for a deviation of 10° the resulting effect is very narrowly elliptical light—axis in the ratio twelve to one—with the major axis forming very nearly the same angle with the plane of the rhomb as the plane polarized light which results when the plane of the rhomb coincides with the major axis of the elliptical light coming from the mirror. This allows some latitude in the setting of the rhomb provided the angle between the planes of rhomb and Nicol is maintained.

Mount Wilson Solar Observatory
February 1909

SOME CONVENIENT FORMS OF COMPARISON-PRISMS

BY LOUIS BELL

Everyone who has used the spectroscope much, and especially everyone who has used forms of spectrophotometer requiring a comparison-prism over the slit, has been annoyed by the black band between the spectra produced by the edge of the comparison-prism. In spectrophotometry particularly the presence of this band is more than inconvenient. It is fatal to precision. Having recently had built a spectrophotometer of the Nichols form for use on the photometer-bar in place of the regular sight-box, the writer encountered the familiar difficulty and set about finding means of remedying it. The resulting device for overcoming the difficulty is here put on record for the benefit of others. Fig. 1 shows the familiar arrangement of prisms ordinarily used in the Nichols spectrophotometer, together with the paths of the rays from the prospective sides through the prisms and into the slit. The new device was built from this same pair of prisms. One of them, *A*, was cut down, retaining the right angle, so that its new hypothenuse dimension was equal to its old perpendicular. The other prism, *B*, was treated as follows: Its hypothenuse face was silvered densely enough to present a smooth, opaque coating. A cut through this coating centrally in the long axis of the hypothenuse face was made by a sharp ivory blade, and one-half the silver coating was completely removed, leaving the hypothenuse face half clear and half densely silvered. The arrangement of this hypothenuse face is shown in Fig. 3. Then the cut-down prism *A* was cemented with Canada balsam to the hypothenuse face of *B* as shown diagrammatically in Fig. 2. This balsam cementing should be done at a very moderate temperature lest the silver be started if the baking is too hard. In Fig. 2 the ray from the right side enters the perpendicular face of the prism *B*, just as it would in Fig. 1, but on arriving at the silvered face, that part of the beam which meets the balsamed face of clear glass passes straight through, almost unimpeded, since the balsam has very nearly the same index of refraction as the glass of the prism. The rest of the beam is reflected at that part of the hypothenuse face which is protected from the balsam by the silvering, and passes into the slit. The ray from the other direction enters the external portion of the prism *A*, is reflected at its rear face, which for symmetry and the interception of stray light should also be silvered, and, in so far as the silvering permits, passes through the balsamed surface of junction of *A* and *B* and enters the slit. That part of the beam which meets the silvering is reflected out of the prism. The paths of the two rays are shown diagrammatically in Fig. 2. The line of demarkation between the two spectra provided by this comparison-prism is due to the sharp edge that limits the silvering on the cemented face. In making up the prism for regular use it would probably be best to cut through the silvering with a very fine diamond point and then clear it away from this line as a boundary. The line of separation of the two rays is so sharp and its plane can be brought so close to the slit that, when the instrument is properly adjusted, the dark band usually seen with the comparison-prisms has practically disappeared entirely. In the spectrophotometer in which the experimental prism was tried, the focal length of the collimator is only 12 cm, and even under this trying condition the joining of the two spectra is very good indeed. With a collimator of such focal length as is more commonly used, even better results can be obtained. In fact with lenses of a meter or more in focus, the difference in focus for the slit and for the silvered face can be made hardly more than the difference in focus due to the secondary spectrum.

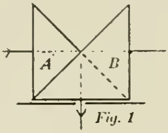

Fig. 1

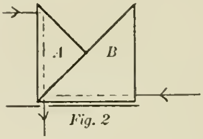

Fig. 2

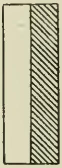
Fig. 3

Fig. 4

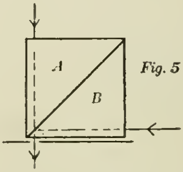

Fig. 5

Obviously one can remove the silvering in strips as shown in Fig. 4, and in this way can get an even more delicate comparison of brightness than with the two fields given by the form shown in Fig. 3.

Where it is merely necessary to bring in a comparison-spectrum from a position at right angles to the axis of the collimator, the very obvious modification shown in Fig. 5 can be used to advantage. Here two entirely similar prisms in which the hypothenuse face of one is silvered in one or more stripes are balsamed together. The direct beam then comes straight through from the front of the slit except where it is deflected by the silvering, and the lateral beam comes in by reflection at the silvered surface just as tin the case of Fig. 2. The use of either of these forms of comparison-prisms requires careful adjustment to obtain the maximum of clearness between the contact of the two spectra, but once the adjustment is made, the result is excellent. With a collimating lens of so short focus as that employed in the spectrophotometer mentioned, it is advantageous to have screw adjustments so arranged that the lens can be slightly raised, lowered, or tipped so that line spectra thrown simultaneously in from the two directions can be matched with precision. Once this is accomplished, the adjustment can be readily preserved. The prisms in this instance were of common crown glass, but in making up special prisms for a spectrophotometer it might be desirable to pick out a crown which would match the index of refraction of the balsam with special accuracy. This would require an index of nearly 1.54, since the index of balsam rises materially as the terpenes are baked out.

Boston, Mass.
March 1909

THE BINARY VARIABLE STAR *RZ CASSIOPEIAE*

By JOHN STEIN

The investigations of this variable star made by J. A. Parkhurst and F. C. Jordan[1] make it of interest to see how well the light-curve, as determined by their "absolute scale" of photographic magnitudes, can be represented by the satellite theory.

In what follows I restrict myself to the simplest form of the theory. Two bodies, with radii 1 and x, respectively, the first bright, the second dark, revolve in a circular orbit of radius a. The plane of the orbit forms a small angle i with the line of sight. Consider in place of the bright star a uniformly illuminated disk of total intensity 1, in place of the satellite a dark disk; then the light-intensity J for the time t, reckoned from minimum, is given by the formulae:

$$J=1-\frac{M}{\pi}\;;$$

$$M=\tfrac{1}{2}\{x^2\,(2\phi-\sin 2\phi)+(2\phi'-\sin 2\phi')\}\;;$$

$$\sin\phi'=x\sin\phi\;;\quad \cos\phi'=\frac{\rho^2+x^2-1}{2x\rho}\;;$$

$$\rho^2=a^2\sin^2 nt+a^2\sin^2 i\cos^2 nt\;.$$

In these equations M is the part of the disk of the principal star hidden by the satellite, ρ the apparent distance of their centers, ϕ' and ϕ the angles under which the common chord of the two disks is seen from the centers of the principal star and the satellite. The magnitude difference $m-m_0$ (m_0 being the normal light) is then given by

$$m-m_0=-2.5\log J\;.$$

To determine the approximate elements we have the data:

(*a*) Period of *RZ Cassiopeiae* $=1^{\mathrm{d}}.195258$.

(*b*) Normal light $6^{\mathrm{m}}.43$; minimum $7^{\mathrm{m}}.64$; minimum intensity $J_{\mathrm{m}}=0.3281$.

(*c*) Duration of eclipse $2t_0=0^{\mathrm{d}}.23=5^{\mathrm{h}}\,32^{\mathrm{m}}$.

(*d*) The minimum is sharply defined, no trace of a secondary minimum.

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[1] *Astrophysical Journal*, **26**, 251–254, 1907.

Theoretical light-curves were calculated on two hypotheses:

(I) $x = 1$. Then

$$M = 2\phi - \sin 2\phi \ ; \quad \cos \phi = \frac{\rho}{2} \ ;$$

$$J_{\mathrm{m}} = 1 - \frac{2\phi_{\mathrm{m}} - \sin 2\phi_{\mathrm{m}}}{\pi} \ ; \quad \cos \phi_{\mathrm{m}} = a \sin i \ .$$

Also

$$a^2 \sin^2 nt_0 + a^2 \cos^2 nt_0 \sin^2 i = (1 + x)^2 .$$

Therefore we find

$$a = 3.437 ; \ i = 8° 43' .$$

(II) $1 - x^2 = J_{\mathrm{m}}$; $x = 0.820$. This is the minimum value of x. Since there is no stationary period at the minimum, the disks must be in internal contact when the centers are at their minimum distance. Therefore

$$a \sin i = 1 - x \ .$$

These equations, combined with those derived from the duration of the eclipse, give

$$a = 3.191 \ ; \ i = 3° 14' .$$

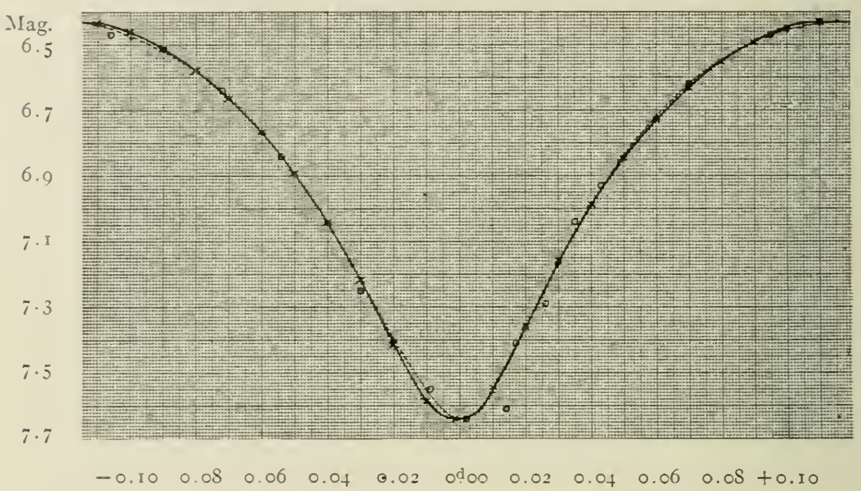
Since the light-curve C_0, given by P. and J., is not symmetrical the quantities for equal times before and after the minimum were combined into the mean values C_{m}. In the following table this mean curve is compared with the curves C_{I} and C_{II} calculated with the hypotheses I and II.

t	C_{m}	C_{I}	C_{II}	$C_{\mathrm{m}} - C_{\mathrm{I}}$	$C_{\mathrm{m}} - C_{\mathrm{II}}$	C_{III}	$C_{\mathrm{m}} - C_{\mathrm{III}}$
±0ᵈ00	7ᵐ64	7ᵐ640	7ᵐ640	0ᵐ000	0ᵐ000	7ᵐ640	0ᵐ000
.01	7.555	7.569	7.580	−.014	−.025	7.567	−.017
.02	7.38	7.428	7.403	−.048	−.023	7.385	−.005
.03	7.195	7.259	7.213	−.064	−.018	7.189	+.006
.04	7.02	7.090	7.044	−.070	−.024	7.014	+.006
.05	6.87	6.940	6.899	−.070	−.029	6.871	−.001
.06	6.745	6.810	6.778	−.065	−.033	6.749	−.004
.07	6.64	6.700	6.676	−.060	−.036	6.649	−.009
.08	6.565	6.608	6.592	−.043	−.027	6.566	−.001
.09	6.505	6.533	6.524	−.028	−.019	6.501	+.004
.10	6.47	6.476	6.472	−.006	−.002	6.455	+.015
±0.11	6.44	6.438	6.438	+.002	+.002	6.430	+.010

The change from hypothesis I to II decreases the residuals, but they retain the same sign. The values of x between 1 and 0.82 are therefore excluded.

The differences $C_m - C_{II}$ nearly disappear if a slight decrease is assumed in the duration of the eclipse. Since Professor Müller at Potsdam, in his provisional determination of the light-curve,[1] finds a considerably shorter duration, this change is probably justified. A few trials proved that a duration of $0^d22 = 5^h\ 17^m$ (a decrease of only a quarter-hour) gave the best curve, C_{III} in the above table. As before, x was assumed as equal to 0.82, and therefore $a = 3.317$ and $i = 3°\ 7'$.

As the extremely small differences $C_m - C_{III}$ show, the theory represents the *mean* light-curve with great accuracy. But, as before remarked, the original curve C_o is not symmetrical with respect to the minimum. This asymmetry, however, is in large part only apparent. If the minimum of the curve be assumed 0^d0016 earlier, the asymmetry almost disappears. Only in the immediate vicinity of the minimum can it still be seen, and there it is probably not real.



In the following table the curve C_o is compared with the theoretical curve, after the latter is shifted -0^d0016.

In the accompanying illustration the theoretical curve C_{IV} is shown by the full line; the observed curve C_o, where it does not coincide with the former, is shown by the broken line. The small circles

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[1] *Astronomische Nachrichten*, **171**, 359, 1906.

t	C_0	C_{IV}	C_0-C_{IV}	t	C_0	C_{IV}	C_0-C_{IV}
$-0^{\mathrm{d}}11$	$6^{\mathrm{m}}44$	$6^{\mathrm{m}}433$	$+0^{\mathrm{m}}007$	$+0^{\mathrm{d}}11$	$6^{\mathrm{m}}44$	$6^{\mathrm{m}}430$	$+0^{\mathrm{m}}010$
.10	6.48	6.461	+ .019	10	6.46	6.449	+ .011
.09	6.52	6.510	+ .010	09	6.49	6.492	− .002
.08	6.58	6.578	+ 002	.08	6.55	6.554	− .004
.07	6.66	6.664	− 004	.07	6.62	6.634	− .014
.06	6.77	6.767	+ 003	.06	6.72	6.731	− .011
.05	6.90	6.802	+ .008	05	6.84	6.850	− .010
.04	7.05	7.040	+ .010	04	6.90	6.988	+ .002
.03	7.22	7.219	+ .001	.03	7.17	7.159	+ .011
.02	7.40	7.414	− .014	.02	7.36	7.359	+ .001
.01	7.55	7.587	− .037	.01	7.56	7.549	+ .011
∓0.00	7.64	7.640	000	±0.00	7.64	7.640	.000

are the normal points which P. and J. find from the photographic plates. The residuals are as follows:

t	O.−C.	t	O.−C.	t	O.−C.	t	O.−C.
$-0^{\mathrm{d}}106$	$+0^{\mathrm{m}}03$	$-0^{\mathrm{d}}030$	$+0^{\mathrm{m}}03$	$+0^{\mathrm{d}}017$	$0^{\mathrm{m}}00$	$+0^{\mathrm{d}}043$	$-0^{\mathrm{m}}01$
− .072	− .01	− .009	− .05	+ .026	+ .04	+ .070	− .01
− .054	.00	+ .002	+ .01	+ .035	− .03	+ .095	.00
		[+ .014	+ .13]				

With the single exception of the normal point inclosed in the parenthesis,[1] the observations are satisfactorily represented by the theory.

We therefore find as the most probable elements:

Radius of the principal star = 1

$$x = 0.820$$

$$a = 3.317$$

$$i = 3^{\circ}\,7'$$

$$(2t_0 = 0^{\mathrm{d}}.22)$$

Even a considerable excentricity in the orbit has so slight an effect on the light-curve of an *Algol*-type star, that the relative size

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[1] NOTE by translator.—The normal point at phase +0.014 depends upon only three images, as follows:

Plate	Exp.	Phase	Mag.	C_{IV}	O.−C.
R 1672...	5	0.012	7.61	7.49	$+0^{\mathrm{m}}12$
R 1650...	2	0.014	7.71	7.48	+0.13
UV 210...	7	0.017	7.52	7.47	+0.05

The large difference O.−C. seems mainly due to failure completely to reduce the observations with the reflector to the system of the more reliable extra-focal images.—J. A. PARKHURST.

and positions of the axes cannot be well determined from photometric data. It is hoped that the definitive reduction of the radial-velocity plates taken at the Yerkes Observatory[1] and Potsdam[2] will give more accurate values of these quantities.

According to Hartmann's provisional reductions, the distance a of the bright body from the center of gravity is 1,170,000 (:cos i) km. If we assume that the orbit is circular and that the two components have the same mean density d, we derive the following elements:

Distance of centers = 3,297,000 km = 0.022 astronomical units.
Combined mass of the system = 1.002 sun's mass.
Mean density $d = 0.222$ sun's density.
Radius of the bright body = 994,000 km = 1.43 sun's radius.
Mass of the bright body = 0.646 sun's mass.
Radius of the satellite = 815,000 km = 1.17 sun's radius.
Mass of the satellite = 0.356 sun's mass.

SPECOLA VATICANA
March 1909

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[1] *Astrophysical Journal*, **25**, 59, 1907.

[2] *Astronomische Nachrichten*, **173**, 101, 1906.

THE PHOTO-HELIOMETER

By CHARLES LANE POOR

In 1905 the writer published the first of a series of articles dealing with the "Figure of the Sun." In this paper were given the results derived from the measurement of a series of solar photographs taken by the late Lewis M. Rutherfurd in his private observatory in the years 1870–72. These early photographs were taken with a 13-inch lens photographically corrected and were made on collodion plates. These excellent photographs of Rutherfurd's, which have hardly been excelled at the present day, gave images of the sun with sharp, clean-cut, and easily measurable limbs. A determination of the polar and equatorial diameters gave results which were very accordant and which in precision compared very favorably with the heliometer measures made by the German observers of the transits of *Venus*. Several short series of more modern plates were measured, but on the whole their results were not as satisfactory as those obtained from the early plates of Rutherfurd. There is not a dearth of solar photographs in the world, for several observatories have for many years photographed the sun on each and every clear day, yet these photographs ordinarily are not suitable for accurate measurement. They were mostly made with horizontal telescopes, and were taken to supply data for investigations as to the number, size, and location of sun-spots. The mirror of such horizontal instruments introduces errors and makes the image unsymmetrical, and in the taking and developing of the photographs but little attention was paid to sharpness of edge.

In order to determine the best methods of applying photography to the determination of the size and shape of the sun, a series of experiments was undertaken and was carried out at the Yerkes Observatory by Mr. Philip Fox, under the direction of Professor Frost. A 4-inch photographically corrected lens of 40 feet focal length was mounted on the tube of the 40-inch Yerkes telescope. With this many photographs were taken, but none of them equalled in clearness and sharpness of edge the early photographs of Rutherfurd. To a large extent this was due to the use of modern rapid dry plates, but some of the distortions of the image were undoubtedly caused by layers and currents of heated air near the tube of the great telescope.

In the summer of 1907 the 4-inch was dismounted and a pair of 2-inch photographically corrected lenses of 25 feet focal length were mounted in its place. These lenses were placed side by side in the same cell, so as to give overlapping images of the sun, and the cell was arranged so as to revolve about the collimation axis in a manner similar to the cell of a heliometer. The line joining the centers of the two lenses could readily be placed parallel to the equatorial or to the polar diameter of the sun, and the whole apparatus formed what might be termed a 25-foot photographic heliometer. Some trial photographs were made with the twin lenses by S. A. Mitchell of Columbia University (who was spending the summer at Yerkes) and by Mr. Fox.

There are two methods of utilizing the photographs made with such a heliometer, the first involving the measurement, on the overlapping images, of the width of the lune, the second requiring the measurement of the common chord.

1. The first method directly determines the length of the diameter, which passes through the centers of the two images. For, neglecting refraction, this diameter is equal to the known distance between the centers of the lenses added to the measured width of the lune. This is the simplest and most direct method of using a photographic heliometer; it obviates the use of a micrometer screw on the instrument and replaces an eye-estimate of contact by the measurement of a small distance on a photographic plate. The correction for refraction is extremely simple and may be made by the ordinary differential formulae.

2. In the second method the length of the diameter is only indirectly determined, and the corrections for refraction are somewhat troublesome and complicated. On each photograph the length of the chord, common to the two images, is a measure of the radius passing through the extremity of the chord, and slight changes in the radius produce large changes in the chord. As will be shown later, a change, Δr,

in the radius will produce a change, Δd, in the semi-chord approximately equal to

$$\Delta d = \frac{r}{d} \Delta r\ ,$$

where r and d are the respective lengths of the radius and the chord. Thus, if the chord be one-third the radius, an error of measurement of 1″ in the chord would produce an error of only 0″.3 in computed length of the radius. This is the great advantage of the second method.

The ultimate choice of the two methods depends upon the relative accuracy with which the different measures can be made. In order to test this point, the chord and the lune were measured on twelve test plates. The measures of one of these were rejected as on that plate the width of the lune could be only approximately determined, because "the background was too dark to distinguish the edges of the lune." The probable errors of a single measure were found to be:

	Lune	Chord
Harpham............	±0.011 mm	±0.011 mm
Davis...............	±0.015 mm	±0.011 mm

This would seem to show that the chord can be measured with as great accuracy as the lune and would therefore indicate a decided superiority in this method. This chord not only furnishes an extremely accurate and delicate measure of the radius, but it is also extremely easy of measurement. The measures can be made far more rapidly than those necessary on a direct photograph of the solar disk. Unfortunately, however, as has been noted, the reductions for refraction are rather complicated, and special formulae are necessary.

Aside from the general effect of bad seeing outside the 40-inch tube, the photo-heliographic method suffers special defects from bad seeing. When watching the solar image, pulsations of bad definition can be seen to sweep across the solar disk. Entering at one limb, they can be followed across the disk by the rippling along the limb, so that at any moment one part of the limb may be steady, and other parts unsteady. In the overlapping images of the photoheliograph, two points of one limb fall upon two points of the opposite limb, and to have a perfect photograph these four points of the limb must have sharp definition. It is not difficult to get one image in the common area sharp, but to get both sharp requires unusual steadiness. Any distortion of one image in the points of intersection will increase or decrease the length of the common chord, and will give an erroneous value for the radius. The distortion will be a change in length of the solar radius, and this will increase or decrease the length of the chord. As the measured length of the chord is a delicate test of the length of radius, according to the formula:

$$\Delta d = \frac{r}{d} \Delta r,$$

any error in its length will multiply the error in determining the radius, and in just the ratio $\frac{r}{d}$.

In measuring a simple solar image, such local errors from bad seeing are not magnified, and may moreover be eliminated by taking the mean of several measures of the diameter near the equatorial and axial diameters. Here we have not this possibility, and to offset the disadvantage, more plates must be measured than in the older method. On account of the simplicity of measurement and reduction, however, this is not a costly expedient.

REFRACTION FORMULAE

Let $L'L''$ be the two lenses mounted in a single cell at a fixed distance apart and with the optical axes $L'C'$ and $L''C''$ parallel. The cell with the two lenses can be revolved about the common collimation axis LC.

Each lens will form an independent image of the sun, and these images will appear on the photographic plate, P, with their respective centers at C' and C''. If the adjustment of the lenses is perfect, each image will be an exact duplicate of the other, and if there be no refraction they will be sensibly circular. The distance between the centers of the two images, which will be equal to that between the optical centers of the two lenses, is one of the constants of the instrument, and will be denoted by $2a$. From the measured value of the common chord, $2d$, the radius of the image, passing

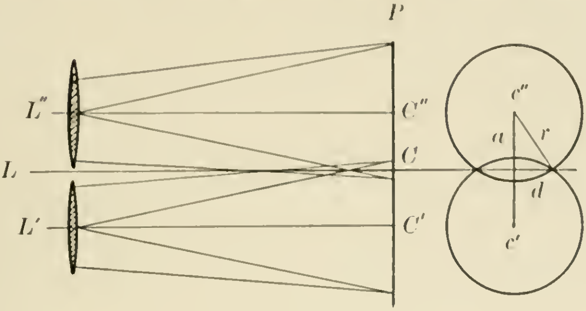

FIG. 1

through the extremity of the chord, can be readily found by the formula,

$$R=\sqrt{a^2+d^2}\,. \tag{1}$$

Refraction deforms the apparent disk of the sun into a figure of unknown shape which, however, is symmetrical with respect to the

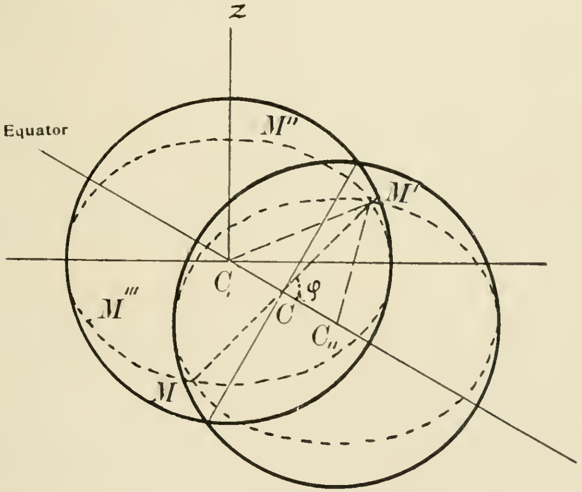

FIG. 2

vertical axis through the center. The real images which appear on the photographic plate are therefore two symmetrical but unknown curves (approximately ellipses) whose centers are at $C_{,}$ and $C_{,,}$. These deformed images are shown in Fig. 2, which represents the case when the line joining the centers of the two lenses is parallel to the equator of sun. The image formed by lens I has its center at $C_{,}$, the image formed by lens II is identical with that formed by I, but it has been moved, as it were, along an axis parallel to the sun's equator until its center is at $C_{,,}$. The common chord of the two images is the line MM', which passes through C, the point in which the collimation axis of the instruments cuts the focal plane. The ends of the chord, M and M', do not correspond to real points in the heavens, they are merely the intersections of the two curves in the focal plane, or on the photographic plate. The points of the two curves, which coincide on the image, and form the the point M, in reality lie on opposite sides of the sun. It is thus incorrect to speak of "the refraction" of the point M; yet this point of intersection is changed by the effects of refraction, changed by the combined effects of the refraction of the points M and M'''.

If the sun be photographed at a very small altitude, then the refractions of the four points M, M', M'', and M''' will be different. The radius CM, for example, is not only shortened by refraction, but it is also turned through a minute angle. The entire refraction effect is differential, and formulae for finding the actual amount of the differential refraction both in radius and position angle were developed by Bessel. These formulae require the use of the position angle at the middle point of the distance measured and, therefore, are not entirely suitable for photographic plates. Jacoby made a slight modification in these formulae and adapted them for photographic work. They are:

$$\sigma - s = sx\,[\tan^2 \xi \cos^2 (p-q) + 1] + \text{terms of second order}$$

$$\begin{aligned}\pi - p = &-x \operatorname{cosec} 1'' \tan^2 \xi \sin (p-q) \cos (p-q)\\ &-x \operatorname{cosec} 1'' \tan \xi \sin q \tan \delta\\ &+ \text{terms of second order},\end{aligned}$$

in which s and p are the measured distance and position angle, and σ and π the same corrected for refraction. These formulae of Jacoby are identical with those of Bessel, with the exception of the terms of the second order, which terms are usually so small as to be entirely inappreciable. The term:

$$-x \operatorname{cosec} 1'' \tan \xi \sin q \tan \delta\,,$$

which affects all the position angles alike on any given plate, may be neglected and will therefore be omitted from the above formula.

At opposite ends of a diameter the second-order terms are equal, but of contrary sign. They only begin to become appreciable, however, when the altitude of the sun is less than 26°, at which point these terms may amount to as much as $\pm 0''.013$. At altitudes greater than this, therefore, the refractions of M', and M''' are sensibly equal, as are also those of M'' and M. Hence at altitudes greater than 26°, refraction deforms the solar image (assumed to be a circle) into a curve which is sensibly symmetrical with respect to the horizontal as well as the vertical axis.

At about 17° altitude the second-order terms become as great as $\pm 0''.05$, and at smaller altitudes the terms increase very rapidly. The figure is no longer symmetrical with respect to the horizontal axis and the common chord MM' no longer bisects the line $C'C''$. But for all practical altitudes it may be assumed that these two lines mutually intersect; the error in this assumption being only of the third order. Denoting the refraction of the radii drawn to M, M', M'', and M''' by r, r', r'' and r''' respectively, we shall have for the true lengths of these radii,

$$\begin{gathered} C_{,}M \;\; + r \\ C_{,}M' + r' \\ C_{,}M'' + r'' \\ C_{,}M''' + r'''. \end{gathered}$$

And without sensible error we may take,

$$\begin{aligned} r &= r'' \\ r' &= r''' . \end{aligned}$$

Further, if the sun itself be regarded as a figure of revolution about its polar axis, then radii symmetrically placed with respect to the equator will be equal, and the four above-mentioned radii will all be equal. We have, therefore,

$$C_{,}M' + r' = C_{,}M'' + r'' = R\,;$$

but $C_{,,}M'$ is $C_{,}M''$ moved into a new position by the action of the second lens. Hence we have

$$C_{,}M' + r' = C_{,,}M' + r'' = R\,, \qquad (2)$$

where R is the true length of the sun's radius.

From this we find at once

$$C_{,}M'-C_{,,}M'=r''-r'. \tag{3}$$

Now in the two triangles $C_{,}M'C$ and $C_{,,}M'C$ the sides $C_{,}C$ and $CC_{,,}$ are equal, each being one-half the distance between the optical centers of the two lenses. This is a constant of the instrument and has already been denoted by a. The distance CM' is one-half the measured chord and is denoted by d. The angle at which $C_{,}C_{,,}$ and $M'M$ intersect will be denoted by ϕ; it differs but slightly from a right angle.

From trigonometry we have:

$$\overline{C_{,}M'}^2=a^2+d^2+2ad\cos\phi$$

$$\overline{C_{,,}M'}^2=a^2+d^2-2ad\cos\phi\ ,$$

whence by addition,

$$\overline{C_{,}M'}^2+\overline{C_{,,}M'}^2=2(a^2+d^2)\ . \tag{4}$$

By solving equations (3) and (4) the values of $C_{,}M'$ and $C_{,,}M'$ can be found as follows:

$$C_{,}M'=\tfrac{1}{2}(r''-r')+\sqrt{a^2+d^2-\tfrac{1}{4}(r''-r')^2}$$

$$C_{,,}M'=-\tfrac{1}{2}(r''-r')+\sqrt{a^2+d^2-\tfrac{1}{4}(r''-r')^2}\ . \tag{5}$$

Whence from (2) the length of the radius will be:

$$R=\tfrac{1}{2}(r''+r')+\sqrt{a^2+d^2-\tfrac{1}{4}(r''-r')^2}\ , \tag{6}$$

and this determines the corrected radius in terms of the refractions and the measured chord.

The radius thus found, however, is not the equatorial radius, nor is it any single radius. It is the mean value of two radii, symmetrically placed on each side of the equator. The angle at which these radii are inclined to the equator may be found with sufficient accuracy by taking

$$\tan\theta=\frac{d}{a}\ . \tag{7}$$

This assumes the chord to be perpendicular to the line joining the centers, and therefore neglects the slight tilting effect of refraction. This tilting effect amounts to only a few minutes of arc and the consequent error in θ is small.

If the centers between the two lenses should be separated to such an extent that the two images become tangent, then the chord vanishes and the instrument becomes a heliometer of the ordinary form. In this case the refractions r' and r'' are equal and formula (6) reduces to,

$$R=a+r'. \tag{8}$$

In this case a is variable and is measured by a micrometer screw. An error in the measurement of a will directly effect R to its full value.

In the photographic method a is constant and d is measured. The effect of an error in measuring d upon the resulting value of R can be found by differentiating (6), regarding a and the refractions as constant as follows:

$$\Delta R=\frac{2d\cdot\Delta d}{2\sqrt{a^2+d^2-\frac{1}{4}(r''-r')^2}}\,.$$

But, neglecting refraction, the value of the denominator of the above fraction is equal to R: hence, as heretofore given, we have:

$$\Delta R=\frac{d}{R}\Delta d \text{ (approximately).} \tag{9}$$

Therefore an error in measuring d will affect R only in the ratio $\frac{d}{R}$. If the half-chord subtend an angle of $5\frac{1}{2}°$ then $\frac{d}{R}=0.1$, and an error of measurement will affect the result by only $\frac{1}{10}$ of its amount. In the test photographs made at the Yerkes Observatory the scale was such that one millimeter was approximately equal to $27''.0$ and the chord was one-third the radius. Hence an error of measurement of 0.01 mm in the chord would cause an error of only 0.0033 mm, or $0''.10$ in the computed value of the radius. On good photographs quantities of this size can be easily measured.

EXPERIMENTAL RESULTS

As an example of the reductions I will refer to the plates taken at the Yerkes Observatory by Mr. Fox on November 12, 1907. The data, as furnished by him, follow:

Plate	Σe 137	Σe 138
Date	November 12, $10^{h}\ 58^{m}.7$	$11^{h}\ 2^{m}.3$
Diameter	Equatorial	Polar
Barometer	29.33 in.	29.33
Att. Thermometer	$48°.5$	$48°.5$
External Thermometer	31.0	31.0

The plates were measured with the Repsold measuring machine of

Columbia University, on different days and by different observers, with the following mean results:

	Σe 137	Σe 138
Harpham	32.686	32.560
Poor	.670	.540
Davis	.698	.547
Mean	32.6847	32.5490
d	16.3424	16.2745

From measures on a series of plates a preliminary value of the distance between the centers of the two lenses was obtained. This gave for the half distance, or instrumental constant,

$$a = 31.9424$$

and from the formula,

$$R = \sqrt{a^2 + d^2},$$

an approximate value of the apparent radius may be calculated for each plate. Then from the given data can be computed the zenith distance of the center of the sun at the time each plate was taken, and thence, with the approximate values of the apparent radius, the refractions for the points of intersections. We thus find:

	Σe 137	Σe 138
Zenith distance	60° 47′ 10″	60° 40′ 3″
Parallactic angle	— 8° 21′ 19″	— 7° 36′ 28″
θ	27° 5′ 17″	27° 3′ 45″
Approximate radius	35.8803 mm	35.8493 mm
Refraction, r''	+ 0.0330 mm	+ 0.0422 mm
" r'	+ 0.0102 mm	+ 0.0200 mm
$(r''-r')$	+ 0.0228 mm	+ 0.0222 mm
$\frac{1}{2}(r''-r')^2$	+ 0.0001 mm	+ 0.0001 mm
$\frac{1}{2}(r''+r')$	+ 0.0216 mm	+ 0.0311 mm

From formula (6) now are found the corrected values of the radii, and reducing the results to distance unity, we have finally:

Equatorial radius, plate Σe 137 = 35.5296 mm
Polar radius, plate Σe 138 ..... = 35.5082 mm
Mean radius .... = 35.5189 mm
Polar minus equatorial .... = −0.0214 mm

The scale value was approximately determined as:

$$1 \text{ mm} = 27.''06.$$

And this, combined with the above value for $P-E$, shows that on this date the radius measured on the equatorial plate exceeded that on the polar plate by

$$0''.57 .$$

These radii cut the surface of the sun in latitudes 26° and 64° respectively.

A series of similar plates taken on various days during October and November give the following results:

DATE	MEAN RADIUS	RESIDUALS		$P-E$ ARC
		mm	Arc	
Oct. 18......	35.538	+033	+0″.89	−0″.81
21......	.469	−036	−0.97	−1.16
22......	.497	−008	−0.22	−1.43
23......	.536	+031	+0.84	−1.08
Nov. 8......	.469	−036	−0.97	−0.62
12......	.519	+014	+0.43	−0.57
Means........	35.505			−0.95

CONCLUSIONS

The photographic tests made at the Yerkes Observatory establish the value of the photographic method and of the photo-heliometer, but they showed that in order to obtain the most satisfactory results several essentials must be observed:

1. The results derived from the use of the photo-heliometer are more satisfactory and accurate than those derived from direct photographs of the sun. The measurement of the chord common to the overlapping images can be made more easily and rapidly than can the measurements of the diameter on a direct image of the sun. And again, the reductions and computations connected with a heliometer measure are very much more simple than are those necessary to determine the relative lengths of the diameters from measures upon a direct photograph of the solar disk. It takes far longer to measure and reduce a number of diameters on a direct photograph of the sun, than it does to measure and reduce an equal number of heliometer images. The defects from bad seeing, heretofore mentioned, can be eliminated by taking the mean of a number of plates made on the same day. Hence for the purpose of determining the relative lengths of various diameters of the sun, and of possible variations therein, the

photo-heliometer would seem to be more efficient than the direct photographic telescope.

2. The photographic lenses, or half-lenses of the heliometer, must be carried on a separate and specially designed equatorial mounting, so constructed that the shutter is independent of the mounting. If attached to the mounting, the shutter will almost invariably cause jars and consequent distortions of the image. To obtain images of sufficient size for accurate measurement the focal length of the objective must be comparatively great; from twenty-five to fifty feet. To satisfactorily mount and house such a large instrument is a complicated and very expensive matter. The use of such a large and cumbersome instrument might be overcome by the use of a fixed magnifying system, somewhat similar to tele-photo lenses. By making the optical system fixed, so that the same magnification would always be obtained, many of the objections to the use of a magnifying eyepiece would be overcome, and the total length of the instrument kept within reasonable and handy limits. Simple photographic telescopes, using this principle of magnification and made by Steinheil, have been in use for some time.

3. Wet plates should be used. On the modern rapid dry plates the edges of the image are not as sharp and clear as those on the old collodion plates.

Columbia University
1908

MINOR CONTRIBUTIONS AND NOTES

NOTE ON ATMOSPHERIC TRANSMISSION BY NIGHT AND BY DAY

M. Ch. Nordmann, of the Paris Observatory, announces (*Transactions of the International Union for Co-operation in Solar Research,* **2**, 48, 1908) that "in many cases, especially when it is very clear, the distribution of atmospheric absorption in the spectra of the stars appears to be notably different from what it is when studied in the solar spectrum." He says: "I am able to state in the majority of cases and when the sky is clear, that for zenith-distances less than about 70° (because beyond this point the phenomenon returns to the diurnal type), the violet and blue parts of the spectrum are notably less absorbed at night than by day. I have found that, in certain cases, this part is less absorbed than the red part itself." He concludes that the atmospheric absorption is different at night from what it is in the daytime.

In regard to M. Nordmann's hypothesis of a difference in the absorption of short waves by day and by night, the following considerations show that it cannot be admitted: The human eye has an extraordinary range of sensitiveness for different colors. For example, my own eye is 790,000 times as sensitive for green light of wave-length 0.5 μ, as it is for deep-red light, $\lambda = 0.768\ \mu$. Moreover, although the change is not as rapid in the brighter part of the spectrum, there is still a variation of sensitiveness in the ratio 1:10 in passing from $\lambda = 0.60\ \mu$ to $\lambda = 0.55\ \mu$. Consequently, differences in the distribution of the more intense groups of absorption lines, such as occur in the spectra of stars of different types, may shift the point of maximum brilliancy into regions of the spectrum for which the eye has very diverse degrees of sensitiveness. A comparatively slight change in the wave-length of the ordinate of maximum brightness in the spectral luminosity-curve, due to absorption of the rays by the atmospheres of either the star or the earth, may thus correspond to a much more considerable change in the star's apparent brightness on account of the unequal sensitiveness of the eye to the combined rays when those wave-lengths to which the eye is especially sensitive are either deficient, or present in excess. Thus the change of apparent brightness for the same difference of altitude may vary considerably in the case of two stars whose original light-distribution is not the same in the region of the spectrum which chiefly contributes to the visual impression. Much larger changes in other parts of the spectrum, such as the violet, which experience strong absorption, but do not affect the eye much, will have relatively little effect on the apparent magnitude of the star obtained by eye estimate, using the sum of all visible rays. Thus the apparent transmission of star light, determined photometrically, is much larger than the combined transmission of solar luminous rays deduced from spectrobolometric observations at high and low altitudes, the reason being that, while the stellar observation ostensibly relates to luminous rays of every wave-length, it actually or virtually refers to a limited region of the visible spectrum which has a larger transmission-coefficient than the average for all kinds of light. Some incongruity of light-distribution in the region of the spectrum to which the eye is most sensitive is probably responsible for discrepancies which have been noted in photometric observations of stars of different types at high and low attitudes.

The visual phenomena described above, however, do not affect photometric comparisons made in the spectrum. Here another cause is at work; for by night and in feeble light, the eye is much more sensitive to violet than by day, and is relatively much more sensitive to light of enfeebled intensity in the violet than to equally enfeebled green or yellow light, whenever the luminous intensities are equally dim. This is due to a physiological cause which appears to be connected with the quite gradual formation of the violet pigment which assists in the absorption and thence in the recognition at the retina of short waves, and which is more or less destroyed by exposure of the eye to strong daylight. Thus a rest of 15 or 20 minutes in complete darkness is required to restore the sensitiveness of the eye to violet light, whereas a much shorter period suffices for the longer light-waves. Hence the apparent increase of atmospheric transmission of violet rays by night, noticed by M. Nordmann, finds its explanation in physiological causes. It will be noted that as the star sinks lower and becomes fainter, the eye becomes relatively more sensitive to violet light; but even this is insufficient to reverse the great absorption of violet rays at very low altitudes of the stars, since M. Nordmann finds that when the zenith-distance exceeds 70°, the ordinary daylight law is restored. In photometric comparisons made at night in the lunar spectrum where there is abundance of light, I find no other change at high and low altitudes of the moon, and no other law of atmospheric absorption than that obtained from solar observations in the daytime, which includes minor variations, but never a reversal of the order of absorption for violet and red.

FRANK W. VERY

WESTWOOD, MASS.
March 31, 1909

THE ARC SPECTRUM OF IRON

The article on the arc spectrum of iron, λ 6855 to λ 7412, by E. J. Evans, published in the March number of this Journal, does not refer to any earlier measurements of the iron lines in the region of longer wave-lengths than λ 6855. I may therefore be permitted to say that a careful measurement of the portion of the iron spectrum in question, extending to λ 7600, was made in a thesis prepared under Professor Kayser's direction, which perhaps escaped the attention of Mr. Evans on account of the place of its publication. The reference is: E. A. Kochen, *Der rote Teil des Eigenbogenspectrums*, pp. 21. Dissertation, Bonn 1909; *Zeitschrift für wissenschaftliche Photographie*, **5**, 285–299, 1907.

H. KONEN

PHYSIKALISCHES INSTITUT
MÜNSTER i/W.

NOTICE OF INCREASE IN SUBSCRIPTION PRICE

With the thirtieth volume of the *Astrophysical Journal*, beginning with the July number, 1909, the annual subscription price will be increased by one dollar, thus becoming five dollars, or $2.50 per volume of five numbers. This action becomes necessary as the result of the increasing cost of production, particularly in respect to printing and engraving. The editors have reluctantly yielded to the necessity of this change, although, in view of the number of pages per year and of the quantity and quality of the engravings, the *Journal* has been relatively much cheaper than its contemporaries foreign or domestic. An increase cannot be asked in the subsidy granted by the University of Chicago, amounting to about 60 per cent. of the cost of publication. The editors believe that the subscribers will prefer to have the subscription price raised rather than to have the size of the *Journal* reduced or the average number and quality of the illustrations diminished.

Particulars as to the price of foreign subscriptions and of single numbers will be found in the publishers' notice elsewhere in this number.

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MEASUREMENTS OF WAVE-LENGTHS OF THE SPARK SPECTRA OF CHROMIUM AND CALCIUM; ALSO OF THE ARC SPECTRA OF CERIUM AND THORIUM; TOGETHER WITH A STUDY OF THE POSSIBLE INFLUENCE OF VARIATIONS OF CURRENT, CAPACITY, ETC., IN THE SPARK CIRCUIT[1]

By FRANK LAWRENCE COOPER

I. MEASUREMENT OF SPARK SPECTRA

INTRODUCTION

The spark spectra of chromium and calcium are of considerable interest since the solar spectrum contains a great number of lines which correspond to those found in the spectra of these metals.

Wave-lengths of the spark spectra of chromium and calcium have been made by Huggins, Thalén, Kirchhoff, Hartley, Adeney, Lockyer, and Liveing and Dewar;[2] but the most recent determinations are those of Exner and Haschek,[3] being a part of their great work on the ultra-violet spark spectra of the elements.

The last observers made all their measuremens from plates taken in the first-order spectrum of a 15-foot concave grating. Their method of measurement, which was not entirely free from objections, was to project an enlarged image of their negatives upon a screen where it fell upon a scale of 5 mm divisions, from which they read off the wave-lengths. Considering these two facts it is reasonable to expect greater accuracy from measurements made with a carefully constructed dividing engine and upon plates taken in the second-order spectrum of a 21-foot concave grating, as was the case with the wave-lengths between λ 2900 and λ 5000 given in the following tables. For these reasons it was thought desirable to make a more careful study of the spectra of these metals.

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[1] Dissertation submitted to the Board of University Studies of the Johns Hopkins University in conformity with the requirements for the degree of Doctor of Philosophy, June, 1907.

[2] Watts, *Index of Spectra.*

[3] *Sitz. der Kais. Akad. der Wiss. in Wien,* 1897, and following years.

THE PRESENT MEASUREMENTS: APPARATUS

The grating employed in this work was, as mentioned above, a 21½-foot concave Rowland grating with 20,000 lines to the inch, mounted as described by Professor Ames in the *Philosophical Magazine* (5), **27**, 369, 1889, and being in all respects the same instrument used by Professor Rowland in his work on the solar spectrum. For the purpose of comparison of spectra, the camera carries a shutter consisting of a long brass plate, capable of revolution around a horizontal axis and having, throughout its length, a horizontal opening of width equal to the thickness of the plate; so that when the shutter is in a vertical position, the photographic plate is exposed along a strip extending over the whole length of the plate and over a width equal to the thickness of the shutter. When the shutter is turned horizontally, the central strip is covered and the remainder of the plate above and below the strip is exposed. The camera-box was fixed rigidly upon the track by means of wedges placed under the wheels carrying the camera-box.

The spark was produced by a 110-volt alternating current of about 30 amperes with a frequency of 60 cycles per second. After being stepped up to about 1000 volts, it was passed through a transformer known as the "Rowland coil," which raised the potential sufficiently to produce a spark about two centimeters in length. The "Rowland coil" consists of a primary of 8 layers of 150 turns each of No. 12 insulated wire, and a secondary of 98 layers of 164 turns each of No. 25 copper wire. The spark terminals were pieces of the metal to be studied, turned down at the spark ends to the shape of a blunt lead-pencil point. The points would wear down slightly during a long exposure, but care was taken to turn them down to the proper shape before each and every exposure. They were given this shape so that the spark would keep on the slit throughout an exposure.

The arc for the comparison spectrum was produced by a 110-volt direct current of about 10 amperes.

Both the arc and spark lamps were placed (the arc nearer the slit) upon a platform which was movable upon two knife-edges in a direction parallel to a line drawn through the centers of the slit and grating respectively. Stops were placed before and behind the platform at such a distance from it, that when it was against one of them the arc was in focus, and when it was against the other the spark was in focus; that is, in both cases the source of light was in precisely the same place at a distance of 130 centimeters from the slit.

The light was focused upon the slit by means of a quartz condensing lens which was never disturbed during the exposure on any one given plate, so that all exposures would have the grating illuminated as near as possible the same. The spark-gap was usually 8 or 9 mm in length and its image on the slit about 6 mm.

In the secondary circuit, there were several condensers consisting of plates of copper foil and glass surrounded with transformer oil. The capacity could be varied from 0.0036 to 0.245 microfarad.

The plates used were Seed's Orthochromatic, Seed's No. 27, Cramer's Isochromatic, and Wrattan & Wainwright's Panchromatic, all cut $1\frac{1}{4}$ by 19 inches, and were developed with a hydrochinone solution according to Jewell's formula.[1] All measurements on plates were made with the dividing engine which is fully described by Humphreys in the *Astrophysical Journal*, 6, 180, 1897, and also by the method described at the same place.

METHOD OF EXPOSURE

As stated before, the arc and spark lamps were on a movable platform. The spark spectrum was usually taken first; the shutter turned, and then the platform, which was independent of the mounting of the grating, was carefully slipped back on the knife-edges, thus placing the arc in exactly the position previously occupied by the spark.

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[1] *Astrophysical Journal*, 11, 240, 1900.

LINES USED AS STANDARDS

The standards used were iron lines whose wave-lengths have recently been determined by Fabry and Perot, that is, for that part of the spectrum extending from λ 2373.737 to λ 6494.994, which were the limits of Fabry and Perot's measurements. For the shorter wave-lengths the standards used were lines whose wave-lengths have been redetermined by Mr. L. E. Jewell, basing his measurements upon the original wave-lengths of Professor Rowland.

CORRECTIONS

A calibration curve was drawn for each and every plate with wave-lengths as abscissae and the difference between readings and the arc standards as ordinates. From these curves the corrections for intermediate lines were read off.

TABLES OF WAVE-LENGTHS

In the tables, *r* means that the line is reversed, *d* that it is doubly reversed, and *tr* that it is reversed three times. *R* and *V* indicate that the line shades off on the red or violet side respectively. *n* means that the line is nebulous; and *br* that it is broad.

In making the estimates of relative intensities between the arc and spark lines, the iron line 4045.969 was taken as standard. This intensity is marked 30 and corresponds approximately to Kayser's mark 10.

CALCIUM

Wave-Length	Intensity		Wave Length	Intensity	
	Spark	Arc		Spark	Arc
2208 82 *br*	0		4526.976 *R*	0	20 *r*
2372.197	1	1	4578.564	4	20
2398.565	0 *r*		4581.405	5	30
2994.960	1	1	4585.876	7 *R*	
2997.338	2	3	4685.872	1 *R*	30
2999.650	1	2	4778.138	2 *R*	3
3000.871	1	1	5041.610	1	2
3006.875	0	2	5188.859	2	3
3009 194	1	1	5260.393	0 *r*	0
3158.880	50 *r*, *R*	10	5261.708	3 *r*	3
3179.342	50 *r*, *R*	15	5262.254	3 *r*	3
3181.270	10 *R*	3	5264.247	3 *r*	3
3487.748	1	2	5265.563	3 *r*	3
3602.09 *br*	1		5270.276	3 *r*	3
3624.136	2	10	5349.450	4 *r*	4
3631.004	3	8	5512.987	4 *r*	4
3644.410	3 *r*	20	5581.978	4 *r*	4
3706.164	50 *R*, *r*	10	5588.780	6 *r*	6
3736.920	50 *R*, *r*	20	5590.225	3 *r*	3
K 3933.786	1000 *r*	500 *r*	5594.474	4 *r*	4
3948.902	0 *r*	8	5598.598	4 *r*	4
3957.035	2 *r*	8	5601.287	3 *r*	13
H 3968.488	500 *r*	300 *r*	5602.765	3 *r*	3
3973.724	1 *V*, *r*	8	4557.456	8 *r*	8
4092.86 *br*, *n*	0	2	5967.560	2 *r*	2
5094.95 *br*, *n*	1	4	6102.824	9	9
4098.53 *br*, *n*	1	4	6122.218	10 *n*	10
4132.55	00		6161.294	4	4
g 4226.751	100 *r*	1000 *r*	6162.176	15 *r*	15
4240.426	0	2	6163.757	3	3
4283.014	20 *r*	50	6166.538	4	5
4289.372	20 *r*	50	6169.036	6	6
4298.995	20 *r*	30	6169.565	6	7
4302.522	50 *r*	100	6439.084	8 *r*	8
G 4307.736	20 *r*	100	6449.820	6	6
4318.645	30 *r*	50	6455.213	1	2
4355.087 *br*, *n*	1	2	6462.577	5	5
4425.440	20 *dr*	*V* 100	6471.665	4	5
4434.965	20 *dr*	100 *r*	6493.784	6	6
4435.687	15 *dr*	50	6499.662	4	4
4454.874	30 *r*	200 *r*	6572.714	1	1
4456.625	15 *r*	20	6717.721	5	5
4456.625	5 *r*	8			

CHROMIUM

2132.749	0		2144.300	0	
2133.568	0		2150.815	0	
2134.651	0		2170.777	0	
2135.490	0		2171.196	0	
2141.285	0		2183.799	0	

CHROMIUM—*Continued*

Wave-Length	Intensity		Wave-Length	Intensity	
	Spark	Arc		Spark	Arc
2185.098	0		2306.940	0	
2190.895	0		2370.278	1	
2191.710	0		2310.101	0	
2198.009	0		2314.729	1	
2198.698	0		2314.729	1	
2203.325	0		2314.820	1	
2208.788	0		2319.150	1	
2211.897	0		2319.489	1	
2213.883	0		2320.200	1	
2217.591	0		2324.985	2	
2218.752	0		2330.103	0	
2219.700	0		2333.551	1	
2226.781	0		2333.970	0	
2231.900	0		2334.330	0	
2233.873	0		2334.508	0	
2235.970	1		2334.618	0	
2237.651	1		2337.817	0	
2241.430	0		2340.551	0	
2241.902	0		2344.645	0	
2243.390	0		2345.407	0	
2243.740	0		2365.290	2	
2244.213	1		2366.891	1	1
2244.970	0		2381.586	1	1
2247.809	0		2389.859	1	1
2248.000	0		2394.100	1	
2248.408	0		2397.848	1	
2248.693	0		2397.848	1	
2249.950	0		2398.600	0	1
2250.098	0		2399.745	0	1
2251.617	0		2400.318	0	
2252.140	0		2404.054 *br*	0	
2256.149	0		2408.764	0	1
2256.860	0		2413.722	0	
2257.250	0		2416.471	1	
2257.002	0		2419.490	0	
2257.896	0		2419.986	0	
2258.147	1		2420.206	0	
2258.250	0		2425.292	0	
2258.750	0		2425.745	0	
2261.829	0		2428.448	0	
2265.020	0		2429.746	0	
2273.502	1		2430.204	0	
2275.601	0		2433.305	1	
2276.560	1		2435.392	0	
2277.578	0		2438.551	1	
2284.580	1		2445.178	0	
2287.300	0		2445.649	0	
2289.309	0		2446.148	0	
2290.755	1		2446.959	0	
2295.640	1		2449.701	1	
2297.270	1		2450.020	1	
2300.589	1		2450.443	0	

CHROMIUM—*Continued*

Wave-Length	Intensity		Wave-Length	Intensity	
	Spark	Arc		Spark	Arc
2452.704	0		2516.000	0	
2454.140	0		2516.717	1	2
2454.545	1		2518.410	2	2
2455.302 *br*	0		2519.646	0	2
2456.87 *br*	0		2520.744	1	
2458.902	0		2522.79	0	
2459.051	0		2523.402	3	
2460.542	0		2523.746	1	
2460.950	0		2527.195	0	2
2462.437	0		2527.516	0	
2465.024	0		2530.057	2	
2465.728	0		2530.304	1	1
2465.865	0		2531.097	1	
2466.334	0		2531.935	0	
2466. 4 *br*	1		2534.410	1	1
2469.222	1		2536.900	0	
2469.605	0		2537.805	0	
2470.944	0		2538.419	4	
2472.955	1		2539.046	0	
2475.028	0		2542.880	0	
2475.770	0		2543.229	1	
2477.025	0		2544.410	0	
2477.852	0		2545.287	0	1
2479.908	1	1	2546.068	0	
2483.184.	1		2246.547	0	
2483.903	2		2547.650 *br*	0	
2485.604	0		2548.146	1	
2486.413	1		2548.668	1	
2486.752	0		2550.127	0	
2487.176	0		2550.416	0	
2489.413	1		2551.697	2	
2489.870	0		2552.001	0	
2490.209	0		2555.647	2	
2490.87 *br*	0	1	2557.149	0	2
2492.747	1	2	2557.570	0	
2492.990	1	2	2558.500	0	
2493.387	1		2559.924	1	
2496.52 *br*	0	2	2560.814	0	2
2497.04 *br*	0		2561.134	0	
2497.983	0		2561.951	0	
2498.945	1	1	2562.603	0	
2501.602	0		2563.448	1	
2504.432	0	2	2563.696	2	
2505.937	0		2564.864	0	
2506.185	0		2566.447	0	
2506.49	0		2566.595	0	1
2510.346	0	1	2566.993	0	
2511.310	0		2567.470	0	
2512.109	0	1	2567.71 *br*	0	
2512.518	0		2568.704	1	1
2513.817	2	1	2569.572	0	
2515.201	2		2570.940	0	

CHROMIUM—*Continued*

Wave-Length	Intensity		Wave-Length	Intensity	
	Spark	Arc		Spark	Arc
2571.896	1	2	2632.693 *br*	0	
2572.239	0	1	2633.746 *br*	0	
2573.658	1	1	2634.447 *br*	0	
2574.285	0		2635.874 *br*	0	
2575.898	0		2636.447	0	
2578.398	1	2	2637.290	0	
2579.204	0	1	2637.601	0	
2582.029	0	1	2639.248 *br*	0	1
2582.350	0		2640.100 *br*	0	1
2583.734	0	1	2640.827	0	
2584.206	1		2641.518	0	
2585.027	0		2641.946	0	1
2585.720	0		2643.669	0	
2587.508	1	1	2647.612	0	
2588.106	0	1	2648.240	0	
2588.306	0	1	2650.947	0	
2589.150	0		2652.215 *br*	1	
2589.806	1		2663.696	3	3
2590.540	0	1	2655.897	0	
2590.865	2	3	2657.710 *br*	0	
2591.944	0		2658.695	2	3
2595.664	1		2659.047	1	3
2596.197	1	1	2659.586	0	
2601.978	0		2659.890	0	
2603.051 *br*	0	1	2660.896	0	
2603.816	0		2661.485	2	
2604.247	0		2661.850	2	1
2605.730	0		2663.147	0	
2606.169	0		2663.588	4	3
2606.640	1		2663.800	2	1
2607.729	0		2665.798 *br*	1	
2608.003	1		2666.195	6	3
2608.247	0	1	2668.076	1	
2608.906	0		2668.835	5	3
2610.194	0	1	2670.215	2	
2610.930 *br*	1		2670.406	1	
2611.103 *br*	0		2671.953	6	3
2611.697	0	1	2672.505	1	1
2612.48	0	1	2672.935	4	3
2613.307	0	1	2675.396	0	
2613.616	0		2675.791	2	
2614.715	1		2676.675	0	
2616.301	0		2677.270	2	4
2616.548	0		2678.874	8	3
2618.854	0	1	2680.005	0	
2619.729	1	1	2680.396	1	1
2620.626	1	1	2683.550	1	
2623.472	1	2	2684.227	0	
2626.146	1		2685.165	0	
2626.885	1	1	2685.252	0	
2628.006	1		2686.105	0	
2631.005	2		2686.490	0	

CHROMIUM—*Continued*

Wave-Length	Intensity		Wave-Length	Intensity	
	Spark	Arc		Spark	Arc
2687.174	5	3	2733.690	0	
2688.394	10	2	2734.859	0	
2689.278	2		2735.828	0	
2689.908	0		2737.140	0	1
2690.504	0	1	2739.499	1	2
2691.147	10	3	2739.841	0	
2692.235	2		2740.172	2	1
2693.615	5		2741.165	1	2
2694.800	0		2742.128	5	2
2696.853	1	1	2743.704	6	
2697.594	3		2744.367	1	
2698.015	4	1	2745.043	2	
2698.515	3	3	2745.486	0	
2698.734	2	3	2746.263	3	
2698.940	3	1	2747.950	0	
2699.496	0		2749.023	7	3
2700.710	0		2749.885	1	
2701.221	1		2750.817	10	3
2701.215	1		2751.965	10	1
2701.788	0		2752.490	0	
2702.115	0		2762.903	0	3
2702.670	0	1	2753.286	0	
2703.087	0		2753.753	0	
2703.645	5	1	2754.000	1	
2703.951	1	1	2754.396	2	
2704.883	0	1	2755.115	0	1
2705.549	0	1	2755.340	1	1
2706.227	0		2756.384	2	
2706.665	0	1	2757.036	1	4
2708.904	4		2757.805	7	3
2709.420	4		2758.723	0	
2711.015	4		2759.075	3	
2711.306	0		2759.496	3	1
2712.413	5	2	2759.825	2	
2715.750	0		2760.145	1	
2717.096	0		2760.406	1	
2717.587	2	1	2760.485	1	
2718.210	1		2760.612	1	
2718.485	6		2760.957	0	
2719.157	0		2761.845	0	3
2720.178	2		2762.701	10	3
2720.365	1		2762.846	1	1
2720.787	0		2763.695	1	
2722.825	4	2	2764.096	0	
2723.688	3		2764.395	1	
2724.121	3		2765.147	0	
2726.362	0		2765.583	1	
2726.583	0	5	2765.700	0	
2727.317	4	1	2766.005	1	
2728.250	0		2766.624	15	4
2729.829	0		2767.353	0	
2731.989	0	5	2767.692	0	1

CHROMIUM—*Continued*

Wave-Length	Intensity		Wave-Length	Intensity	
	Spark	Arc		Spark	Arc
2768.253	0		2826.198	0	
2768.682	3		2826.285	1	
2759.451	0		2826.550	0	1
2760.983	0	6	2828.078	0	
2771.405	0	1	2828.906	0	
2772.032	1		2830.635	20	1
2772.477	0		2832.592	3	
2773.425	2		2833.586	0	
2774.561	2		2834.397	3	
2776.754	1		2835.707	25	4
2778.175	3	1	2836.590	1	
2779.053	0		2838.000	1	
2780.415	4	8	2838.903	4	1
2781.027	1		2839.237	0	
2781.205	1	1	2840.137	7	1
2782.480	2		2840.566	0	
2782.697	0		2842.517	0	
2783.959	0		2842.895	0	1
2784.450	0		2843.350	15 *r*	4
2785.249	0		2846.497	2	
2785.820	7		2846.796	1	
2786.580	2		2848.500	1	
2787.715	3		2849.456	0	
2788.003	0	1	2849.936	10	4
2789.184	0		2850.447	0	
2789.507	3		2850.740	0	
2792.256	10		2851.485	6	1
2793.765	0		2852.390	0	
2795.643	1		2852.795	0	
2798.803	2	1	2853.346	2	
2800.290	1		2853.935	0	
2800.884	10	1	2854.271	0	
2803.447	0		2854.713	0	
2804.143	0		2855.141	1	
2808.137	1		2855.747	10	3
2809.42 *br*	0		2856.413	0	
2809.714	0		2856.856	2	1
2810.250 *br*	1		2857.495	2	1
2811.000	0		2858.065	1	
2811.179	1		2858.742	1	1
2811.568	0		2859.020	3	3
2812.120	10	1	2861.047	4	3
2813.642	0		2862.685	10	3
2814.353	0		2865.214	3	3
2816.917	2		2865.455	1	1
2817.665	0		2865.803	0	
2818.075	0		2866.025	1	
2818.183	7	1	2866.842	4	3
2822.178	4	1	2867.239	1	1
2822.527	10	2	2867.740	4	3
2824.687	0		2868.741	0	
2825.649	1		2870.540	4	2

CHROMIUM—*Continued*

Wave-Length	Intensity		Wave Length	Intensity	
	Spark	Arc		Spark	Arc
2871.559	0		2911.275	0	2
2871.737	0	2	2911.813	2	
2873.602	2	2	2913.655	0	
2873.957	1	1	2913.837	0	1
2874.625	0		2915.345	1	
2875.135	1		2916.187	0	1
2876.054	4	2	2921.342	2	
2876.385	2	2	2921.933	2	1
2876.754	0		2922.608	0	
2878.057	2	2	2923.600	1	
2878.536	1	1	2923.851	2	
2879.307	0	2	2926.279	1	
2880.987	3	1	2927.203	4	
2882.010	3		2928.271	2	
2885.418	0		2928.455	2	1
2886.525	0		2929.548	1	
2887.105	0	2	2930.932	1	
2887.875	1		2932.800	1	
2888.389	2		2933.715	0	
2889.297	2		2934.055	1	1
2889.613	2		2934.380 *br*	1	
2889.955	1		2935.255	3	1
2891.238	3		2937.027	1	
2891.527	0	1	2939.550	1	
2891.985	1		2940.340	1	
2892.890	0		2941.065	0	
2893.075	1		2942.070	2	1
2893.269	0	2	2945.832	0	
2893.625	0		2946.900	2	1
2894.378	1	2	2947.573	0	
2894.945	0		2949.572	1	1
2895.134	0		2949.885	0	
2895.788	0		2950.201	0	
2896.520	4	1	2950.781	0	
2896.878	1	2	2951.519	0	
2897.357	0		2952.060	0	
2897.810	2		2952.539	0	
2898.649	4	1	2953.458	2	1
2899.253	1	1	2953.780	3	
2899.590	2		2954.741	0	
2901.147	0		2955.240	0	
2902.764	0		2956.705	0	
2903.010	0		2957.706	0	
2903.719	0		2959.671	0	
2904.088	1		2960.042	0	
2904.837	0	1	2961.832	3	1
2905.607	0	2	2963.570	0	
2906.275	0	0	2966.147	2	1
2908.417	0		2967.025	0	
2909.165	0	2	2967.730	0	3
2910.775	1		2968.805	0	1
2911.005	0	2	2969.770	0	

CHROMIUM—*Continued*

WAVE-LENGTH	INTENSITY		WAVE-LENGTH	INTENSITY	
	Spark	Arc		Spark	Arc
2971.230	0	3	3039.883	0	1
2972.040	10	2	3040.350	0	
2975.587	0	3	3040.950	10	3
2976.807	1		3041.910	4	
2979.858	10	2	3042.945	0	
2980.915	0	3	3044.060	0	
2984.845	0		3044.385	0	
2985.485	10	1	3047.790	0	0
2986.106 *br*	1	2	3047.915	0	
2986.579	1	3	3050.270	10	0
2986.929	0		3050.830	0	
2988.159	1		3051.680	0	
2988.733	0	3	3052.331	0	0
2989.263	10	1	3053.826	0	
2992.000	0	2	3053.988	1	5
2992.549	1		3055.526	0	
2992.645	0		3056.778 *br*	0	
2994.177	0	2	3057.956	0	
2994.840	1		3058.445	0	
2995.212	0	2	3059.640	1	0
2996.689	1	2	3061.772	0	0
2998.916	0	2	3063.460	0	
2999.426	0		3063.965	0	
3000.150	0		3065.201	0	1
3001.103	0	2	3067.291	1	
3004.045	1		3071.712	0	
3005.160	1	2	3072.570	0	
3010.735	0		3073.354	0	
3011.569	0		3073.784	0	1
3013.169	0	2	3077.370	1	
3013.840	1	3	3077.942	1	1
3014.900	0	3	3079.490	0	
3015.049	0	2	3083.790	0	
3015.300	0	2	3084.682	0	
3015.620	2		3085.521	0	
3017.691	1	2	3088.200	1	
3018.601	0	2	3093.615	2	
3018.919	0	2	3094.120	0	
3020.826	0	2	3095.085	0	
3021.876	1	3	3095.588	0	
3024.465	1	3	3096.209	2	1
3026.755	7		3098.228	0	
3028.180	2		3103.560	1	
3029.261	0	2	3107.658	2	
3030.356	1	2	3108.749	1	
3031.454	0	1	3109.439	0	1
3033.034	1		3111.010	0	1
3034.304	0	2	3112.030	0	
3034.634	0		3113.725	0	
3035.100	0		3115.265	1	
3037.160	0	2	3115.730	1	
3038.170	0		3116.811	1	

CHROMIUM—*Continued*

Wave Length	Intensity		Wave Length	Intensity	
	Spark	Arc		Spark	Arc
3117.351	0		3190.005	0	
3118.200	0		3194.112 *br*	0	
3118.764	10	3	3194.785	0	
3120.481	15	3	3197.225	15	3
3121.151	0		3198.153	0	0
3121.312	0		3200.025	0	
3121.945	0		3200.582	0	
3122.705	2		3201.435	1	
3125.109	20	3	3202.700	0	
3125.579	0	1	3203.646	0	
3128.819	4	2	3205.215	1	
3130.687	0		3208.106	0	
3132.169	20	4	3208.707	1	0
3134.420	2		3209.297	10	1
3135.429	2		3211.421	0	0
3135.852	2		3211.571	0	
3136.822	4	2	3212.560	0	
3137.252	0		3212.965	0	
3137.620	0		3216.610	2	
3138.321	0		3217.505	7	2
3140.041	0		3219.221	1	
3140.330	2		3219.711	0	0
3142.860	0		3219.901	0	
3143.050	0		3225.504	0	
3145.230	1		3229.324	0	0
3145.880	1		3229.560	0	
3147.350	4	2	3230.006	0	
3148.550	0	2	3231.759	0	
3149.935	1		3234.155	5	
3150.221	1		3237.840	0	1
3152.320	2	1	3238.205	0	0
3154.205	0		3238.635	5	
3155.245	0	2	3245.662	0	1
3158.145	1		3249.660	0	
3159.046	0	0	3250.761	0	
3159.225	0		3250.940	0	
3160.000	0	0	3251.719	0	0
3160.247	0		3252.002	0	1
3162.284	0		3252.615	0	
3163.796	0	1	3255.500	0	
3164.286	0		3257.944	0	1
3169.430	1		3258.905	1	
3172.260	2		3260.110	0	1
3173.630	0		3264.421	2	
3178.860	0		3266.440	0	0
3179.453 *br*	1	0	3268.651	0	
3180.821	10	2	3269.309	1	
3181.531	1	0	3269.950	1	
3183.428	3		3270.358	2	
3184.470	0		3271.260	0	
3186.880	0		3273.058	0	
3188.145	0	0	3274.141	0	

CHROMIUM—*Continued*

Wave-Length	Intensity		Wave-Length	Intensity	
	Spark	Arc		Spark	Arc
3276.069	0		3352.101	0	1
3278.931	0		3353.251	2	0
3279.675	0		3355.285	0	
3283.193	1		3357.542	3	
3286.010	1		3358.649	10	2
3286.460	0		3360.485	20	2
3288.160	0		3361.965	3	0
3291.408	0		3363.872	1	
3291.905	2		3364.851	0	
3295.110	0		3367.593	1	0
3295.623	4	0	3368.193	20	3
3298.482	0	0	3368.891	0	
3298.902	0		3369.202	1	
3301.384	0		3371.603	0	
3303.070	0		3372.270	0	
3304.914	0		3375.107	0	
3307.220	7	0	3376.428	0	0
3308.351	0		3376.830	0	
3310.028	0		3377.472	0	
3310.865	3		3378.476	4	0
3312.115	2		3379.514	2	0
3312.380	2		3380.961	4 *R*	2
3313.269	1		3382.825	10	2
3314.240	1		3385.493	0	0
3314.775	2		3386.661	0	0
3315.461	0		3387.852	0	
3316.658	0	0	3388.865	0	0
3322.869	0		3390.920	0	1
3323.710	0		3391.578	4	1
3324.231	2	0	3393.160	4	0
3324.491	3		3393.980	3	0
3326.571	0	0	3394.432	3	2
3328.525	2	0	3395.731	2	
3329.186	0	0	3399.655	1	
3329.620	0		3402.542	3	0
3333.070	0	0	3403.404	15	3
3335.530	6		3405.407 *br*	0	
3336.477	4	1	3408.911	20	3
3337.087	0		3410.715	0	0
3338.062 *br*	0		3411.164	0	
3339.932	10	2	3412.350	0	
3340.838	0		3415.588	0	
3342.110	0		3419.435	0	
3342.717	10	2	3421.353	10	3
3343.425	0	0	3421.770	0	
3344.654	0		3422.892	20	3
3346.154	0	1	3424.779	0	
3346.860	1	1	3426.252	0	
3347.970	5	1	3427.230	0	
3349.135	0	0	3427.745	0	
3349.519	1	0	3428.040	0	
3351.745	0	0	3430.012	0	

CHROMIUM—*Continued*

Wave-Length	Intensity		Wave-Length	Intensity	
	Spark	Arc		Spark	Arc
3430.565	0		3477.320	0	0
3431.415	0	0	3478.291	0	
3431.776	0		3478.889	0	0
3432.149	0	0	3480.439	0	
3432.480	0	0	3481.439	1	1
3433.453	4	1	3481.695	1	1
3433.715	1	3	3482.745	1	
3434.230	1	0	3483.650	0	
3435.819	0	0	3484.289	2	0
3435.962	0	0	3486.636	0	0
3436.332	1	2	3488.578	0	0
3436.878	0		3494.677	0	
3438.570	0		3495.108	0	0
3441.241	0	0	3495.530	2	0
3441.588	1	2	3495.670	1	
3443.939	0		3502.445	0	0
3444.501	0		3503.508	0	
3445.741	1	2	3508.236	0	
3447.149	0	0	3510.686	0	0
3447.567	0	1	3512.000	3	
3447.901	0	0	3513.217	0	
3449.400	0		3518.757		
3450.999	0		3522.262	0	
3453.469	1	2	3523.14	0	
3453.879	0	0	3527.201	0	0
3455.108	2	0	3513.572	0	
3455.336	0		3532.980	0	
3455.738	1	1	3536.615 *br*	0	
3457.78	4		3539.078	0	
3458.249	1	0	3547.180	0	
3459.428	2		3548.86 *br*	0	
3460.568	0	0	3550.431	1	1
3461.441	0		3552.76	1	
3462.869	0		3554.072	0	
3462.720	0		3556.218	0	
3464.151	0		3558.75 *br*	1	2
3464.978	0	0	3559.901	0	0
3465.390	0	0	3562.418	0	
3465.685	0	0	3562.56	0	
3466.40	0		3567.050	0	
3467.132	0	0	3564.410	0	
3467.845	0	0	3564.838	0	
3468.885	0	0	3566.454 *br*	1	3
3469.736	0	0	3569.238	0	
3470.585	0	0	3572.890	0	1
3471.650	0	0	3573.780	0	1
3472.216	2		3574.944	0	1
3472.948	0	0	3575.106	2	1
3473.036	0	0	3578.832	20 *r*	30 *r*
3473.760	0	0	3582.766	0	0
3474.525	0	0	3584.46 *br*	1	0
3475.270	2	0	3585.421	3	

CHROMIUM—*Continued*

Wave-Length	Intensity		Wave-Length	Intensity	
	Spark	Arc		Spark	Arc
3585.637	2		3686.926	1	1
3593.636	20 *r*	30 *r*	3687.454	2	1
3594.500	0		3687.690	2	1
3599.535	0	0	3688.572	1	0
3601.806	2	2	3689.414	0	0
3602.708	0	0	3689.667	0	
3603.919	10	1	3693.128	0	
3605.479	20	30 *r*	3696.807	2	
3608.547	0	0	3698.044	9	
3609.650	0	1	3711.496	1	
3610.218	0	1	3713.946	15	1
3612.789	0	0	3715.164	8	1
3613.109	2	0	3715.436	8	
3613.828	0	0	3716.481	1	1
3615.795	0	1	3723.373 *n*	3	
3617.450	0	1	3727.332 *n*	5	
3619.599	0		3730.314	2	2
3621.649	0		3732.064	3	2
3626.449	0		3737.590	3	
3631.758	9 *r*		3738.435 *n*	7	8
3632.979	1	2	3743.035	2	2
3634.997	0		3743.575	5	2
3634.996	0		3743.952	9	4
3636.728	2	3	3744.525	2	
3639.943	4	4	3747.326	2	
3641.597	0	1	3748.706 *br*	4	2
3641.970	2	2	3749.039	10	4
3643.305	0		3750.524 *br*	3	
3644.802	0		3754.652 *n*	8	
3646.0000	0	0	3757.238	1	1
3647.485	0		3757.739	6	4
3648.669	0	1	3758.107	3	2
3649.137	2	2	3761.748 *n*	2	
3649.920	0		3716.956 *n*	2	
3650.490	2	0	3767.487	1	1
3651.792	0		3768.296	6	5
3654.052	1	2	3768.799	3	2
3656.404	1	2	3778.600	2	
3658.316	0		3783.752	0	
3662.978	0	1	3788.917	1	
3663.346		1	3789.784	1	1
3664.755	1		3790.317	1	1
3666.110	0	0	3790.514	2	1
3666.781	0	1	3791.448	4	2
3668.155	0	0	3792.206	4	2
3676.457	0	0	3793.346	4	2
3677.815	1	0	3793.932	4	2
3677.965	3 *R*	0	3794.669	3	1
3679.170	0	0	3797.198	4	2
3681.838	0	0	3797.757	6	2
3684.370	1		3801.251 *n*	2	
3685.665	0	1	3804.878	10	6

CHROMIUM—*Continued*

WAVE-LENGTH	INTENSITY		WAVE-LENGTH	INTENSITY	
	Spark	Arc		Spark	Arc
3806.898	4	2	3894.022	10	2
3807.983	4	1	3897.657	9	0
3809.556	2		3902.107	0	0
3812.321	4	2	3902.947	2	2
3814.072 *n*	4		3903.188	0	1
3814.690	4	3	3905.668	0	
3815.500	5	2	3907.358	0	
3816.232	4		3907.787	0	0
3817.914	2	2	3908.765	0	4
3818.529	4	1	3911.392	0	
3819.645	6	1	3914.417 *br n*	0	0
3820.520	4	3	3915.469	0	1
3821.581	0		3916.040	0	1
3823.644	0	10 *r*	3916.268	0	2
3825.402	0		3917.180	0	0
3826.412	1	1	3917.589	8	0
3830.052	4	1	3919.167	3	7
3831.162	4	2	3919.968	4	0
3834.772	3		3921.046 *br n*	0	4
3836.102	6	1	3926.637	1	0
3841.282 *n*	1	2	3928.215	0	4
3849.012	1	1	3928.641	1	
3849.462	0	1	3936.128	1	
3849.631	1	1	3937.608	5	
3849.982	1		3938.397	1	
3852.202	10	2	3941.188	0	
3852.604 *n*	1		3941.495	0	3
3853.222	2	1	3943.518	1	
3854.262	1	1	3945.108	1	
3854.862 *n*	2		3945.491	1	0
3855.313	2	0	3945.959	12	0
3855.612	3		3948.858	3	0
3856.229 *n*	1	0	3949.66 *br n*	1	0
3857.659	1	1	3951.077	0	0
3862.583	8		3951.775	0	0
3865.662	1		3952.407	10	0
3866.575	1		3953.181	4	0
3868.299	8		3958.078 *br n*	1	
3870.291	0		3960.760	1	0
3871.462	1		3963.10	10	
3872.607	1		3969.145 *R V*	30	5
3874.588	1		3971.261	1	1
3875.257	8	15 *r*	3972.689	1	0
3789.186	1	1	3976.698	4	5
3881.318 *n*	9		3978.668	0	1
3881.811	0		3979.519 *n*	3	0
3883.327	0		3979.795	0	0
3883.635	1		3981.235	7	1
3885.221	10	3	3983.278	10	0
3886.799	1	3 *R*	3983.331	4	4
3890.867	1		3984.145	1	2
3891.927	4	0	3989.988	1	2

CHROMIUM—*Continued*

Wave-Length	Intensity		Wave-Length	Intensity	
	Spark	Arc		Spark	Arc
3991.192	0	4	4098.484	4	
3991.699	0	2	4099.122	2	0
3992.830	1	2	4099.450	1	
3993.851	0	1	4099.861	1	
3998.899	0		4101.162	2	
3999.677	2	0	4104.862	4	
4001.454	3	1	4106.423	2	
4002.540	4		4109.587	4	0
4003.339	4		4111.010 *br*	8	
4012.490	0	0	4113.259	2	
4014.689 *br n*	4	0	4120.640 *br*	3	0
4016.084	0		4123.327	4	
4018.269	1	0	4121.813	1	0
4022.250	2	1	4122.156	3	0
4025.025	0	0	4123.400	6	0
4025.368	0	1	4125.450	2	
4026.184	3	1	4126.521	8	0
4027.096	3	1	4126.918	2	2
4029.537	2		4127.058	2	0
4031.467	2	0	4127.274	4	0
4033.199	1	0	4127.588	4	0
4037.307	1	0	4128.506	4	0
4038.058	1		4131.355	6	0
4039.102	1	2	4132.398	1	
4042.601	0	0	4134.038	0	
4043.813	1	0	4142.178	4	
4048.767	4	2	4142.477	1	
4049.2 *n*	1		4145.798	10	
4049.766	1	0	4149.482	2	
4051.315	1	0	4151.118 *n*	3	
4052.462	2		4152.755	4	0
4054.377	0		4163.070	2	0
4056.077	0	0	4153.719	5	2
4056.812	0	0	4161.419	7	1
4058.774	2	2	4163.666	7	1
4060.615 *n*	1	0	4165.524	7	1
4064.738	0		4169.857	4	0
4065.720	1	1	4170.220	3	0
4067.530	0	1	4171.702	2	1
4068.403	3		4172.678	4	
4070.963	3		4174.821	4	2
4074.908 *n*	2	0	4175.344	3	0
4076.054	3	1	4175.958	3	0
4077.077	2	0	4177.838	2	
4080.413	0		4179.256	7	2
4081.723	0		4179.418	2	
4082.304 *br*	3		4184.909	1	1
4084.153	1		4186.347	2	0
4086.033	2		4190.038	1	1
4090.327	3		4191.284	3	2
4092.206	1		4191.781	3	
4093.073	1		4192.022	5	0

CHROMIUM—*Continued*

Wave-Length	Intensity		Wave-Length	Intensity	
	Spark	Arc		Spark	Arc
4193.687	2	1	4284.687	1	1
4194.857	1		4284.866	2	1
4195.331 *br*	2	2	4288.456	1	1
4197.241	3	2	4289.731 *R V*	30	30 *R*
4198.421	3	2	4291.984	2	2
4200.112	2	2	4293.560	2	2
4203.581	2	2	4295.960	4	3
4204.209	1	1	4297.048	3	2
4204.472	3	2	4297.754	3	3
4206.909	3	1	4299.756	2	2
4208.364	2	1	4299.966	1	1
4209.371	3	2	4300.516	2	2
4209.714	2	2	4301.171	2	1
4211.360	2	2	4302.791	1	
4212.649	2	2	4305.461	2	2
4213.170	2		4306.952 *br*	3	1
4216.366	2	2	4312.471	1	1
4217.567	0	2	4317.971	0	1
4221.584	3	2	4318.602	0	
4222.737	2	2	4319.640 *br*	3	2
4224.530	2	0	4320.681	1	1
4224.857	2		4321.312	1	1
4230.407	1	1	4321.681	1	1
4232.237	1	1	4323.572	2	2
4233.217	2	1	4325.132	5	3
4234.477	1	1	4337.563	11	10
4238.954	2	1	4338.408	2	1
4240.719	2	1	4338.808	2	1
4042.357	6	1	4339.448	8	5
4248.347	1	1	4339.713	7	4
4248.727	2	1	4340.128	3	2
4252.297	1	1	4341.091 *br*	2	1
4252.657 *n*	4		4343.203	1	2
4254.352 *R V*	30	50 *r*	4344.501	10	10
4255.506	2	1	4345.121	0 *r*	1
4256.157 *br n*	1	1	4346.818	2	3
4261.343	3	2	4351.047	6	8
4261.526	2	1	4351.761	10	15
4262.134	6	1	4353.861	2	1
4262.345	2	1	4356.761	2	1
4263.137	6	3	4357.532	2	1
4268.347	1	1	4359.515	6	8
4268.487	1	1	4363.098	4	3
4269.297 *n*	2		4368.231	2	1
4269.966	2	1	4369.861	0	
4271.086	2	2	4371.273	10	10
4273.987	2	2	4373.246	2	3
4274.805 *R V*	30	50 *R*	4373.641	2	1
4275.547	10		4374.162	6	3
4276.076	1	1	4375.324	3	3
4280.403	3	3	4376.773	3	2
4283.017 *br*	3		4377.546	3	2

CHROMIUM—*Continued*

Wave-Length	Intensity		Wave-Length	Intensity	
	Spark	Arc		Spark	Arc
4378.336	1		4511.911	2	3
4379.776	2	1	4514.510	0	2
4381.110	2	2	4515.338	0	1
4384.980	10	8	4521.152	0	1
4387.494	2	3	4524.848	0	1
4391.760	2	3	4526.115	0	1
4395.426	0	1	4526.480	6	8
4397.236	2	2	4527.462	2	4
4399.871	1	2	4529.852	0	2
4403.368	2		4530.698	1	
4403.497	3	3	4530.742	6	8
4410.299	2	2	4532.782	0	1
4411.076	2	2	4535.142	0	2
4412.251	2	2	4535.711	5	6
4413.837	3	2	4539.778	0	3
4422.286	2	2	4540.476	0	3
4423.266	2	1	4540.712	2	3
4424.293	3	3	4541.068	1	2
4425.156	2	1	4541.522	2	1
4428.557	2	1	4542.517	0	2
4429.966	2	1	4543.732	0	1
4430.486	2	1	4544.610	2	4
4432.166	3	2	4545.339	0	2
4433.875	1	1	4545.961	4	8
4442.266	2		4553.962 *n*	2	
4443.706	2	1	4554.972	0	
4458.426	5	3	4556 138	0	3
4459.351	2	1	4558.659 V	20	1
4462.776	2	1	4563.670	0	1
4464.756	2	1	4564.160	1	1
4464.806	2	1	4565.518	1	
4465.345	3	2	4565.778	1	4
4466.156	1	1	4569.618	1	3
4467.557	2	1	4571.679	2	4
4473.746	1	1	4575.110	1	2
4475.306	1	2	4578.210	1	1
4480.306	2 *r*	1	4580.058	2	8
4481.456	1		4584.100	1	1
4482.885	2	2	4585.13	1	1
4488.044	3	2	4586.220	1	1
4489.466	3	2	4588.240 *R*	15	1
4491.659	1		4591.400	4	8
4491.842	2	1	4592.060 *n*	4	
4492.311	4	1	4592.631	4	
4495.298	2	1	4595.580	1	2
4496.871	10	10	4598.430	1	1
4498.745	0	2	4500.109	1	2
4500.299	1	3	4600.762	4	6
4501.112	1	2	4601.037	1	2
4501.788	0	1	4601.500 *n*	2	
4506.868	0	2	4607.171 *n br*	2	
4510.048	1	1	4613.365	4	4

CHROMIUM—*Continued*

WAVE-LENGTH	INTENSITY		WAVE-LENGTH	INTENSITY	
	Spark	Arc		Spark	Arc
4613.921	2	2	4729.695	0	0
4616.126	5	4	4730.728	2	2
4616.531	1		4737.371	2	2
4618.841	6		4745.331	0	
4619.532	0	1	4752.101	2	1
4621.886	1	1	4756.131	4	4
4621.949	1	2	4764.290	1	1
4622.448	2 *r*	2 *r*	4766.647	0	0
4625.917	0	1	4767.869	0	0
4626.179	5	4	4789.348	3	3
4628.458 *br n*	5		4790.342	1	1
4632.141	6		4792.522	3	2
4633.093	6		4796.193	3	2
4637.173	1	1	4801.043 *n*	0	
4637.759	1	1	4803.430	0	
4639.500	0	1	4807.545	0	
4641.842	3		4812.380	0	
4643.142	3		4816.280	0	
4646.169	6	4	4824.140	6	
4646.498	2	1	4829.371	2	2
4646.784	0	0	4836.864	0	
4648.119	1	1	4849.665	4	
4649.853	0	1	4861.849	1	1
4649.435 *br*	3	2 *r*	4864.250	3	
4651.283	4	4	4870.816	3	2
4652.165	0	0	4876.340	3	
4654.122	6	4	4879.800	0	
4654.734	0	0	4884.640	0	0
4656.189	0	1	4885.875	1	1
4661.752 *n*	0		4885.952	0	0
4663.314	0	1	4887.203	3	2
4663.830	1	2	4888.528	0	0
4664.796	2	3	4901.532	0	0
4665.907	1 *r*	2 *r*	4903.262	0	0
4666.218	0	1	4906.774	3	2
4666.486	2	3	4922.281	0	0
4669.335	1	1	4936.347	1	2
4676.281	0	0	4942.495	0	1
4680.489 *n*	0	1 *r*	4954.828	1	1
4689.371	2	2	4995.787 *n*	0	
4693.956	1	1	5001.387 *br*	4	
4695.162	0	0	5004.384 *br*	4	
4697.061	1	1	5007.307 *br n*	1	
4698.529	4	4	5010.052	0	0
4699.640	0		5013.291	1	0
4700.526	1	1	5021.920	1	0
4705.451 *n*	0		5051.906	0	
4707.896	3	4	5065.900	0	
4708.027	4	4	5067.696	0	
4723.125	0	1	5068.307	0	
4724.413	0	1	5072.936	0	
4727.168	0	1	5090.974	0	

CHROMIUM—*Continued*

Wave-Length	Intensity		Wave-Length	Intensity	
	Spark	Arc		Spark	Arc
5110.782	0		5275.172	0	0
5113.142	0		5275.758	2	
5122.133	0		5276.069	0	0
5123.475	0		5296.711	4	1
5139.651	1	0	5298.294	6	1
5144.664	0	0	5300.768	0	0
5166.271	1	1	5328.144 *n*	3	0
5184.576	0		5345.816	6	1
5191.993	1		5348.336	5	1
5193.507	0		5400.624	0	0
5196.451	2		5409.793	5	1 *r*
5200.193	0		5664.004 *n*	0	
5204.518	6	6 *r*	5787.927	1	0
5206.053	6	6 *r*	5790.960	2	1
5208.434	6	6 *r*	6135.770	1	1
5225.032	1	0	6330.107	0	1
5237.331	0	0	6362.881	1	2
5247.675	2	0	6597.788	0	1
5255.133	0		6630.051	0	0
5264.167	5	1	6661.010	0	0
5265.725	1	0			

II. MEASUREMENTS OF ARC SPECTRA

INTRODUCTION

Measurements of the arc spectra of cerium and thorium have been made by Watts and by Hagenbach and Konen, but only over a limited range of the spectrum. Owing to the purity of some salts of these metals which were prepared by Dr. A. C. Neish of Columbia University and described by him in a paper entitled "A New Separation of Thorium from Cerium, Lanthanum and Didymium by Metanitrobenzoic Acid," it was thought advisable to measure the wave lengths over a more extended region of the spectrum.

APPARATUS

The same grating was used as that employed for the spark spectra. The dividing engine, to which reference has been made before, was constructed by Professor Rowland and was used in his work on the solar spectrum. For plates taken in the second spectrum of a 21-foot grating, the scale reads approximately to Ångström units.

The plates used were the same as those used in the work on spark spectra.

CORRECTIONS

In order to eliminate all lines due to impurities, the spectrum of the carbon arc was taken on each plate for comparison. They were also compared with the spectra of all elements likely to be associated with the substance under examination. This comparison was made by direct superposition of the plates. The lines, like those of spark spectra, were referred to the iron lines as determined by Fabry and Perot, and by Jewell, respectively. A calibration curve was also drawn for each plate as was done in case of the plates of spark spectra.

CERIUM

Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity
2651.125	0	3145.387	1	3233.555	0	3304.972	1
2652.572	0	3146.515	1	3234.301	3	3306.773	0
2660.466	0	3148.585	0	3235.032	1 *r*	3307.365	0
2831.047	0	3149.510	0	3235.801	0	3308.165	1
2854.755	0	3151.265	0	3236.859	2	3309.298	0
2854.987	0	3154.618	0	3242.275	0	3311.031	0
2874.256	0	3155.882	1	3243.499	2	3311.627	0
2908.525	0	3164.383	1	2345.301	0	3312.353	0
2956.047	0	3166.387	0	3245.671	0	3313.439	0
2964.921	0	3166.741	0	3246.812	1	3314.175	0
2972.812	0	3167.415	0	3249.292	0	3314.863	1
2974.706	0	3169.314	1	3249.555	0	3315.338	0
2977.024	0	3171.758	1	3252.649	1	3317.935	0
2977.565	0	3172.458	0	3254.178	1	3319.002	0
2991.003	0	3176.931	0	3256.802	0	3320.499	0
2995.763	0	3177.258	0	3259.013	0	3321.087	0
3002.247	0	3178.887	0	3259.965	0	3323.456	0
3002.863	0	3181.005	0	3261.115	0	3325.498	0
3003.655	0	3183.647	1	3263.543	0	3326.229	0
3008.079	1	3184.346	0	3264.033	0	3327.405	0
3037.849	0	3186.281	1	3265.594	0	3327.812	0
3052.073	0	3187.776	0	3270.276	0	3328.069	0
3055.345	1	3188.899	0	3271.288	0	3329.139	0
3056.857	1	3189.751	0	3271.672	0	3330.625	0
3063.115	2	3190.497	1	3272.135	0	3331.392	0
3068.778	0	3194.936	2	3272.389	2	3331.947	0
3069.776	0	3196.051	0	3272.847	0	3332.361	0
3071.238	0	3199.407	0	3275.001	1	3332.629	0
3071.735	0	3201.243	0	3276.351	0	3333.197	0
3073.025	0	3201.836	2	3279.135	0	3333.815	0
3077.470	0	3203.045	0	3279.981	1	3334.045	0
3079.752	0	3206.071	0	3280.647	0	3334.429	0
3080.049	0	3206.603	0	3281.221	0	3334.605	1
3083.789	0	3207.025	0	3283.401	0	3336.498	0
3084.568	0	3207.728	0	3283.840	0	3336.678	0
3090.503	0	3211.081	0	3284.356	0	3339.659	0
3091.431	0	3212.628	0	3285.349	1	3339.947	0
3096.641	0	3212.707	0	3286.163	0	3341.049	0
3097.041	0	3216.809	0	3286.501	0	3342.043	1
3103.501	1	3218.499	1	3287.537	0	3344.005	1
3107.578	0	3219.057	3	3288.287	0	3344.909	1
3109.069	0	3221.042	2	3288.885	0	3345.551	0
3110.403	0	3221.327	2	3290.503	0	3346.648	0
3111.318	0	3222.547	0	3290.729	0	3348.309	0
3127.612	0	3223.532	0	3293.751	0	3350.105	0
3130.441	0	3224.057	0	3295.078	0	3331.314	0
3130.981	1	3225.831	0	3295.440	1	3351.668	0
3132.725	0	3226.164	0	3296.337	0	3352.441	0
3133.461	0	3227.249	2	3297.040	0	3353.118	1
3136.842	0	3229.251	0	3298.475	0	3353.465	0
3137.731	0	3229.487	1	3300.141	0	3354.108	0
3142.399	0	3231.391	1	3300.289	0	3354.665	0
3144.708	0	3232.425	0	3303.365	0	3355.155	1

CERIUM—*Continued*

Wave-Length	Intensity	Wave Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity
3356.549	1	3416.703	0	3469.152	0	3510.095	0
3357.347	1	3415.815	0	3469.543	0	3510.387	0
3358.589	0	3416.985	0	3470.289	0	3510.039	0
3359.491	0	3317.543	1	3470.562	0	3511.754	0
3359.602	0	3417.985	0	3472 167	0	3518.725	0
3360.703	2	3419.045	0	3474.372	0	3515.913	0
3361.946	1	3420.312	0	3474.938	0	3517.510	2
3364.185	0	3422.612	0	3475.799	0	3518.156	0
3364.493	0	3422.839	2	3476.483	0	3518.540	0
3364.978	0	3423.992	0	3476.985	1	3519.194	1
3365.997	0	3425.465	0	3477.554	0	3510.853	0
3366.708	1	3426.085	0	3478.111	0	3520.665	1
3368.544	0	3426.342	2	3479.147	0	3521.139	0
3368.872	0	3426.703	0.	3479.752	0	3522.039	1
3368.938	0	3428.843	0	3480.463	0	3523.347	0
3369.352	0	3429.997	0	3480.519	0	3524.178	0
3371.352	0	3430.442	0	3481.165	0	3526.815	1
3371.352	1	3431.002	0	3481.329	0	3527.743	0
3373.647	1	3431.649	0	3482.265	0	3527.975	0
3373.910	1	3433.247	1	3482.531	1	3528.168	0
3374.315	0	3433.795	0	3484.892	0	3528.759	0
3375.272	0	3435.346	0	3485.197	2	3529.161	0
3375.939	0	3436.430	0	3486.978	0	3530.160	1
3377.274	1	3437.455	0	3488.665	0	3530.749	0
3379.321	0	3437.955	0	3490.287	0	3531.059	0
3381.658	0	3439.947	0	3491.805	0	3531.749	0
3383.558	0	3441.358	1	3492.647	0	3532.750	0
3383.849	1	3442.513	1	3493.250	0	3533.029	0
3384.102	0	3443.079	0	3493.859	0	3533.759	0
3385.493	0	3443.652	0	3494.798	0	3534.202	2
3387.922	0	3446.348	0	3495.157	0	3534.571	1
3389.802	0	3446.846	0	3495.613	0	3535.708	0
3390.671	0	3448.403	0	3496.075	0	3536.647	0
3391.758	1	3451.758	0	3496.478	0	3536.857	0
3393.746	0	3453.385	0	3497.491	0	3937.293	0
3394.071	0	3454.594	0	3498.759	0	3537.597	0
3394.283	0	3454.957	0	3496.635	0	3538.935	0
3395.187	0	3456.835	0	3500.157	0	3539.235	2
3395.187	0	3456.915	0	3500.832	0	3540.963	0
3395.565	0	3459.998	0	3501.604	1	3541.821	0
3396.882	0	3460.305	0	3501.729	1	3542.072	0
3398.853	0	3461.513	0	3501.957	0	3543.447	0
3399.115	0	3461.923	0	3502.803	0	3543.682	0
3400.405	0	3463.360	0	3503.057	0	3545.748	1
3403.749	0	3463.902	1	3503.195	0	3545.978	1
3404.268	0	3464.319	1	3506.399	0	3546.325	1
3405.047	0	3464.989	0	3506.895	0	3546.798	0
3405.951	0	3465.102	0	3507.485	0	3547.146	1
3406.102	1	3466.018	0	3508.108	1	3547.955	0
3406.347	1	3467.108	0	3508.615	0	3548.289	0
3408.549	0	3467.925	0	3508.848	0	3548.978	0
3412.450	0	3468.255	0	3509.397	0	3551.603	0
3415.751	0	3469.071	0	3509.888	1	3551.853	1

CERIUM—*Continued*

Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity
3552.249	0	3619.410	0	3667.759	0	3727.968	1
3552.872	1	3619.984	0	3667.964	2	3728.421	2
3554.780	0	3621.187	0	3668.705	0	3729.014	0
3555.949	0	3622.194	1	3671.931	0	3729.920	0
3556.497	0	3622.446	0	3672.161	0	3730.359	0
3557.354	0	3623.854	2	3672.799	1	3731.909	0
3558.865	0	3624.176	0	3673.650	1	3733.587	1
3559.473	0	3625.295	0	3674.091	0	3739.732	0
3561.002	3	3628.290	0	3676.127	0	3740.162	0
3561.704	0	3628.638	0	3679.124	0	3741.050	0
3562.238	0	3629.816	0	3679.411	0	3741.421	0
3563.945	0	3630.184	0	3679.911	0	3741.759	0
3566.204	0	3630.450	0	3680.454	0	3744.066	0
3568.247	0	3631.232	1	3680.141	0	3746.387	1
3568.487	0	3632.136	0	3680.869	0	3748.067	1
3569.448	0	3633.414	0	3681.397	0	3750.136	0
3569.975	0	3636.086	0	3682.100	0	3751.066	1
3571.132	0	3637.534	0	3687.810	0	3751.509	2
3572.548	0	3637.764	0	3688.665	0	3752.407	0
3573.846	0	3638.018	0	3689.174	0	3752.466	0
3576.349	1	3638.241	0	3693.457	0	3752.913	0
3577.579	2	3639.486	0	3693.716	0	3753.816	0
3580.725	0	3640.726	0	3694.918	1	3755.473	1
3580.941	0	3641.157	0	3695.957	0	3755.780	1
3583.775	0	3642.618	0	3696.097	1	3756.327	0
3584.497	0	3642.827	0	3697.693	0	3757.266	0
3586.927	0	3643.464	0	3698.128	0	3757.897	0
3587.383	0	3644.336	0	3698.368	0	3759.190	0
3587.779	1	3645.235	0	3698.659	0	3759.780	0
3588.270	0	3645.451	0	3699.928	0	3760.426	0
3588.593	1	3646.650	1	3702.818	0	3760.735	0
3590.778	1	3646.869	2	3705.016	1	3763.006	1
3593.329	0	3647.531	0	3706.859	0	3763.637	0
3594.219	0	3647.764	1	3702.408	0	3764.136	2
3596.257	0	3649.703	0	3702.648	0	3765.053	1
3596.878	0	3650.137	0	3709.962	3	3765.906	0
3597.392	0	3650.910	1	3710.714	0	3766.063	0
3598.347	1	3652.1	0	3713.995	0	3766.533	0
3600.137	0	3652.292	0	3714.801	0	3768.012	0
3600.742	2	3653.111	1	3715.148	0	3768.283	0
3601.826	0	3653.640	1	3715.469	0	3768.777	1
3604.216	1	3654.908	1	3716.369	2	3769.075	0
3606.167	0	3655.823	2	3716.919	0	3769.073	0
3607.659	1	3659.230	1	3717.498	0	3770.816	1
3607.887	0	3659.869	0	3718.195	1	3771.637	1
3609.735	2	3660.037	0	3718.371	2	3772.686	0
3610.947	1	3660.637	1	3722.119	0	3773.256	0
3611.386	1	3661.711	0	3722.310	0	3773.483	0
3611.759	0	3662.859	0	3722.757	0	3776.118	0
3612.438	0	3663.699	0	3723.657	0	3776.630	1
3613.746	1	3664.707	0	3724.647	0	3777.088	0
3615.574	0	3665.137	0	3725.676	1	3777.665	0
3616.241	0	3666.012	0	3726.969	1	3779.613	0

CERIUM—*Continued*

Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity
3781.120	0	3834.754	0	3893.782	0	3930.810	0
3781.643	2	3835.732	0	3894.325	0	3931.097	2
3782.527	2	3836.076	1	3895.144	2	3931.390	1
3783.026	0	3837.182	0	3895.460	0	3931.844	1
3783.523	1	3838.257	2	3896.832	2	3932.148	1
3786.628	3	3838.542	2	3897.410	0	3932.984	0
3787.161	0	3843.739	0	3898.392	2	3934.741	0
3787.527	0	3848.559	2	3898.692	0	3935.231	0
3788.166	0	3849.525	0	3898.969	1	3935.907	0
3788.467	0	3852.110	0	3899.500	0	3937.124	0
3788.763	2	3852.395	1	3900.198	0	3937.614	0
3790.857	0	3853.164	2	3901.310	0	3937.804	0
3791.700	0	3854.154	2	3901.644	0	3938.058	2
3792.319	1	3854.275	2	3903.350	1	3939.521	0
3792.592	0	3855.263	2	3903.932	0	3939.613	0
3793.610	0	3856.335	0	3904.199	0	3940.224	2
3793.851	0	3856.629	0	3904.361	1	3940.607	0
3794.700	0	3856.964	1	3904.601	0	3940.945	1
3795.266	1	3857.209	0	3905.306	0	3942.163	4
3796.170	0	3857.591	1	3906.952	1	3942.714	2
3796.670	0	3857.840	0	3907.341	1	3943.111	0
3797.920	0	3860.392	0	3907.414	1	3943.450	0
3799.071	0	3860.799	0	3908.106	0	3943.917	3
3800.237	1	3862.428	1	3908.400	1	3944.876	0
3801.554	4	3866.785	0	3908.516	1	3946.689	0
3802.777	0	3868.092	1	3909.024	0	3947.850	1
3803.080	2	3868.464	1	3909.307	1	3949.377	2
3807.674	0	3870.863	0	3909.759	0	3949.790	0
3808.10	2	3874.674	1	3909.841	0	3950.133	0
3809.199	1	3875.017	1	3910.706	0	3950.812	0
3809.461	0	3876.104	0	3911.314	0	3951.642	0
3810.910	0	3876.857	2	3912.206	0	3952.205	0
3812.211	1	3878.352	2	3912.462	2	3952.613	8
3814.844	0	3881.658	0	3914.004	0	3953.647	1
3815.777	1	3881.897	0	3914.962	0	3954.960	2
3817.425	1	3882.454	1	3915.534	1	3955.372	1
3818.679	0	3883.563	0	3916.157	2	3955.941	0
3819.009	0	3883.967	0	3916.668	0	3956.012	0
3820.862	1	3884.206	0	3916.912	0	3956.235	3
3821.252	1	3885.220	0	3917.255	1	3956.898	2
3821.711	1	3885.724	0	3917.639	0	3957.180	0
3823.905	1	3885.978	0	3918.278	3	3958.860	0
3824.407	0	3887.164	0	3919.812	2	3958.214	1
3825.215	0	3888.365	0	3921.714	2	3958.870	2
3827.155	0	3888.984	0	3923.146	2	3959.613	1
3829.346	1	3889.289	0	3924.664	1	3959.846	1
3829.662	1	3889.498	0	3924.805	0	3960.412	0
3829.985	1	3890.019	4	3927.008	0	3960.941	2
3831.078	1	3890.511	0	3927.405	0	3962.102	0
3832.258	0	3890.760	0	3927.580	0	3963.414	0
3832.720	0	3891.005	1	3928.743	0	3964.217	0
3834.204	1	3891.794	0	3929.130	0	3964.522	1
3834.515	1	3893.246	0	3929.997	0	3967.174	2

CERIUM—*Continued*

Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity
3967.266	1	4010.186	0	4061.418	0	4095.840	0
3967.517	0	4011.597	0	4061.306	0	4098.160	0
3970.074	0	4012.195	0	4061.245	1	4099.10	1
3970.462	0	4012.430	5	4062.957	2	4099.769	0
3970.662	0	4014.929	2	4063.959	0	4100.918	1
3971.784	1	4015.878	0	4064.818	0	4101.787	2
3972.108	1	4018.609	0	4065.176	0	4102.300	0
3973.188	0	4019.059	0	4066.548	0	4104.459	0
3974.000	0	4019.297	0	4066.918	0	4105.518	1
3974.247	0	4019.485	0	4067.299	1	4106.159	0
3975.098	0	4019.907	0	4068.487	1	4106.958	2
3976.738	0	4020.558	0	4068.858	2	4107.406	3
3977.529	0	4022.299	2	4070.127	0	4107.819	0
3977.797	0	4023.306	0	4070.866	0	4108.257	0
3978.668	2	4024.508	3	4071.128	0	4108.748	0
3979.959	0	4025.155	1	4072.956	1	4109.566	0
3980.938	1	4025.909	0	4073.509	2	4110.400	3
3981.917	0	4027.057	0	4073.859	1	4110.879	1
3982.218	0	4027.676	0	4074.667	0	4111.417	2
3982.929	2	4028.700	3	4075.728	2	4111.960	1
3983.109	1	4029.298	0	4075.908	3	4113.749	1
3984.677	2	4029.758	0	4076.286	1	4114.157	0
3986.158	0	4030.227	0	4076.879	0	4114.928	1
3986.417	0	4030.396	2	4077.465	1	4115.270	2
3989.447	1	4031.339	3	4078.337	1	4117.000	1
3990.128	1	4033.809	0	4078.557	3	4117.299	1
3990.459	0	4037.387	0	4079.038	0	4117.588	1
3990.718	0	4037.696	1	4079.319	0	4118.156	3
3991.238	0	4038.258	1	4079.705	1	4119.019	1
3992.157	0	4039.898	0	4080.456	2	4119.847	7
3992.297	3	4040.777	5	4080.560	2	4120.858	2
3992.906	1	4041.289	0	4081.258	3	4121.608	0
3993.188	0	4042.146	0	4083.260	2	4123.256	1
3993.957	2	4042.588	4	4083.535	0	4123.523	1
3994.605	0	4043.459	0	4083.659	0	4123.886	3
3995.450	0	4045.229	2	4085.270	2	4124.840	2
3996.488	0	4046.246	3	4086.418	1	4125.458	0
3997.485	0	4047.317	1	4087.359	1	4125.787	0
3997.747	0	4048.358	0	4087.590	0	4126.656	0
3999.278	5	4049.018	0	4088.589	0	4127.380	3
4000.749	0	4049.806	0	4088.876	1	4127.816	1
4001.087	0	4050.359	0	4088.967	0 *R*	4128.108	1
4001.575	2	4051.419	2	4089.788	0	4128.287	2
4001.776	1	4052.008	1	4089.846	0	4129.160	1
4002.858	0	4053.516	2	4090.520	1	4130.739	2
4002.987	0	4055.048	2 *R*	4090.968	1	4131.129	2
4003.198	0	4055.818	0	4091.879	0	4132.358	0
4003.789	3	4056.907	0	4092.120	0	4132.556	0
4004.606	0	4058.258	0	4092.749	0	4133.840	7
4005.655	2	4058.759	0	4093.327	0	4135.147	2
4007.598	1	4060.216	0	4094.000	1	4135.996	0
4008.677	0	4060.499	0	4095.165	0	4136.907	1
4009.096	0	4060.737	0	4095.467	0	4137.489	2

CERIUM—*Continued*

Wave-Length	Intensity	Wave Length	Intensity	Wave Length	Intensity	Wave Length	Intensity
4137.668	3	4202.833	4	4288.646	1	4386.366	0
4138.107	1	4203.491	0	4289.915	3	4386.776	5
4138.370	1	4204.730	0	4292.615	1	4388.000	1
4139.459	0	4205.119	0	4292.796	0	4390.286	0
4139.836	0	4205.830	1	4294.797	1	4391.666	3
4140.518	0	4209.369	1	4296.075	1	4393.197	0
4140.797	1	4209.971	0	4296.366	0	4394.789	0
4142.430	2	4213.040	0	4296.726	7	4396.598	0
4142.846	1	4214.028	2	4299.364	2	4398.789	0
4144.529	2	4215.499	0	4300.345	3	4399.228	2
4145.018	3	4217.560	2	4304.296	0	4400.536	0
4146.250	2	4221.188	0	4304.746	0	4400.867	0
4148.169	0	4222.618	4	4305.147	1	4405.487	0
4148.907	0	4223.870	0	4306.738	3	4407.286	0
4149.956	10	4224.529	0	4309.736	2	4408.849	1
4150.976	2	4227.730	2	4310.748	0	4410.678	2
4152.076	3	4228.260	0	4311.647	0	4412.036	0
4153.165	1	4231.729	1	4315.395	1	4413.197	0
4154.457	1	4232.020	1	4317.316	0	4413.807	0
4155.266	0	4232.558	0	4320.736	2	4416.886	1
4155.555	1	4233.182	0	4324.817	0	4418.758	3
4159.048	2	4234.171	1	4326.837	0	4419.286	0
4160.166	1	4234.682	0	4330.405	1	4423.437	0
4161.187	1	4239.881	4	4331.786	0	4423.662	0
4162.671	0	4241.992	0	4332.700	1	4427.081	2
4163.548	1	4243.694	2	4334.871	0	4427.931	2
4165.229	3	4243.626	0	4336.342	2	4428.433	2
4166.677	1	4245.917	4	4337.792	3	4429.262	3
4166.906	2	4246.375	0	4339.336	2	4429.991	0
4167.802	1	4246.687	2	4340.577	0	4437.517	0
4169.816	4	4248.647	3	4342.188	0	4439.237	0
4172.145	0	4251.866	1	4342.487	0	4440.886	0
4174.287	0	4253.345	2	4343.597	0	4443.748	0
4175.245	0	4255.754	2	4345.936	2	4444.357	2
4176.056	0	4256.146	1	4346.436	0	4444.718	2
4176.715	1	4257.106	0	4349.788	3	4449.326	3
4179.297	1	4259.737	0	4352.637	2	4450.757	2
4181.116	1	4261.154	0	4353.408	1	4455.666	0
4185.345	2	4263.415	1	4359.086	0	4457.812	0
4186.578	10	4264.386	0	4360.186	0	4460.231	7
4187.287	2	4267.206	1	4360.478	0	4461.143	2
4189.166	0	4268.297	0	4361.677	0	4463.422	2
4189.607	1	4269.248	0	4364.666	3	4464.691	0
4190.635	1	4270.186	2	4366.998	0	4467.547	1
4193.084	3	4270.735	2	4367.577	0	4471.242	5
4193.256	2	4271.756	1	4368.236	0	4472.710	1
4193.855	1	4273.446	0	4369.249	0	4474.740	1
4194.897	1	4275.517	2	4372.428	0	4479.359	3
4196.216	2	4278.856	1	4373.836	2	4483.908	3
4197.616	3	4280.145	0	4375.936	2	4484.830	0
4197.974	1	4281.017	1	4380.087	0	4485.530	0
4198.643	10	4281.137	0	4381.799	0	4486.909	4
4201.242	2	4285.468	1	4382.157	3	4494.247	0

CERIUM—*Continued*

Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity
4495.398	0	4523.100	4	4551.338	1	4579.270	1
4496.247	0	4527.358	4	4558.620	0	4582.519	2
4497.859	1	4528.489	4	4560.320	3	4591.150	0
4500.368	0	4532.510	1	4560.908	2	4593.948	3
4506.448	0	4536.897	0	4562.368	7	4606.439	2
4508.109	0	4538.902	1	4565.279	0	4615.220	1
4509.198	1	4539.070	0	4565.867	2	4634.930	1
4511.660	0	4539.749	3	4572.298	4	4628.178	4
4515.870	0	4544.968	0	4572.730	0	4632.339	1
4518.029	0	4548.880	1	4578.790	1	4646.150	0
4519.598	0	4550.336	0				

THORIUM

Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity
3503.758	0	3532.031	0	3557.603	1	3600.558	0
3503.909	0	3534.663	0	3559.585	2	3600.975	0
3504.157	0	3536.115	0	3560.109	1	3601.206	2
3505.653	0	3537.245	1	3561.015	0	3603.372	2
3507.008	0	3537.568	0	3561.928	0	3603.518	1
3508.270	1	3538.349	0	3563.527	0	3603.765	0
3509.240 *br*	1	3538.507	0	3567.186	0	3604.208	0
3510.679	0	3538.903	0	3567.401	1	3604.846	0
3510.887	0	3539.425	2	3567.845	0	3605.802	0
3511.286	0	3539.711	3	3568.115	0	3607.536	0
3511.760	4	3540.475	0	3569.765	0	3608.530	0
3512.875	0	3541.753	0	3571.723	0	3609.362	0
3513.368	0			3572.531	3	3609.608	3
3513.895	1	3542.351	0	3573.382	1	3610.207	0
3514.656	1	3542.638	0	3573.669	0	3610.553	0
3515.096	0	3542.765	0	3575.443	2	3610.947	0
2515.847	0	3544.158	2	3576.559	0	3612.579	1
3516.463	0	3545.097	2	3576.682	0	3613.025	0
3516.689	0	3545.404	2	3579.460	1	3613.946	0
3516.947	0	3546.359	0	3580.371	0	3614.142	0
3518.535	0	4547.489	0	3582.148	1	3615.267	2
3519.008	0	3548.047	0	3584.286	0	3616.834	0
3519.805	1	3548.275	0	3587.115	1	3617.207	3
3520.814.	0	3548.593	1	3588.375	0	3618.525	5
3521.200	0	3549.048	1	3589.253	0	3619.863	0
3521.386	1	3549.479	1	3589.465	1	3620.521	1
3522.058	1	3549.726	0	3589.865	0	3621.282	2
3523.656	0	3549.858	0	3591.203	0	3622.476	0
3525.758	0	3550.419	0	3591.578	0	3622.549	0
3526.148	0	3550.893	0	3592.937	1	3624.138	0
3526.760	1	3551.538	0	3593.992	0	3624.640	0
3526.858	1	3552.003	0	3594.247	0	3625.095	3 *R*
3527.445	0	3552.878	0	3595.448	0	3625.790	3
3528.967	1	3553.259	1	3595.752	0	3626.085	1
3529.065	1	3553.538	0	3597.629	0	3632.765	0
3530 435	0	3555.175	2	3598.247	1	3634.358	0
3531.590	0	3555.837	0	3599.476	0	3634.725	1
3531.754	1	3556.443	0	3599.885	0	3635.396	1

THORIUM—*Continued*

Wave Length	Intensity	Wave Length	Intensity	Wave Length	Intensity	Wave Length	Intensity
3636.080	1	3681.351	0	3724.875	0	3774.348	0
3637.701	0	3682.038	0	3725.553	0	3776.425	0
3638.457	0	3682.652	0	3726.890	2	3781.162	0
3638.779	0	3685.057	1	3727.306	0	3781.513	0
3639.896	2	3687.126	1	3727.428	0	3783.175	1
3641.241	0	3688.115	0	3728.049	1	3783.447	2
3642.375	3	3688.903	2	3730.048	1	3785.486	0
3643.641	0	3690.259	0	3730.532	1	3785.763	3
3644.473	0	3690.649	1	3730.905	0	3787.021	0
3647.484	0	3690.768	1	3731.565	0	3788.488	0
3647.786	1	3691.560	0	3734.728	0	3789.255	2
3648.287	0	3691.782	0	3737.645	0	3790.948	1
3648.535	0	3692.044	1	3738.847	1	3791.443	0
3649.378	2	3692.233	1	3741.001	0	3792.502	0
3649.865	1	3692.714	1	3741.343	4	3794.379	0
3650.945	1	3693.841	0	3742.407	1	3794.477	0
3651.705	0	3694.007	2	3743.054	2	3794.850	0
3652.317	2	3694.489	0	3743.641	0	3796.883	0
3652.684	1	3695.472	0	3744.870	0	3800.505	0
3653.701	1	3696.153	1	3745.753	0	3803.224	2
3654.613	0	3696.807	0	3746.105	2	3805.969	0
3655.248	0	3697.172	0	3747.693	2	3808.046	1
3656.350	0	3698.253	0	3750.648	0	3808.280	0
3656.899	0	3698.440	1	3751.247	1	3809.984	0
3658.206	2	3700.017	0	3752.728	4	3811.157	0
3658.326	2	3700.484	0	3753.387	0	3813.187	1
3658.941	0	3700.921	0	3754.182	0	3821.563	2
3659.654	2	3701.153	0	3754.740	1	3823.208	0
3659.773	2	3703.020	0	3755.336	0	3823.705	0
3661.750	1	3703.386	0	3756.007	0	3827.068	0
3663.352	1	3703.950	0	3756.438	0	3828.539	1
3663.847	2	3704.192	2	3757.820	2	3829.535	0
3665.821	0	3705.007	0	3758.612	0	3833.203	0
3667.134	0	3706.151	1	3758.888	0	3836.648	2
3668.272	1	3706.932	1	3759.443	0	3838.006	1
3670.152	2	3707.147	0	3759.551	1	3839.853	2
3670.735	0	3707.606	0	3760.448	0	3842.115	1
3671.684	0	3708.907	0	3761.235	0	3854.687	3
3672.448	0	3711.463	2	3761.647	0	3856.513	0
3672.663	0	3711.769	0	3762.473	1	3873.965	1
3673.447	2	3712.715	0	3763.035	3	3874.980	2
3673.949	1	3715.734	0	3763.439	0	3875.389	1
3674.203	1	3715.987	0	3764.450	0	3884.675	0
3675.049	0	3716.703	0	3765.354	0	3884.963	0
3675.287	0	3717.965	0	3666.241	1	3885.897	0
3675.692	2	3718.275	0	3768.035	1	3887.053	0
3676.818	0	3718.765	0	3768.565	0	3887.143	0
3677.850	0	3719.547	2	3769.750	0	3891.205	0
3678.163	2	3720.404	2	3770.208	1	3891.896	1
3678.591	0	3721.990	3	3771.514	2	3892.455	0
3679.275	0	3722.302	2	3772.358	0	3893.247	1
3679.348	2	3723.440	0	3772.803	0	3893.568	1
3680.623	1	3723.798	0	3773.901	2	3894.573	1

THORIUM—*Continued*

Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity
3895.054	0	3951.673	1	4025.752	0	4106.112	1
3895.540	1	3952.909	0	4026.259	0	4108.597	2
3898.614	0	3955.301	1	4027.138	0	4109.523	1
3900.268	0	3956.003	1	4028.785	0	4110.795	1
3900.716	0	3956.743	1	4029.086	1	4111.048	1
3901.027	1	3956.832	1	4030.137	0	4112.887	1
3901.289	0	3959.439	0	4030.985	0	4115.947	1
3901.811	0	3960.483	0	4032.085	1	4116.878	1
3903.252	0	3962.548	0	4032.263	1	4123.748	1
3904.236	1	3963.355	0	4034.407	0	4127.606	0
3905.237	1	3963.610	0	4035.042	0	4131.200	0
3906.948	0	3965.008	0	4036.175	1	4131.618	1
3911.136	0	3967.547	1	4036.702	1	4132.943	1
3912.090	0	3969.153	0	4040.021	0	4134.269	1
3912.438	0	3972.286	0	4041.359	0	4140.410	1
3913.157	0	3972.794	0	4043.276	0	4141.808	0
3913.965	0	3973.370	0	4043.573	0	4142.900	2
3915.365	0	3973.756	1	4051.048	0	4148.312	1
3916.534	0	3974.378	1	4053.705	0	4150.187	1
3916.879	0	3976.532	0	4059.403	0	4152.418	1
3917.385	0	3979.175	0	4060.038	0	4155.525	2
3918.186	0	3980.239	0	4063.547	1	4156.407	0
3918.402	0	3980.901	0	4064.475	0	4162.865	0
3918.621	0	3981.256	0	4065.789	1	4163.732	0
3919.143	0	3982.027	1	4067.601	1	4164.248	0
3922.350	1	3982.239	1	4069.364	2	4165.187	0
3922.756	1	3984.506	0	4069.610	1	4165.501	0
3923.939	0	3984.899	0	4071.925	0	4165.870	0
3925.247	0	3987.365	0	4072.785	0	4167.748	0
3925.338	1	3987.854	0	4075.683	0	4168.729	0
3926.838	0	3988.710	1	4075.887	0	4170.575	0
3927.308	0	3988.	1	4080.475	1	4171.463	0
3927.561	0	3990.646	0	4080.860	0	4171.803	0
3929.803	2	3991.903	0	4081.537	0	4178.087	2
3932.155	0	3992.447	0	4082.429	1	4179.721	1
3932.376	0	3994.701	2	4083.605	0	4180.011	0
3933.037	0	3996.212	1	4085.178	2	4181.015	0
3935.715	0	3998.015	1	4085.576	0	4193.078	0
3937.153	0	4000.453	0	4086.673	3	4202.005	1
3938.059	0	4001.237	0	4088.847	1	4209.003	3
3938.868	1	4001.864	0	4089.278	0	4216.065	0
3941.387	1	4003.246	0	4091.525	1	4217.247	0
3942.184	0	4003.458	1	4093.511	0	4218.234	0
3942.790	0	4005.202	0	4094.907	3	4218.572	0
3943.503	1	4005.679	0	4097.483	0	4220.101	0
3943.818	0	4006.515	0	4097.887	0	4224.208	0
3945.636	1	4007.163	1	4099.093	1	4224.565	0
3945.941	0	4008.331	1	4100.518	2	4229.472	0
3946.279	1	4009.175	1	4001.088	1	4230.441	0
3947.477	0	4011.891	1	4102.817	1	4233.357	0
3948.175	1	4012.620	2	4103.785 *br*	1	4234.358	0
3949.006	2 *R*	4019.271	4	4104.553	1	4235.046	0
3950.540	1	4022.233	0	4105.502	1	4235.337	0

THORIUM—*Continued*

Wave Length	Intensity	Wave Length	Intensity	Wave Length	Intensity	Wave Length	Intensity
4240.700	0	4298.896	1	4359.393	0	4436.016	0
4242.773	0	4299.850	0	4361.292	0	4436.239	0
4244.082	0	4300.801	0	4361.740	0	4436.420	0
4247.739	0	4306.389	0	4365.893	0	4439.148	1
4248.015	1	4307.188	0	4366.936	0	4440.548	0
4249.710	0	4309.070	3	4369.257	0	4440.856	1
4250.318	2	4313.000	0	4373.825	0	4443.075	0
4253.562	0	4313.307	1	4374.137	0	4447.700	1
4255.781	0	4315.336	0	4374.788	1	4454.507	0
4256.147	0	4318.346	2	4377.837	1	4458.031	0
4256.262	0	4319.100	0	4378.162	0	4461.162	0
4257.534	0	4320.146	2	4380.926	1	4465.352	2
4260.425	0	4320.505	1	4381.890	10	4469.511	0
4261.305	0	4327.146	1	4384.686	0	4472.301	0
4263.402	0	4328.697	0	4387.077	0	4474.090	0
4264.155	0	4329.519	0	4391.029	10	4480.829	0
4270.351	0	4331.979	0	4393.008	1	4481.642	0
4271.122	0	4333.965	0	4394.888	0	4485.783	1
4272.910	1	4334.727	2	4396.500	0	4486.101	0
4272.379	3	4337.386	2	4398.897	0	4486.892	0
4273.018	1	4338.120	0	4399.069	0	4487.487	1
4274.350	0	4341.036	0	4400.372	0	4488.648	2
4276.818	0	4342.285	0	4401.616	0	4492.225	0
4276.927	1	4342.434	1	4402.938	1	4493.336	1
4277.340	4	4343.612	0	4408.867	0	4496.307	0
4278.321	0	4343.991	1	4410.427	0	4498.934	0
4280.610	0	4344.333	1	4412.535	1	4499.966	0
4281.096	2	4344.661	2	4412.776	6	4505.198	0
4281.446	1	4346.450	0	4414.505	1	4510.527	2
4282.055	3	4347.742	0	4416.187	0	4512.477	0
4283.546	1	4349.883 *br*	1	4418.524	0	4513.676	0
4284.960	0	4353.299	0	4421.536	0	4524.848	0
4286.219	0	4354.481	0	4422.047	0	4531.699	0
4288.035	0	4355.300	1	4423.885	1	4532.288	0
4291.846	0	4357.585	0	4427.636	1	4533.307	1
4295.099	0	4358.310	1	4432.237	0	4534.118	0
4297.357	0	4358.559	0	4432.945	3	4535.236	0

III. EFFECT OF VARIATION OF CURRENT AND CAPACITY UPON THE WAVE-LENGTHS OF SPARK SPECTRA

INTRODUCTION

Owing to the fact that the spark is always used for comparison in all line-of-sight work, it is of importance to know whether or not the wave-lengths of spark spectra remain constant under varying external conditions of current, capacity, etc. Although considerable experimenting has been done along this line during the last few years, yet there is a diversity of opinion among observers. As the spark and arc lines did not coincide on several of the plates of spark spectra, this part of the investigation was undertaken to determine whether or not this displacement was due to mechanical causes.

HISTORICAL REVIEW

It has been observed by Hemsalech,[1] Schenck,[2] and others that when small amounts of self-induction were introduced into the circuit of a spark discharge, there was at once a complete extinction of all the air lines from the spark spectrum, and at the same time, a narrowing of all metallic lines. As the self-induction was increased, certain others, called "short lines," disappeared, while certain lines called "long lines," also present in the arc, remained visible or increased in intensity. There was also a third class which were absent in the ordinary condenser spark, being present in the arc, but which appeared on introducing self-induction in the circuit. But the function with which we are particularly concerned here is the effect of self-induction, capacity, and current, etc., upon the wave-lengths of the lines. Exner and Haschek,[3] in the course of their measurements of the ultra-violet spark spectrum of the elements, observed that in many instances the wave-lengths of the lines in the spark spectrum differed considerably from the wave-lengths of the corresponding lines in the arc, this difference being often as much as 0.5 of an Ångström unit. They considered this difference as due to pressure in the spark, and with data taken from the tables of Humphreys and Mohler[4] they estimated the pressure in the spark to be about 25 atmospheres.

Haschek and Mache[5] then attempted to show the existence of such a pressure. A spark was produced in an air-tight vessel which was filled with air and connected to a manometer. On discharging the spark they observed a sudden rise of the manometer and interpreted it as due to a pressure in the spark; but the value thus obtained was much greater than that calculated from the difference between the wave-length of the arc and spark lines. They also found[6] that this effect increased to a maximum and then decreased as they increased the amount of capacity across the discharge circuit.

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[1] *Comptes Rendus*, **129**, 285, 1899; *Journal de Physique* (3), **8**, 652, 1899.

[2] *Astrophysical Journal*, **14**, 116, 1901.

[3] *Sitz. der Kais. Akad. der Wiss. in Wien*, 1897, and following years.

[4] *Astrophysical Journal*, **3**, 114, 1896, and **4**, 175, 1897.

[5] *Sitz. der Kais. Akad. der Wiss. in Wien*, 1898.

[6] *Astrophysical Journal*, **19**, 351, 1899.

Eder and Valenta,[1] however, found no displacement whatever, in case of the lines studied by Exner and Haschek.

Kent[2] repeated some of Exner and Haschek's work and observed that in case of titanium the arc and spark lines did not coincide, due to capacity, but that the displacement did not amount to more than 0.04 of an Ångström unit for lines in which Exner and Haschek observed as much as 0.13 of an Ångström unit.

Middlekauff,[3] working in this laboratory with the 21-foot grating, observed no shift in case of the spark lines of iron.

Keller,[4] working at Bonn, observed a shift but found that it was due to the manner in which the light was allowed to fall upon the slit.

THE PRESENT INVESTIGATION: APPARATUS

The same grating was employed as that used for photographing the arc and spark spectra of the elements.

For electrodes, rods of Norway iron, and also metallic chromium, manganese, and calcium were used. These were turned down to the form of a blunt lead-pencil as stated before.

METHOD OF EXPOSURE

One of the greatest difficulties encountered was the mechanical shift which appeared in a greater or less degree upon nearly every plate. At first this was taken to be a true shift. In order to avoid any possibility of a shift due to a jar of the camera when the shutter was turned, the shutter was detached entirely and held in place by two iron clamps which rested on the floor.

The middle of the plate was generally used for photographing the spark, then the shutter was turned and the arc spectrum obtained on either side, thus making it possible to observe any shift that might occur. Plates were also taken in which the arc spectrum was on the center of the plate.

RESULTS

A large number of plates were taken by the method described above, and invariably it was found that no shift whatever occurred.

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[1] Kayser, *Handbuch der Spectroscopie*, Vol. II, p. 308.

[2] *Astrophysical Journal*, **14**, 201, 1900.

[3] *Ibid.*, **21**, 116, 1895.

[4] *Zeitschrift für Wiss. Phot.*, **4**, 209–231, 1906.

However, in the case of a few lines which were reversed either in the arc or spark, there was an apparent shift due to an unsymmetrical reversal. This is illustrated by the following wave-lengths of some of the chromium lines taken from the plates. In case of the fourth and fifth lines there was an unsymmetrical reversal of the arc lines.

λ Spark	λ Arc	
3823.644	3823.644 (*r*)	0.000
3875.257	3875.257 (*r*)	.000
4254.352	4254.354	.002
4645.853	4645.875	.024
4680.489	4680.505	.016

The effect of increasing the current in the primary of the induction coil was greatly to increase the intensity of the spark, but at the same time, to cause more lines to reverse. The latter was especially true in the case of calcium and manganese.

The current was varied from twelve to forty amperes, this being the limit of the coil. When forty amperes current was used, almost every line in the spark spectrum of both calcium and manganese was reversed, while many were doubly reversed and a few of the manganese lines were reversed three times.

Hence the conclusions to be drawn from the present investigation are the following:

1. In the case of a spark discharge in air, there is no change produced in the wave-length by variations in the circuit conditions up to 60 amperes except that due to unsymmetrical reversal of the lines.
2. There is no appreciable difference in wave-length between the arc and spark lines.

In conclusion, the author takes pleasure in acknowledging his great indebtedness to Professor Ames, at whose sugggestion the work was undertaken, and under whose immediate supervision it was carried out. He also wishes to thank both Mr. L. E. Jewell for his interest and valuable suggestions during the progress of the work, and Dr. A. C. Neish of Columbia University, who prepared and furnished the salts of cerium and thorium.

Johns Hopkins University
January 1909

STATIONARY METEORIC RADIANTS. THE SIZE OF METEORS

BY WILLIAM H. PICKERING

The existence of meteoric showers of several months' duration, coming from the same point in the sky, was first described at length and in detail by Mr. Denning, in *Monthly Notices*, **38**, 111, 1878. The apparent radiant of a meteor depends in general nearly as much on the true direction of motion of the earth, as it does on the true direction of motion of the meteor. Consequently if the same meteor, moving in a straight line, could be met by the earth in two different parts of its orbit, we should find the meteor apparently coming from very different parts of the sky. Thus the radiant of the Perseids from July 8 to August 21 shifts in right ascension from 11° to 55° (Denning's Catalogue, *Memoirs of the Royal Astronomical Society*, **53**, 234, 1899), or a change of 44° in as many days. Consequently the statement that a meteoric shower can persist for five or six months, with the meteors apparently continuing to come from identically the same point in the heavens appears to be well-nigh unbelievable.

No thoroughly satisfactory explanation of this phenomenon has been offered since Mr. Denning first called attention to it. Indeed, in *Monthly Notices*, **45**, 93, 1884, he says: "The fact of stationary radiants exhibiting visible activity during several months is a phenomenon so unaccountable and so utterly opposed to the approved theories as to the orbits of shooting stars, that it must receive a most crucial examination before it can be accepted."

Professor C. A. Young in his *General Astronomy*, §787*, 1899, says: "No satisfactory explanation of such fixity as yet appears, and though Mr. Denning is perfectly confident of the genuineness of his discovery, and though it is very generally accepted as a fact, some very high authorities, Tisserand, for instance, still question it, as being 'incredible and unaccountable.'"

Professor H. H. Turner, in *Monthly Notices*, **59**, 147, 1899, after offering an explanation of the stationary radiants, which he himself later criticizes, says: "In spite of this difficulty, I venture to publish this suggestion, in the hope that it may perhaps at least draw attention to the important problem of stationary radiants." Again he says, referring to another suggestion, that these radiants are due to a confusion of different meteoric streams: "My own opinion is that this interpretation of Mr. Denning's observations is impossible, and that we are face to face with another of those cases in which observation supplies us with facts apparently inexplicable by theory at present, for which a theoretical explanation will yet be found."

The two objections to Professor Turner's explanation which he has himself pointed out, and which strike the writer most forcibly, are: (*a*) That the meteors must move extremely slowly with regard to the earth—this does not seem to be the fact; (*b*) the explanation implies a uniform relative velocity of the meteors and the earth throughout the duration of the shower—this is contrary to the observations.

In the present paper an endeavor has been made to show clearly what assumptions are necessarily involved in any effort to explain the phenomenon described by Mr. Denning. The three hitherto irreconcilable facts stated by him, *Monthly Notices*, **45**, 111, 1884, are: (*a*) the radiants are stationary; (*b*) the showers are of several months' duration: four showers last for six, one shower for seven months; (*c*) the velocity of the meteors changes noticeably in different portions of the earth's orbit. When the apex of our motion is near the radiant the relative speed is high; when remote from it, the speed is low.

Starting with first principles, it is certain that the meteors are moving in an orbit around the sun, rather than around the earth, because of this variable velocity. The orbit must be a conic section, and is almost certainly an ellipse. The direction of its major axis may vary, depending on the portion of the earth's orbit where the meteors are encountered—and this in fact is the opinion of Bredikhine, who considers the phenomenon as due to a series of separate meteoric streams. Denning, on the other hand, believes that all the meteors coming from the same radiant belong to the same shower. This is equivalent to assuming that the orbits of all these meteors have a common major axis. It is our object to explain, as far as possible, the existence of stationary radiants, accepting this condition.

Let us now consider the inclination of the orbit, and let us first suppose that it is perpendicular to that of the earth. If the intersecting node coincides with either the aphelion or the perihelion, the direction of the meteors will also be perpendicular to the plane of the earth's orbit. Under these circumstances the position of the radiant in longitude will depend on the earth's motion, and be independent of that of the meteors. The longitude of the radiant will coincide with that of the apex of this motion, and consequently will increase 1° per day.

As an illustration of this case, let us consider the Perseid shower, which somewhat approximates to these conditions. The latitude of its perihelion is +25°. Its perihelion distance is 0.96, and the inclination of its orbit 116°. When the shower is at its maximum, on August 10, the longitude of the sun is 137° and the longitude of the earth's apex 47°. The meteors should therefore seem to come from this longitude. Their observed radiant in fact on this date is 44°. The radiant advances on the average, as we have already seen, 1° a day in right ascension. Their duration is 44 days, which means that the breadth of the stream is about sixty millions of miles (100,000,000 km). In fact their long duration compared with that of the Leonids indicates, quite as clearly as their uniform distribution along their orbit, the great antiquity of the stream in its present position.

Now let us still assume that the orbit of the meteors is perpendicular to that of the earth, but that the node does not coincide with the aphelion or perihelion. The only difference between this case and the last is that the vertical component of the meteor's motion now has a nearly constant horizontal component combined with it. This will change the direction of the radiant in longitude, but will not materially affect its daily rate of change. It therefore appears that orbits giving stationary radiants cannot be highly inclined to the ecliptic.

Let us now imagine that the earth and a meteoric stream revolving in the same plane, but in opposite directions, should cross one another at intervals of every six months. In the simplest case, where the motions of the two bodies were diametrically opposed, the difference in longitude of the two radiants would be 180°. In every case the motion of the meteors would tend to increase that component of the radiant due to the earth's motion. This component is the one which induces a change in longitude, and is therefore the one whose influence in certain places we must seek to counteract. As far as inclination is concerned, therefore, meteors with stationary radiants can only be revolving in orbits but slightly inclined to the elliptic, and with a direct revolution.

Let us next consider the velocity of the meteors. The Leonids when they reach the outer limits of our atmosphere are, as we know by the elements of their orbit, moving at a speed relatively to the earth of 44 miles (71 km) per second. The Andromedids on the other hand are moving at a speed of only 11 miles (18 km). How fast either of them is moving when they later become visible to us, we have no precise means of measuring, but we have no difficulty in recognizing the difference in their speed, and we are able to form an impression of how a meteor looks that has had a speed of 44 miles a second when it struck our atmosphere, and how one looks that has been going only 11 miles per second.

Describing the meteors from stationary radiants Mr. Denning says in *Monthly Notices*, **45**, 444, 1885: "The meteors falling from the stationary radiants referred to in my paper read at the December meeting, appear to be of ordinary character, and their motions cannot differ essentially from the parabolic velocities exhibited by the planetary streams." From this statement we conclude that their orbits cannot nearly coincide with that of the earth, as suggested by Professor Turner in *Monthly Notices*, **60**, 455, 1900, but that their major axes must be of the same order of size, at least, as those of the periodic comets.

In Mr. Denning's latest catalogue, to which reference has already been made, it is interesting to note that out of 278 different radiants he records only two of which the motion has been definitely established, one, the Perseids already mentioned, and the other, the Lyrids of April 20. The inclinations of the orbits of the comets with which they are associated, 1862, III, and 1861, I, are 114° and 80° respectively. It will thus be seen that in both these cases the orbit is very highly inclined to the ecliptic.

In the catalogue, however, under the head of the Leonids, are found some very interesting observations made by Professor F. P.

Brackett and nine assistants on the six days extending from November 12 to 17, 1898. On one of these dates the longitude of the radiant shows a large deviation from the others, due apparently to the existence of what Professor Brackett in his original paper terms subradiants (*Popular Astronomy*, **7**, 34, 1899). The five other dates show a fairly regular progression in longitude at the rate of 1° per day, as follows: 147° 30′, 148° 45′, 150° 42′, 150°, ——, 152° 30′. The longitude deduced for the radiant on the date indicated by a dash was 147°. The total number of Leonids observed was 177, of which 109 were seen upon November 15. This change of the radiant is precisely what we should expect theoretically for a retrograde stream both in direction and amount.

All the other meteoric streams mentioned in the catalogue, for aught we can tell to the contrary, therefore, may have stationary radiants. Indeed, on p. 204 Mr. Denning definitely states, referring to these radiants: "Certain (in fact the great majority) of these displays are not confined to limited periods, but extend their activity over several weeks, and in many cases over several months." In endeavoring to explain stationary radiants, therefore, we are trying to make clear not the exceptional occurrence, but the usual phenomenon of a meteoric encounter.

Of the 45 recognized periodic comets belonging to our system, but three have retrograde orbits. The great majority of these comets were rendered periodic by the influence of the planet *Jupiter*, and their orbits are generally inclined at small angles to the ecliptic. We may probably safely assume that the meteoric radiants are due, not to parabolic or hyperbolic orbits, but to elliptic orbits whose major axes are of moderate length, and that the meteors themselves are in fact in the majority of cases the visible remains, the ghosts, so to speak, of defunct periodic comets. We may properly expect then that their orbits also will be inclined at small angles to the ecliptic.

Let us now construct a figure representing such an orbit, which we will for the present assume to lie exactly in the plane of the ecliptic. In Fig. 1 let *S* be the sun, let the circle represent the earth's orbit, and the line *P A* the direction of the major axis of the orbit of the meteors. Let us call the direction of *A* as seen from *S* longitude 0°. If the earth encounter the meteors at their aphelion *A*, their orbit will be small; if it encounter them at their perihelion *P*, their orbit will be large; but in either case the longitude of their radiant will be 90°. The reason of this is because, when at the same distance from the sun, the meteors will move faster than the earth, if at their perihelion, but slower than the earth if at their aphelion.

Let us now assume an ellipse of known major axis and eccentricity intersecting the earth's orbit at *T*. Let *E T H* be a tangent to the circle and *M T O* a tangent to the ellipse. Lay off *M T* and *T H* equal respectively to the velocities of the meteor and the earth, complete the parallelogram *T H R M*, and prolong *R T* to *D*. Then *R T* will give the relative velocity of the two bodies, and *R D F* the longitude of the radiant. These two quantities are readily deduced by solving the triangles *T S F* and *T H R*. It is a curious fact that except in the immediate vicinity of *A*, no matter what ellipse we may construct along the major axis *P A*, prolonged if necessary, provided the aphelion is to the right, the angle *R D F* will depend only on the location of *T*. This statement is not rigorous, but it is unexpectedly exact.

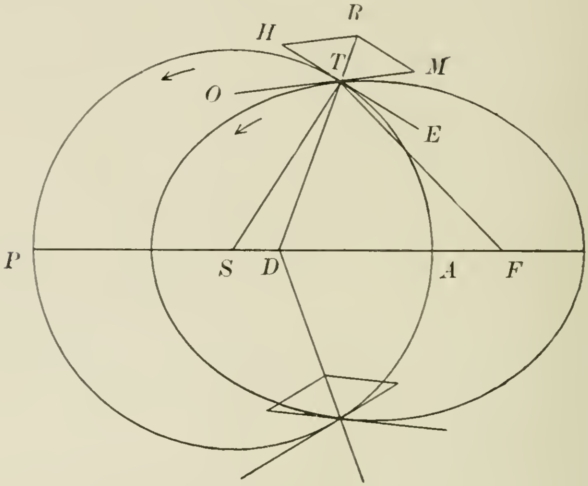

Fig. 1

The reason of it may be seen by an inspection of Fig. 2. Let *T H* represent the velocity of the earth along a small portion of its orbit, and *M T* that of a meteor moving in an orbit of large mean semi-diameter. If this meteor is followed by another having the same major axis, but moving in an orbit of small mean semi-diameter, its direction *m T* will lie at a somewhat larger angle with *T H* but it will also have a smaller velocity relatively to the sun. The unexpected feature is that these two changes should so nearly compensate one another that the angle *R T H* remains practically constant.

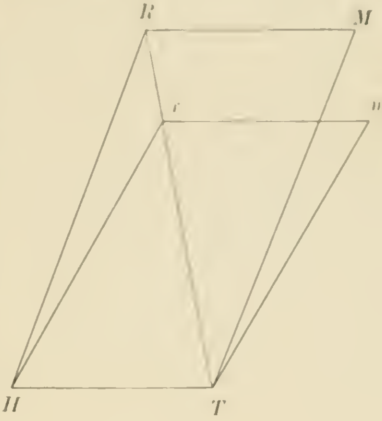

FIG. 2

To illustrate this fact the following table has been constructed, where the first column gives the aphelion distance of the assumed meteor, the second the mean semi-diameter of its orbit, the third its eccentricity, the fourth the longitude of the earth at the intersection of the two orbits, the fifth the longitude of the resulting radiant, and the last the velocity of the meteor relatively to the earth

LONGITUDE OF RADIANTS

Aphelion	*a*	*e*	Long. of Earth at Intersection	Long. of Radiant	Velocity Relative to Earth
1.00	0.900	0.111	0°.0	90°.0	0.057
1.02	0.510	1.—	0.0	78.8	1.020
1.02	0.773	0.320	16.8	73.1	0.200
1.02	0.981	0.040	58.9	74.8	0.036
1.02	1.005	0.015	108.7	98.9	0.014
1.02	1.008	0.012	131.3	106.1	0.010
1.02	1.009	0.0106	150.1	109.0	0.007
1.02	1.010	0.0099	180.0	90.0	0.005
1.04	0.520	1.—	0.0	74.5	1.038
1.10	0.550	1.—	0.0	66.9	1.087
1.10	0.670	0.640	19.2	67.1	0.502
1.10	0.834	0.320	38.1	68.7	0.265
1.10	0.948	0.160	61.6	76.4	0.152
1.10	1.000	0.100	84.2	87.1	0.100
1.10	1.019	0.080	98.8	94.1	0.079
2.20	1.340	0.640	70.7	80.0	0.522
6.19	3.525	0.756	132.4	105.6	0.509
6.06	3.525	0.720	165.7	100.0	0.332
45.08	23.000	0.960	146.8	105.7	0.522
45.77	23.000	0.990	56.8	71.6	1.267

at the time of collision. Numerous orbits of small aphelion distance are given for comparison with orbits where the aphelion distance is larger. The last two aphelia are comparable to those of the orbits of the Perseids, the two previous ones to those of the Andromedids.

These results are plotted in the curves represented in Fig. 3. The abscissas at the top represent degrees of longitude and are taken from

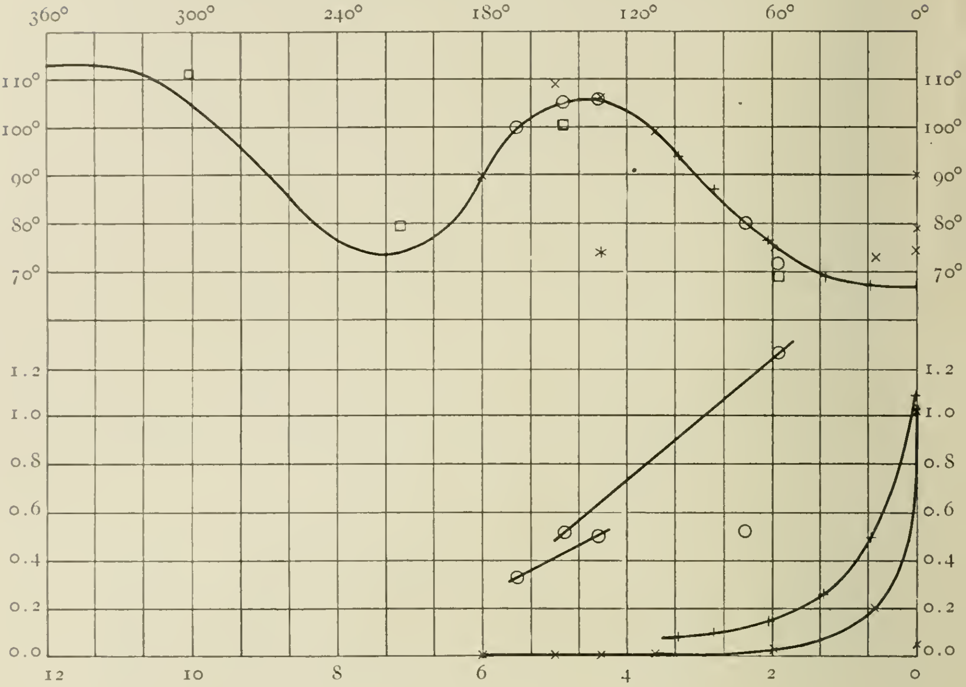

FIG. 3

the fourth column of the table. The abscissas at the bottom represent time, and divide the year into twelve equal parts. The upper series of ordinates gives the longitude of the radiant taken from the fifth column of the table. The lower series of ordinates gives the velocity of the meteor relatively to the earth, and is taken from the last column. The unit velocity is that of the earth in its orbit. Where the aphelion distance is less than 1.10 the result is indicated on the figure by an ×, where it is exactly 1.10 it is indicated by a +, and when it is 2.20 or over, it is indicated by a circle. The half of the curve beyond longitude 180° is exactly like the other half except that it is reversed. By the lower curves we see that unless the aphelion distance is large, the relative velocities between longitudes 90° and 180° are extremely small. It has not been thought necessary to carry the lower curves into the higher longitudes, since the results would be obviously identical with those shown.

From an inspection of the figures we can see at a glance how closely the circles and crosses follow the upper curve except near longitude 0°. We next notice that for an interval of eight months, or between longitudes 60° and 300°, the longitude of the radiant does not vary more than 16° from its mean position. In other words where the inclination is 0°, every radiant must remain within 16° of its mean position for eight months of the year, no matter how the orbit is constructed. Moreover, no radiant of an unperturbed orbit, no matter how it is constructed, can vary less than this amount, if it is observed continuously for eight months.

We may look at these facts from another point of view. Taking up the various elements of every possible orbit of the meteors in succession, we find that the date of perihelion passage is given all possible values, since the meteors are supposed to revolve in a ring. The longitude of the perihelion and the inclination are fixed by the premises. The longitude of the node is not affected in the case considered. Variations in the eccentricity and mean semi-diameter, which control the perihelion distance, are found to neutralize one another, and so produce no effect. These six quantities are the only ones which can effect an unperturbed orbit. Therefore without introducing perturbations from other bodies, particularly the earth, no other variations in the orbits are possible.

The presence of the earth is capable of producing two effects on meteors that soon after encounter it, and so become visible. One is to deflect them, the other to accelerate their velocity. The deflective effect always tends to cause the apparent radiant to approach the zenith. In Fig. 4 the direction of the radiant, as deduced from the upper curve of Fig. 3, is represented in each of the four quadrants by the arrow-heads. During the first quadrant counting from *A*, most of the meteors coming from the given radiant are met after midnight, when the general effect will be to increase the longitude of the radiant. During the second quadrant the meteors are approaching perihelion and traveling faster than the earth, therefore they should mostly be seen before midnight when the longitude of their radiant will be diminished. During the third quadrant the only meteors seen will be shortly after sunset, when the longitude of the radiant will be increased. Finally, in the fourth quadrant the only meteors seen will be shortly before sunrise, when the longitude will be diminished. By reference to Fig. 3 it will be seen that each of these effects tends to straighten out the curve.

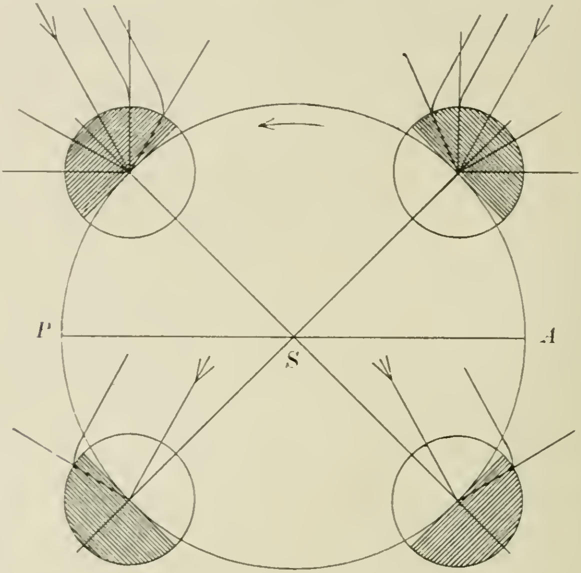

FIG. 4

The maximum acceleration that a meteor can experience with a stationary earth, as has been shown by Turner (*Monthly Notices*, **59**, 141, 1899), is 0.07 of its own velocity. The effect of this acceleration on a meteor whose aphelion distance is a little over 45 units, in longitude 56°8, is to reduce the longitude of the radiant by 2°9. In longitude 146°8 the reduction will amount to 5°0. These results are shown by the little squares in Fig. 3. If the aphelion distance is less, the effect is much more striking. Thus, with an aphelion distance of 1.02 units, in longitude 131°3, an acceleration of only

1 per cent. will reduce the longitude of the radiant by 32°.0. This result is indicated in Fig. 3 by the (*). The effect in the second and third quadrants is nearly twice as great as it is in the first and fourth, but this ratio is still further greatly increased if we consider that the earth is not stationary, but moving. This change makes little difference in the first and fourth quadrants, where the earth and meteor are moving at a high angle with regard to one another. In the second and third quadrants, however, the two bodies are moving in more nearly parallel courses and the meteor is following and overtaking the earth. Under these circumstances the accelerating force lasts for a much longer time, with a correspondingly greater effect.

We thus see that both the deflecting effect and the accelerating influence of the earth tend to straighten out the curve between longitudes 90° and 270°, and the former tends to straighten it in the other half of its course, as may be seen by inspection of Fig. 4. We also see that the mass of the earth is quite sufficient to accomplish this straightening process, provided the aphelion distance of the meteors is not too great. The duration of the shower will of course depend on the breadth of the stream, which in turn depends on the variation in the eccentricity and mean semi-diameter of the orbits of its components.

Although it is true that the earth is quite capable of straightening out the curve, or even of reversing the direction of its curvature, there is no evident reason why it should leave it exactly straight, as should be the case if the longitude of the radiant was invariable. However, the observations are admittedly not very exact when the meteors are scarce. Mr. Denning states (*Monthly Notices*, **38**, 111, 1878): "that a radiant of a feeble system can seldom be relied upon, except in special cases, to within five or seven degrees." When the meteors are more frequent the accuracy is much greater. In the *Monthly Notices*, **45**, 102–105, 1884, he gives a series of tables of his own, and of others' observations, for six different meteoric streams which he considers to exhibit stationary radiants. Observations by a single experienced observer are naturally more accordant than those made by a group of different persons. But even in his own case the monthly means have a range of three or four degrees in right ascension, although in declination they are appreciably more accurate.

It therefore appears that the observations do not require that the line in Fig. 3 should be made exactly straight and horizontal.

There now only remains the general case of a meteoric stream inclined at a small angle to the plane of the ecliptic. Under favoring conditions there is no reason why the ascending nodes of the orbits composing some of the streams should not be distributed throughout the whole 360° of longitude. The conditions may then be represented by a series of prolate ellipsoids with a common major axis, the meteors proceeding from pole to pole over their surfaces and moving in different directions. This would create no confusion, however, for those meteors moving in one direction would have one radiant, and those moving in the opposite direction would have another. It would thus be possible for meteors from a single swarm to move from north to south throughout the year.

It is not likely, however, that any stream persists continuously for many months. It is obvious that geometrical difficulties would arise in such a case. But there seems to be no reason why a stream should not persist for several weeks from approximately the same radiant, and then cease, and reappear in the same place, or near it, a few months later. Such, in fact, appears to be the case with most of the stationary radiants that Mr. Denning has described. It therefore appears that the general principle of radiants, at least approximately stationary, for certain intervals of time, far from being "incredible and unaccountable" with our present knowledge of meteoric orbits, is exactly what this very knowledge would lead us to expect.

Let us now see what further evidence we have for associating the meteors from stationary radiants, that is to say, the majority of meteors, exclusive of the Perseids and Leonids, with the planet *Jupiter*. It is a well-known fact that the perihelion of the comets belonging to *Jupiter's* family are very irregularly distributed in longitude. More than half of them, or 17 out of 33, lie within an arc of 79° or between longitudes 334° and 53°. Their mean longitude is therefore 13°.5. It is also a fact of common knowledge, even omitting the brilliant showers of the Perseids, Leonids, and Andromedids, that the great majority of our meteors are seen between the middle of July and the middle of December. Denning states that meteors are about twice as frequent during this portion of the year as at other times.

Now the heliocentric longitude of the earth in mid-July is 293° and at mid-December 84°, mean value 8°.5. It therefore appears that we encounter the greatest number of meteors when the earth is passing those longitudes where the perihelia of the comets of *Jupiter's* family are most thickly distributed. The mean perihelion distance of these 17 orbits is 1.35, so that in these longitudes their orbits and that of the earth most nearly approach tangency. The cometary orbits attached to the planets lying beyond *Jupiter* lie at all inclinations, and the longitude of the maximum density of their perihelia is about 60°, so that we find no evidence of connection between these comets and the meteor swarms possessing stationary radiants.

If in Fig. 4 we consider *P* to lie in longitude 13°.5, the mean longitude of perihelion of *Jupiter's* comets, then, from what we have already seen, we might expect the radiants of their associated meteors to lie in longitude 283°.5, or 90° preceding. This would undoubtedly be the case if we could see all the meteors that fell during the whole twenty-four hours, and also if the perihelion distance of *Jupiter's* comets was unity. Since neither of these is the case, however, both of these causes tend to increase the longitude of the radiant. Furthermore, in the early morning hours we encounter other meteors traveling both faster and slower than ourselves, while in the evening hours we encounter only those that are traveling faster. The greater number of meteors seen in the morning in this case is due to the fact that all meteoric streams do not have their perihelia in the longitude of *P*; moreover some of their orbits are highly inclined, and others retrograde. As a result of these various facts the longitude of the mean radiant is advanced over 90°, so that the greatest number of meteoric radiants lies between the longitudes of 0° and 60°, mean 30°, as shown by Denning in his catalogue.

An examination of the lower curves in Fig. 3 shows that those meteors moving with cometary velocities and indicated by circles would encounter the earth' at perihelion, longitude 180°, with a velocity of not less than about 0.3 that of the earth in its orbit, of 5.5 miles (9 km) per second. Now there are many very slowly moving fireballs, so called, that even at great altitudes obviously move at much slower rates of speed. There are also some very rapidly moving fireballs, such as those associated with the Leonids, for instance.

Among fainter objects, however, it is a curious fact that those moving slowly through long arcs are not often noticed.

These slowly moving fireballs necessarily cannot have their aphelia located very far beyond the earth's orbit. Mr. Denning, on p. 212 of his "Catalogue," says: "The disposition of certain radiants to supply fireballs is very well marked; and it is a notable circumstance that these fireballs make their apparitions at a period when the radiants are far removed from the apex of the earth's way, and usually placed in the western sky. The positions are nearly all on the ecliptic." It would seem then that their orbital motion must be direct, and their perihelion distance about unity, so that we encounter them when they are overtaking us. Their velocities would be well represented by the two lower curves shown at the bottom of Fig. 3. The perihelia of the fireballs as a whole are distributed in longitude according to the same law that affects *Jupiter's* comets, for nearly twice as many are seen during the last half, as during the first half of the year.

The comparatively slow speed at which fireballs often move is one of the reasons for the fact stated by Mr. Denning (*Monthly Notices*, **50**, 413, 1890) that the average height at which they disappear is 30 miles (48 km), while the ordinary shooting stars disappear at an altitude of 54 miles (87 km). Since the density of the atmosphere is halved for every three and a half miles that we ascend, its density where the fireballs are last seen is 128 times as great as where the shooting stars disappear.

We often see the statement made that the average mass of a shooting star is at most but a few grains. This intrinsically improbable statement is based on some very doubtful assumptions with regard to the amount of energy converted into light by a candle, and the amount of energy similarly converted by a meteor. It was shown in the *Annals of Harvard College Observatory*, **41**, 140, in connection with a study of the Leonid shower of 1897, that if the intrinsic brilliancy of a meteor of the third magnitude was about that of the incandescent portion of the carbons of an electric arc light, the meteor itself must be 6 or 7 inches (15 or 18 cm) in diameter. This intrinsic brilliancy was considered a fair estimate since either iron or stone would volatilize at a lower temperature than the electric arc. If on the other hand the meteor weighed only one grain, its intrinsic brilliancy must be 4000 times that of the incandescent carbons, or 40 times that of the surface of the sun itself.

Mr. Charles C. Trowbridge in the *Astrophysical Journal*, **26**, 95, 1907, shows that the persistent train, often lasting many minutes, left by a large fireball is usually a mile (1.6 km) or more in diameter, by ten to thirty miles in length. This train can be formed only within certain limits of gaseous pressure, and therefore can only exist at a particular height in the atmosphere. The mean height found is 54 miles, or about the level at which shooting stars disappear. The material of these bodies is therefore mostly exhausted before they reach the atmospheric level, suitable for the production of such a luminous train. If we consider that the average fireball is of magnitude -2, and the average shooting star of magnitude 3, then this implies that the former gives out 100 times the light, and has 1000 times the mass of the latter. If the average shooting star weighs but a few grains, it is difficult for us to believe that even if it were capable of exciting in the air about it an electric glow like that of a fireball, this glow could produce luminosity throughout a volume of one-thousandth part of twenty cubic miles or let us say a volume of air a mile square by 100 feet in depth.

On the other hand assuming the average shooting star to be 6 or 7 inches in diameter, the average fireball must measure 5 or 6 feet, and be sometimes much larger. Bodies of these dimensions sometimes reach the surface of the earth and they may have been very much larger still when at an altitude of thirty to fifty miles.

It is not necessary to suppose, however, that the whole luminous mass is solid. The only photographic spectrum ever secured of a flying meteor, *Harvard Circular*, No. 20, shows that most of the light came from bright lines in the spectrum, which incidentally indicates that the nucleus was not very intensely incandescent. It can be shown mathematically that on account of the great rarity of our atmosphere at these altitudes, the pressure in front of a meteor cannot exceed a few atmospheres. The tremendous quantity of heat generated by the blast of air must be devoted chiefly to vaporizing the front surface of the meteor, which we may conceive to be cut away when at high velocities at a linear rate of two or three feet per second. The temperature would be constantly kept down by the evaporation, so that at ordinary pressures it would be impossible for a high incandescence to exist. In general the phenomenon is so short-lived that conduction of heat can play very little part in the process of destruction.

Harvard College Observatory
March 4, 1909

A FURTHER STUDY OF THE H AND K LINES OF CALCIUM[1]

BY ARTHUR S. KING

The electric furnace[2] in the Pasadena laboratory has been used for additional observations of the calcium spectrum by way of supplementing the writer's former papers on this subject.[3] The new features in the present investigation have been the use of higher dispersion in the blue and violet, giving more definite data concerning the width of the lines, and a comparison of the changes in the spectrum when the furnace is operated in vacuum, in hydrogen atmosphere, and in air at atmospheric pressure.

The vertical Littrow spectrograph was used with the objective of 13 feet (4 m) focal length. The third order then gave a dispersion of 1.37 Ångström units to the millimeter in the blue, permitting a good comparison of the widening of the spectrum lines.

When an atmosphere of hydrogen was used in the furnace chamber, the gas was prepared from zinc and sulphuric acid and carefully purified. The furnace was pumped out to about 5 mm, flushed once or twice with hydrogen, and then filled with the latter to atmospheric pressure. The hydrogen was kept slightly above atmospheric pressure during the operation of the furnace. It was found necessary to use a different jacketing material about the furnace tube when hydrogen was employed, as the carborundum previously used, which gives very efficient heat insulation, appears to trap a certain amount of air when the chamber is pumped out. This air is given off when the carborundum becomes heated and is at once removed if the pump is in connection, as is the case for work in vacuum. When hydrogen was used, however, the residue of air caused small explosions, besides contaminating the gas. The change made was to use two-inch graphite blocks as jacketing material, these being cut to size and placed vertically side by side perpendicular to the furnace tube, a hole in each block allowing space for the tube. This jacketing was successful except that as heat insulation it proved far inferior to carborundum, the heat being carried away so rapidly that the current which usually gave nearly 3000° C. gave barely 2500°. The temperatures were sufficient, however, to compare the effect of hydrogen with the other furnace conditions, runs being made for comparison with the graphite jacketing both with the furnace in vacuum and with air present, though the carborundum insulation was used with the last two arrangements to obtain the higher temperatures.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 38.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 28; *Astrophysical Journal*, **28**, 300, 1908.

[3] *Astrophysical Journal*, **27**, 353, 1908; *Contributions from the Mount Wilson Solar Observatory*, No. 32; *Astrophysical Journal*, **28**, 389, 1908.

The present investigation has verified in all essential points the conclusions arrived at in the previous papers, while the results with the furnace in hydrogen and in air throw interesting lights on the effects observed in the vacuum experiments. The use of hydrogen at atmospheric pressure not only excludes oxidation, which is accomplished well enough by pumping out the chamber, but alters the rate of diffusion of the metallic vapor in the tube and so affects the vapor-density. The loss of heat by convection was found to be very considerable with the hydrogen atmosphere and accounts in large measure for differences observed between these spectra and those obtained in vacuum with the same current through the tube.

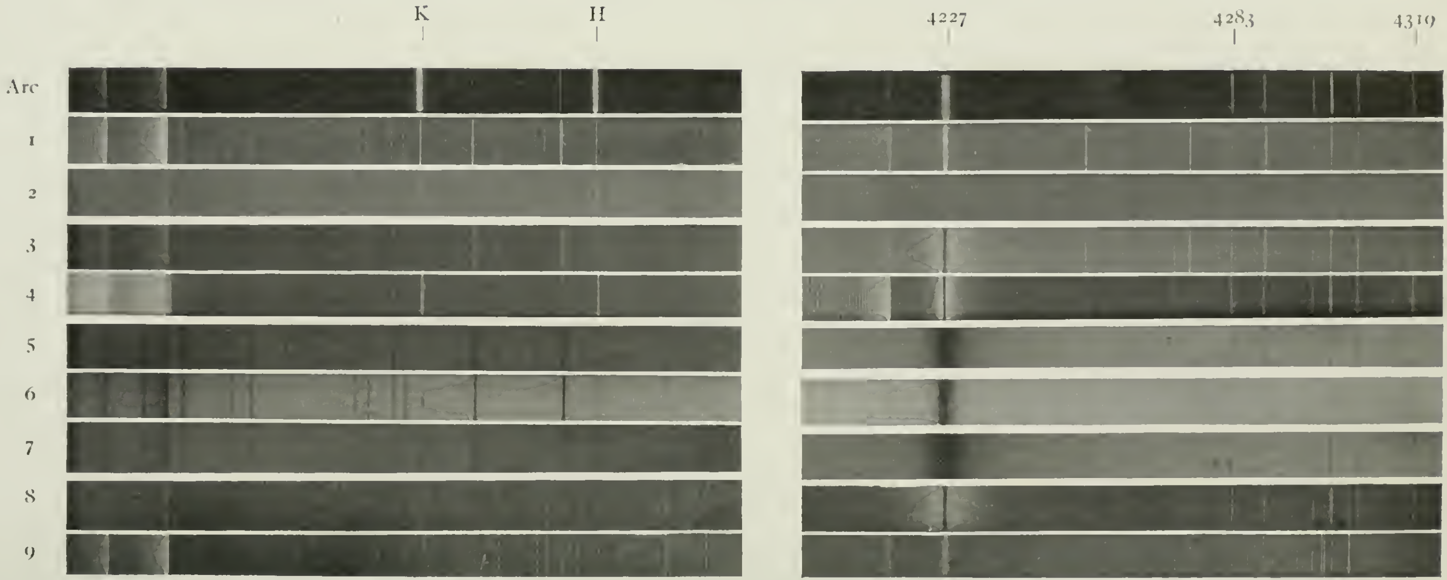
The presence of hydrogen in the furnace is very favorable to the fluted spectrum in the red referred to in papers by Olmsted[1] and the writer[2] and now being measured by the former from large-scale photographs. This was observed during the present work both visually and by photographs made with red-sensitive plates. If these flutings arise from an actual compound of calcium with hydrogen, it would mean that hydrogen introduces a chemical action of its own. The formation of the compound, however, is by no means certain.

To use the furnace in air, one of the outlet pipes was usually left open. As was to be expected, a lively combustion took place in the tube when air was present, although the tube itself was consumed but

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 21; *Astrophysical Journal*, 27, 66, 1908.

[2] *Contributions from the Mount Wilson Observatory*, No. 35; *Astrophysical Journal*, 29, 190, 1909.

PLATE XXII



Electric Furnace Spectra of Calcium, Given by the Furnace in Vacuum, Air, and Hydrogen, with Comparison Arc Spectrum slowly and would last for several runs, there being no active circulation of air. There should have been more convection of heat than when hydrogen was used, but the oxidation more than balanced this loss, and brilliant spectra were obtained. At the higher temperatures the continuous ground soon became strong enough to give all lines in absorption. The assumption in the former papers, that the continuous spectrum from the furnace is produced through the reflection of light from the white-hot walls of the tube by the suspended particles of vapor, appears to be correct; as when air was present and a high current was thrown on the tube, a series of exposures, each lasting a few seconds, showed that the mass of vapor which quickly arose gave a strong continuous spectrum in the earliest stages, while the bands were developing and before any lines appeared. With the furnace tube in vacuum or in hydrogen a large quantity of calcium in the tube as well as a high temperature was required to give the lines in absorption. With the tube in air, the vigorous oxidation gave this condition with a very moderate supply of calcium.

At least two runs of the furnace were made under each set of conditions, to eliminate effects of any irregularities in the action of the apparatus. Toward the end of the experiments a high-reading ammeter was installed, so that for some of the photographs the corresponding currents through the tube were measured. Pyrometer readings were frequently taken by sighting the instrument on as much of the wall of the tube as the small aperture of the furnace window permitted to be seen. These readings should give values close to the correct temperatures.

Plate XXII gives a series of third-order spectra for different temperatures and under the three conditions used during this work. A large portion of the spectrum between H and λ 4227 is omitted from the plate. A number of other photographs were taken, not always favorable for reproduction, which gave good comparisons of special conditions and will be referred to in the discussion.

Furnace in vacuum.—The experiments with the furnace in vacuum were carried far enough to show that the higher dispersion fully confirms the conclusion of the former paper that H and K do not appear much below 2500° C. (perhaps 2400° is a better limit), that they are strengthened by higher temperatures but widen very slowly within the range of temperature given by the furnace; while λ 4227 appears at low temperature and depends for its width almost entirely on the amount of vapor present. Plate XXII gives first an arc spectrum for the identification of the furnace lines. Furnace spectrum No. 1 is at about the limit of visibility of H and K as regards temperature but is made fairly strong through long exposure (30 minutes). The amount of calcium in the tube was small for both this and No. 2. A higher temperature gave No. 2, which was obtained with two minutes' exposure. The continuous ground is rather strong, but both temperature and vapor-density were low enough to prevent the spectrum passing into absorption. The relative strength of H and K in No. 2 compared to the group of arc lines near λ 4300 is to be noted in connection with results under other furnace conditions.

Furnace in air.—The use of air in the furnace chamber gave a very vigorous combustion and, if much calcium was present, a strong continuous spectrum. The image of the interior of the tube appeared brilliant white on the slit when a high temperature was employed. All lines and the cyanogen bands then appeared in absorption. Nos. 3, 4, 5, 6, and 7 of Plate XXII show different conditions of the spectrum with air present. No. 6 is typical of the high temperature condition (2800° to 2900° C.) with strong vaporization. This condition was photographed several times during different runs of the furnace. The width of λ 4227 could be made greater with more calcium present and of course appeared greater if the exposure were so timed as to give a weaker continuous ground. This state is shown in No. 7, taken during another run, the continuous ground being weaker and the reversed lines wider. This was taken at an earlier stage of the run, when the amount of vapor was probably greater than for No. 6, the temperature being about the same (accurate pyrometer measurements were difficult during these short exposures). No. 5 was taken at lower temperature than either Nos. 6 or 7.

Nos. 3 and 4 represent lower temperature conditions with air present, the spectrum then showing bright lines. No. 3 was taken after No. 6 with an exposure twenty times as long as for the latter. The temperature for No. 4 was intermediate between Nos. 3 and 6, measuring about 2600° C. The amount of calcium was small for No. 4, the furnace having been used for a run the day before, and was now heated without renewing the supply of calcium. The strength of the cyanogen bands in No. 4 is doubtless due to the tube being worn thin in parts, as it was now used for the sixth successive run.

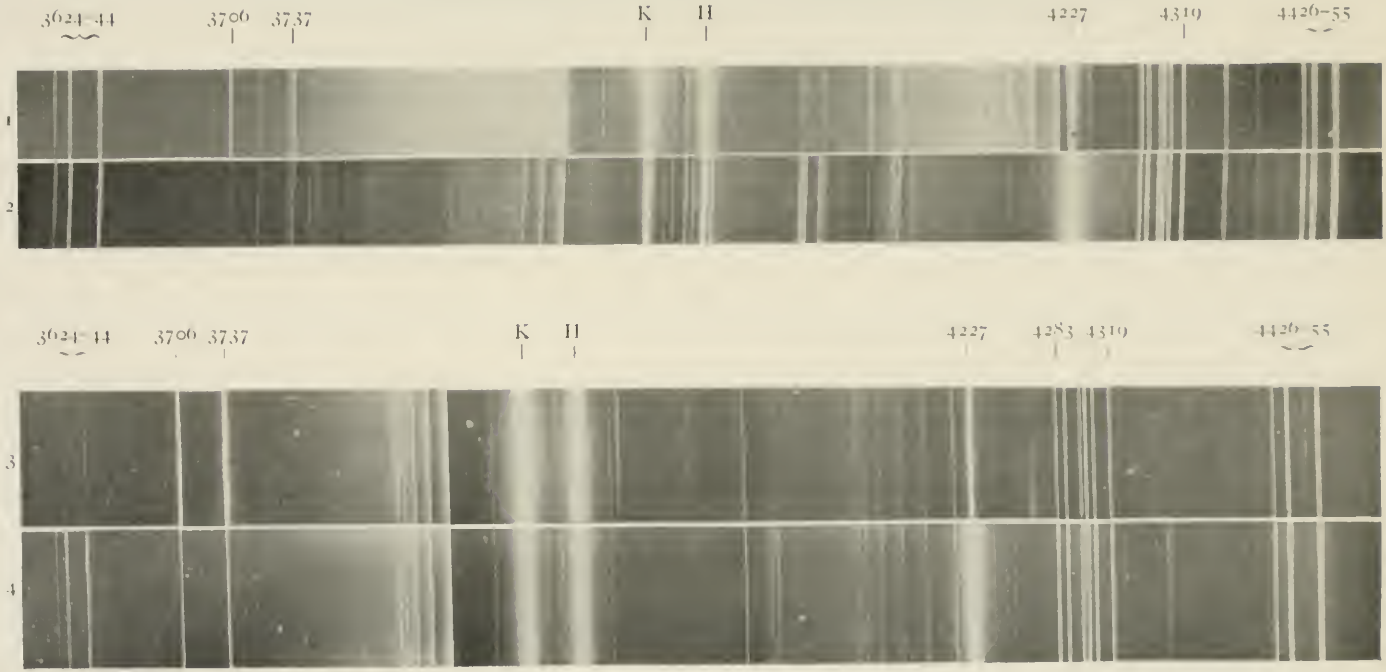
It is evident that H and K can be decidedly strengthened both in emission and absorption by increasing the temperature, especially when assisted by the strong chemical action given when air is present. As to how much they depend upon temperature can best be seen by comparing them with the group of arc lines from λ 4283 to λ 4319. These lines remain fairly narrow and change but slowly with increase of temperature. Taking emission spectra, a comparison of Nos. 3 and 4 shows the K line somewhat weaker than λ 4283 in the low-temperature spectrum, while at high temperature K is decidedly the stronger. The same kind of plate (Seed's "27") was used for all of these photographs. Exactly the same relation is found for the lines in absorption by comparing the low-temperature absorption spectrum No. 5 with either No. 6 or No. 7. It is plain that H and K strengthen much more rapidly with increasing temperature than the average arc line. A comparison of Nos. 1 and 2 shows the same relation for the furnace in vacuum.

Furnace in hydrogen.—The use of hydrogen in the furnace gave results which at first seemed to indicate that the presence of this gas suppressed the H and K lines. They were scarcely visible with the same currents through the tube which gave them very strong in vacuum and in air. Further investigation showed, however, that this effect was in all probability due solely to a lower temperature produced by the convection of the hydrogen at atmospheric pressure, combined with the absence of oxidation. Three runs of the furnace were made with vacuum, hydrogen, and air successively in the chamber, the graphite jacketing being used and all conditions of operation made the same as nearly as possible. Pyrometer measurements showed that for corresponding stages in the different runs the temperatures were nearly the same for vacuum and for air, while for hydrogen they registered 150° to 200° lower. Another test was made by taking a photograph with hydrogen at atmospheric pressure, the temperature being high enough to give H and K distinct though weak. This photograph is reproduced in No. 8 of Plate XXII. The furnace was then pumped out and fresh hydrogen admitted to about 3 cm pressure, after which No. 9 was taken with exposure time and other conditions as nearly the same as possible. H and K show nearly as strong as would be expected in vacuum under the same conditions. The rapid change of H and K with temperature as compared to the arc lines near λ 4300 is very clearly brought out in these two photographs, H and K being very much weaker than the arc lines in No. 8, while in No. 9 they are stronger than the strongest of the λ 4300 group. The effect on λ 4227 of change of vapor-density is very decided in Nos. 8 and 9. The temperature for No. 9 was considerably higher, but the furnace had partially cooled during the changing of the gas and the supply of calcium was doubtless much less than for No. 8. Two other photographs, not reproduced, give the same effect as Nos. 8 and 9 for a smaller temperature interval.

Dependence of λ 4227 on vapor-density.—All of the photographs support the view that the intensity of λ 4227 is governed by the supply of calcium vapor. Visual observations also were very convincing. The spectrum was usually closely watched during the exposures by means of a direct-vision spectroscope of considerable dispersion. At the beginning of a run, with fresh calcium in the tube, λ 4227 was broadly reversed at moderate temperatures. It became narrower as time went on, and after a while the use of a much higher temperature would not widen the line appreciably. The widening of a line of this character refers to the total width, not to the width of the reversal, the latter depending upon the distribution of heated vapor between the middle and ends of the tube.

H and K in the arc and spark.—It may supplement the furnace results to consider the effects upon the H and K lines of conditions which are perhaps the most pronounced in their ability to alter the spectra of the arc and spark respectively. These are the change from the core to the outer layers or "flame" of the arc, and the use of self-induction with the condensed spark. The changes brought about by these influences are well illustrated by Plate XXIII. Nos. 1 and 2 are enlargements of prismatic spectra of the core and flame of the arc which were photographed by Dr. Olmsted in the Mount Wilson laboratory. The spectrograph was a Fuess instrument which gave very bright spectra, permitting short exposures, one-half second being used for the core and 15 seconds for the flame. A num-

PLATE XXIII



The Spectrum of Calcium: (1) and (2), Core and Flame of Arc, (3) and (4), Spark with Capacity and with Self-Induction

ber of typical arc lines, such as the 4300 group and the series triplets at λ 3624–44 and λ 4426–55, are of nearly the same intensity in the two spectra. H and K are very much stronger in the spectrum from the core of the arc.

Nos. 3 and 4 of Plate XXIII are enlarged from a portion of the spark spectrum of calcium photographed by the writer with a one-meter concave grating in the University of Bonn and discussed in a paper[1] published at that time. No. 3 was taken with the strongly condensed spark and No. 4 with self-induction in circuit. The relative strengthening of H and K as compared to the arc lines by the stronger discharge conditions is very decided. The enhancement of the pair of "spark lines" at λλ 3706, 3737 is still greater than for H and K.

Discussion.—A line of reasoning which can perhaps be developed into a working hypothesis seems to follow from the fact distinctly brought out by the furnace experiments, that the H and K lines show a very definite behavior when compared with representative arc lines, such as the group near λ 4300 and the series triplet at λ 4426–55 (shown on the plate with the previous paper[2]). These arc lines are of fair strength at a temperature just high enough for H and K to become visible. As the temperature rises, H and K increase in intensity (though without decided widening) much faster than do the arc lines, surpassing at the higher furnace temperatures the strongest lines of the λ 4300 group (see No. 4 of Plate XXII). If this rate of increase is maintained at higher temperatures, the strength of H and K in the arc may easily follow. Although, as was remarked in the former paper, the difference between their intensities in the arc and furnace seems very large to be caused by the four or five hundred degrees' difference in the measured temperatures of the furnace and the positive terminal of the arc, we do not know even approximately the temperature which would correspond to the strength of the energizing influence in the arc which gives the light-vibration. To account on this hypothesis for the strong broadening of H and K when a large amount of calcium is used in the arc, we should have to assume that this increases the combined electrical and chemical action of the arc in a way that corresponds to higher temperature. The rapid increase of H and K when the furnace is operated in air shows to some extent how this may come about. It is then unnecessary to assume a dependence of H and K upon vapor-density, which in the furnace does not seem to hold. It is to be noted that the widening in the arc is a rapid shading-off from a strong central maximum, the intensity-curves for these lines being very different from that for λ 4227. In both arc and spark the spectra in Plate XXIII show that the relative intensity of H and K increases rapidly with the strength of the discharge conditions and thus with the "electrical temperature."

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[1] *Astrophysical Journal*, **19**, 225, 1904.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 32; *Astrophysical Journal*, **28**, 389, 1908.

H and K have long been regarded as having to some extent the character of "spark lines," but the present work helps to bring out their relation to such typical "spark lines" as λ 4481 of magnesium, λλ 5339, 5379 of cadmium, and λλ 4912, 4925 of zinc. The H and K lines show the same ready response as these lines to changing conditions in the source, but *the range of temperature for their production extends much lower.*

It seems reasonable to suppose that temperature, if forced sufficiently high, could supply the energy to produce those conditions of the spectrum which can be given, in terrestrial sources, only by the electric discharge. It may then be possible to select a list of lines whose strength, relative to other lines, will serve in some measure to indicate the temperature corresponding to the radiating energy of any light-source, which often, as in the flame and electrical sources, would not agree with any thermal measurements to be made with our present instruments. H and K would have their place in such a list as being lines which first appear when the average arc lines have reached a considerable intensity and are changing slowly with increased temperature. Having once appeared, H and K are apparently very sensitive through a long range of temperature and are reinforced for this purpose later by the group of typical "spark lines," some of which require less energy of discharge for their production than others.

A comparison with solar phenomena is of interest in this connection. If the sensitiveness of H and K to increased temperature continues at the solar temperatures, this would seem to be responsible in large measure for the great width of these lines in the spectrum of the body of the sun, their weaker condition in sun-spots, and their appearance at high levels in prominences, where the rarity of the vapor should not greatly affect their intensity if the temperature is high.

Passing to stellar spectra, it may be said in general that H and K are strong in the spectra of those stars which show strong "spark lines." These are the hotter stars if the "spark lines" furnish a valid basis of temperature classification. If their increase in intensity in some cases does not seem to be the same as for other "enhanced lines," the possibility remains that H and K may lose their sensitiveness to some extent at extremely high temperatures, in the same way that we have seen arc lines lose their ready response much lower down.

Mount Wilson Solar Observatory
April 1909

NOTICE OF INCREASE IN SUBSCRIPTION PRICE

With the thirtieth volume of the *Astrophysical Journal*, beginning with the July number, 1909, the annual subscription price will be increased by one dollar, thus becoming five dollars, or $2.50 per volume of five numbers. This action becomes necessary as the result of the increasing cost of production, particularly in respect to printing and engraving. The editors have reluctantly yielded to the necessity of this change, although, in view of the number of pages per year and of the quantity and quality of the engravings, the *Journal* has been relatively much cheaper than its contemporaries foreign or domestic. An increase cannot be asked in the subsidy granted by the University of Chicago, amounting to about 60 per cent. of the cost of publication. The editors believe that the subscribers will prefer to have the subscription price raised rather than to have the size of the *Journal* reduced or the average number and quality of the illustrations diminished.

Particulars as to the price of foreign subscriptions and of single numbers will be found in the publishers' notice elsewhere in this number.

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